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Trajectory planning for multi-robot coordinated towing system based on stability①

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Abstract

Given the unconstrained characteristics of the multi-robot coordinated towing system, the rope can only provide a unidirectional constraint force to the suspended object, which leads to the weak ability of the system to resist external disturbances and makes it difficult to control the trajectory of the suspended object. Based on the kinematics and statics of the multi-robot coordinated towing system with fixed base, the dynamic model of the system is established by using the Newton-Euler equations and the Udwadia-Kalaba equations. To plan the trajectories with high stability and strong control, trajectory planning is performed by combining the dynamics and stability of the towing system. Based on the dynamic stability of the motion trajectory of the suspended object, the stability of the suspended object is effectively improved through online real-time planning and offline manual adjustment. The effectiveness of the proposed method is verified by comparing the motion stability of the suspended object before and after planning. The results provide a foundation for the motion planning and coordinated control of the towing system.

Key words: towing system, unconstrained system, trajectory planning, dynamic stability

0 Introduction

The multi-robot coordinated towing system with fixed base is composed of multiple robots with fixed base and rope-driven parallel system, because it contains the rope-driven parallel mechanism's strong flexibility and large workspace advantages, but also has the towing system's bearing capacity and modular design characteristics, so it has a wide range of applications in military, aerospace and industrial production fields[1-2].

The multi-robot coordinated towing system is a rigid-flexible coupled unconstrained system. Due to the flexibility of the rope and the limitation of the system on the suspended object, the dynamic response in the coordinated towing operation is not controllable to a certain extent, which will become the research difficulty in system dynamics modeling and trajectory planning. Liu et al.[3] studied the minimum rope tension distribution of a rope-driven parallel robot in the workspace, but the influence of the minimum tension distribution on the stability of the moving platform is not discussed. Wang et al.[4] analyzed the kinematics of the rope parallel mechanism, the workspace is established by particle swarm optimization algorithm, and the robust control of the mechanism is discussed. Refs [5,6] established the static balance workspace of a multi-robot coordinated towing system, and the stability judgment method of force-position-pose mixing is proposed, but the freedom of the suspended object is not analyzed. Hong et al.[7] analyzed the stiffness of the unconstrained rope-driven mechanism, and the mechanism optimization design is carried out based on the stiffness analysis. Refs [8,9] discussed the spatial structure of the three-degree of freedom (DOF) rope-driven parallel mechanism, and its stability is evaluated by using the motion sensitivity of the position and pose of the moving platform, but the degree of freedom of the unconstrained system is not discussed. Adel and Olivier[10] used the cubic spline curve interpolation to control the trajectory of the robot based on the Euler equation. Mei et al.[11] selected the B-spline curve for trajectory planning of the robot, and the results show that this method can reduce the vibration of the robot. Gasparett and Zanotto[12] discussed the trajectory plan-

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ning of the robot based on the integral of acceleration squared, which improves the motion stability of the robot. Refs [13,14] established the dynamics of a multi-helicopter transport system based on the Udwadia-Kalaba(UK) equation, and the stability of the system is analyzed by using a sliding mode controller. Zhang et al. [15] established the dynamic equation of the rigid manipulator based on the UK equation, and the trajectory tracking control of the manipulator is carried out. Lan and Lai [16] used the self-disturbance control for the anti-pendulum control of a single unmaned aerial vehicle (UAV) hanging system, so that the motion of the suspended object remains stable.

Due to the low rigidity of the rope, it can only provide a unidirectional constraint force on the suspended object. During dynamic towing, the towing system has a weak ability to resist external interference, which makes it difficult to control the system in terms of dynamic trajectory. The real-time dynamic stability of the motion trajectory is treated on a judgment basis based on the instability deviation between the actual trajectory and the expected trajectory of the suspended object, and the trajectory planning of the suspended object is performed by using online real-time planning and offline manual adjustment. By comparing the trajectories before and after the planning, it is verified that this method can improve the dynamic stability of the suspended object. Therefore, as a new research direction, the system not only has high engineering application value, but also has practical significance for the related theoretical research of rope traction parallel mechanism.

The outline of the paper is as follows. In Section 1, the configuration of the multi-robot coordinated towing system with fixed-base is introduced. In Section 2, the dynamic model of the towing system is established by using the UK equations. In Section 3, the motion stability of the towing system is analyzed, which provide a reference for the trajectory planning of the towing system. In Section 4, through online real-time planning and offline manual adjustment, the planned trajectory of the towing system is simulated. In the last section, some remarkable conclusions are given.

1 Structure of the towing system

The multi-robot coordinated towing system with fixed-base consists of multiple robots with fixed bases and the rope-driven parallel towing system. Fig. 1 shows the spatial configuration of the proposed towing system.

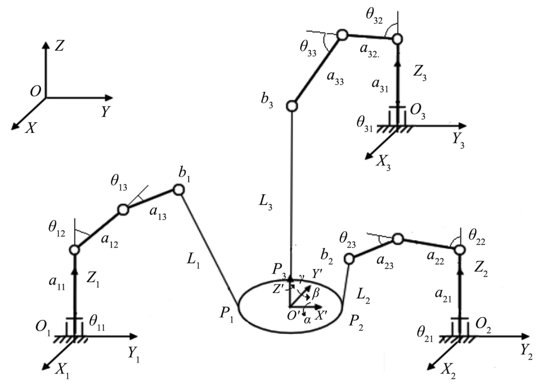


Fig. 1 The structure of the towing system

The coordinate system O-XYZ is established on the horizontal plane, the coordinate system O_i-$X_iY_iZ_i$ is established at the bottom of the robot, and the coordinate system O'-$X'Y'Z'$ is established at the center of the suspended object. In the coordinate system O-XYZ, the connection point between the robot end and the rope is denoted as $\boldsymbol{b}_i$, the connection point between the rope and the suspended object is denoted as $\boldsymbol{P}_i$, the position vector of the rope is denoted as $\boldsymbol{L}_i$, and the length of the rope is denoted as l_i. The rod length of the robot is (a_{i1}, a_{i2}, a_{i3}) and the joint angle is $(\theta_{i1}, \theta_{i2}, \theta_{i3})$. Since the whole system is composed of three towing robots, so $i = 1,2,3$. The position and pose of the suspended object are denoted as $\boldsymbol{r}(x, y, z, \alpha, \beta, \gamma)$, the position and pose of the suspended object is controlled by adjusting the joint angle of the robot and the length of the rope.

Due to the complex structure of the towing system, the following assumptions are made on the towing system without affecting its analytical results.

(1) Regardless of the radius of the end of the robot, the connection between the end of the robot and the rope is regarded as a point-like hinge.

(2) The connection point between the rope and the suspended object and the barycenter of the suspended object is in the same plane.

(3) Regardless of the weight of the rope itself, the rope will not deform when it is stressed.

2 Dynamic analysis

The UK equation is a nonlinear differential equation widely used in robotics and control engineering, and it can be used to establish the dynamic equation. It applies to a wide class of constraints, whether constrained system or unconstrained system.

The multi-robot coordinated towing system is an unconstrained system with rigid and soft coupling, the suspended object is subject to the driven force of the rope and its gravity, where the driven force of the rope is the constraint force and its gravity is the active power. The towing system is divided into the constrained system and the unconstrained system by the D′Alembert principle. Firstly, without considering the constraint forces provided by the ropes, the dynamic model of the unconstrained system in the UK equation is established by the Newton-Euler method, and then the dynamic model of the constrained system is established by the UK equation[17].

The end of the robot and the suspended object are regarded as unconstrained rigid bodies, the displacement of the robot end, the position and pose of the suspended object are regarded as time variables, and the generalized coordinates of the towing system that are unconstrained by external forces are defined as $\boldsymbol{q}_u$.

$$\boldsymbol{q}_u = [\boldsymbol{\lambda}_{b_i} \quad \boldsymbol{\tau} \quad \boldsymbol{\eta}]^{\mathrm{T}} \tag{1}$$

where, $\boldsymbol{\lambda}_{b_i} = [x_{b_i} \quad y_{b_i} \quad z_{b_i}]^{\mathrm{T}}$ is the displacement of the endpoint of the robot, $\boldsymbol{\tau} = [x \quad y \quad z]^{\mathrm{T}}$ is the position of the suspended object, and $\boldsymbol{\eta} = [\alpha \quad \beta \quad \gamma]^{\mathrm{T}}$ is the pose of the suspended object.

The constraint force provided by the rope is not considered at first, the dynamic equations of the towing system in the UK equation is established by using the Newton-Euler method.

$$\boldsymbol{M}\ddot{\boldsymbol{q}}_u = \boldsymbol{Q} \tag{2}$$

where, $\boldsymbol{M} = [\boldsymbol{M}_{b_i} \quad \boldsymbol{M}_o \quad \boldsymbol{I}_o]$ is the mass matrix of the towing system, $\boldsymbol{M}_o$ and $\boldsymbol{M}_{b_i}$ are the mass matrix of the suspended object and the mass matrix of the end of the robot, respectively, $\boldsymbol{I}_o$ is the inertia matrix of the suspended object, and $\boldsymbol{Q}$ is the active power matrix of the towing system.

Depending on the kinematics of the towing system, the position vector of the rope in the coordinate system O-XYZ is

$$\boldsymbol{L}_i = \boldsymbol{b}_i - \boldsymbol{P}_i = \boldsymbol{b}_i - \boldsymbol{R}\boldsymbol{P}'_i - r \tag{3}$$

where, $\boldsymbol{P}'_i$ is the position of the connection point between the rope and the suspended object in the coordinate system O'-$X'Y'Z'$, and $\boldsymbol{R}$ is the transformation matrix of the coordinate system O'-$X'Y'Z'$ with respect to the coordinate system O-XYZ.

$$\boldsymbol{R} = \boldsymbol{R}_z(\varphi_3)\boldsymbol{R}_y(\varphi_2)\boldsymbol{R}_x(\varphi_1)$$

$$\begin{bmatrix} \mathrm{c}\varphi_1\mathrm{c}\varphi_2 & \mathrm{c}\varphi_1\mathrm{s}\varphi_2\mathrm{s}\varphi_3 - \mathrm{s}\varphi_1\mathrm{c}\varphi_3 & \mathrm{c}\varphi_1\mathrm{s}\varphi_2\mathrm{c}\varphi_3 + \mathrm{s}\varphi_1\mathrm{s}\varphi_3 \\ \mathrm{s}\varphi_1\mathrm{c}\varphi_2 & \mathrm{s}\varphi_1\mathrm{s}\varphi_2\mathrm{s}\varphi_3 + \mathrm{c}\varphi_1\mathrm{c}\varphi_3 & \mathrm{s}\varphi_1\mathrm{s}\varphi_2\mathrm{c}\varphi_3 - \mathrm{c}\varphi_1\mathrm{s}\varphi_3 \\ -\mathrm{s}\varphi_2 & \mathrm{c}\varphi_2\mathrm{s}\varphi_3 & \mathrm{c}\varphi_2\mathrm{c}\varphi_3 \end{bmatrix}$$

where, s represents sin, c represents cos.

Assuming that the length of the rope is constant in the multi-robot coordinated towing system, the constraint equation of the system is

$$g_i(\boldsymbol{q}_u, t) = \| \boldsymbol{L}_i \|^2 - l_i^2 = (\boldsymbol{L}_i^{\mathrm{T}}\boldsymbol{L}_i) - l_i^2 \tag{4}$$

The second derivative of the constraint equation is obtained as follows.

$$2\ddot{\boldsymbol{L}}_i^{\mathrm{T}}\boldsymbol{L}_i + 2\dot{\boldsymbol{L}}_i^{\mathrm{T}}\dot{\boldsymbol{L}}_i = 0 \tag{5}$$

The transformation matrix is used to obtain the derivative relation, and the $\dot{\boldsymbol{L}}_i$ and $\ddot{\boldsymbol{L}}_i$ is obtained respectively from Eq. (3), and then substituted into Eq. (5) to obtain the constraint equation. After the classification, the constraint equation is converted to the standard form:

$$\boldsymbol{J}(\boldsymbol{q}_u, \dot{\boldsymbol{q}}_u, t)\ddot{\boldsymbol{q}}_u = \boldsymbol{c}(\boldsymbol{q}_u, \dot{\boldsymbol{q}}_u, t) \tag{6}$$

where, $\boldsymbol{J}$ is the Jacobian matrix of the constraint equation, and $\boldsymbol{c}$ is the polynomial that does not include the acceleration term in the constraint equation.

Based on the UK equation, the dynamic equation of the towing system in the fully constrained state is

$$\boldsymbol{M}\ddot{\boldsymbol{q}} = \boldsymbol{Q} + \boldsymbol{Q}_c \tag{7}$$

where, $\boldsymbol{Q}_c$ is the constraint force generated by the external force on the system, namely the tension of the rope.

$$\boldsymbol{Q}_c = \boldsymbol{M}^{1/2}(\boldsymbol{J}\boldsymbol{M}^{-1/2})^{-}(\boldsymbol{c} - \boldsymbol{J}\ddot{\boldsymbol{q}}_u) \tag{8}$$

where, q is the generalized coordinate of the system under the constrained state, and

$$\ddot{\boldsymbol{q}} = \ddot{\boldsymbol{q}}_u + \boldsymbol{M}^{-1/2}(\boldsymbol{J}\boldsymbol{M}^{-1/2})^{-}(\boldsymbol{c} - \boldsymbol{J}\ddot{\boldsymbol{q}}_u) \tag{9}$$

When the towing system is in a constrained state, Eq. (8) is used to calculate the rope tension, and Eq. (9) is used to calculate the acceleration of the suspended object.

Since the rope can only provide a unidirectional constraint on the suspended object, which increases the control difficulty of the multi-robot coordinated towing system, it is a prerequisite for analyzing the trajectory planning of the towing system to establish the correct dynamic model of the system. The dynamic model of the towing system can be established by using UK equation, which can solve the movement trajectory of the towing system.

3 Trajectory planning

3.1 Stability analysis

In the study of the rope-driven parallel system, the stability analysis is mainly concerned with the ability of the system to resist external forces under static conditions, or with the ability of the control system to recover stability after external disturbances. However, these two stability analysis methods do not provide help for trajectory planning, so it is necessary to analyze the motion stability of the towing system, and provide a reference for the trajectory planning of the towing system based on the judgment.

The multi-robot coordinated towing system is unconstrained, when the number of ropes is 3, only 3-DOF of the suspended object can be controlled. At this time, the motion trajectory of the suspended object will fluctuate in each degree of freedom, such as swinging in position freedom and overturning in the pose freedom. To reduce the impact of such fluctuations on the towing accuracy, the quantified value of fluctuations is taken as the basis to plan the movement of the endpoint of the robot and the length of the rope, so that the actual motion trajectory of the suspended object can track the desired trajectory and realize the control of the suspended object.

The fluctuations in the actual trajectory of the suspended object in the towing system are regarded as the performance of dynamic instability, while the expected trajectory is regarded as the performance of stability. Therefore, the concept of dynamic stability is proposed. At any moment, the absolute value Δ of the difference between the actual motion trajectory and the expected trajectory of the suspended object is taken as the instability deviation, and the maximum theoretically generated instability deviation is defined as Δ_{max}, then the dynamic stability of the suspended object is

$$\sigma = 1 - \frac{\Delta}{\Delta_{max}} \tag{10}$$

It can be seen that the stability of the suspended object is the set of stability at all times. The dynamic stability varies with the movement of the suspended object, and the stability may be different at different times. Dynamic stability is used as a stability index to judge the stability of the motion trajectory at every moment, and the results can provide a basis for trajectory planning of suspended object.

3.2 Trajectory planning based on dynamic stability

In actual working conditions, the towing task has corresponding requirements for the movement of the suspended object on each degree of freedom, and the desired movement of the suspended object is achieved by planning the position of the endpoint of the robot and the length of the rope, that is, trajectory planning.

In trajectory planning for the towing system, the planned trajectory is not unique to achieve a given desired motion. In the process of dynamic solution of the towing system, the calculation result of the N cycle is regarded as the initial value of the $N + 1$ cycle for iterative calculation, which makes the system can only conduct real-time adjustment of the current cycle according to the dynamic stability of the previous cycle. As a result, the system is unable to respond in time, making the trajectory of the suspended object deviate from the expected trajectory. To plan the trajectories with high stability and strong control, combined with the dynamics and stability of the towing system, a real-time trajectory planning method is proposed, the specific procedure of which is shown in Fig. 2.

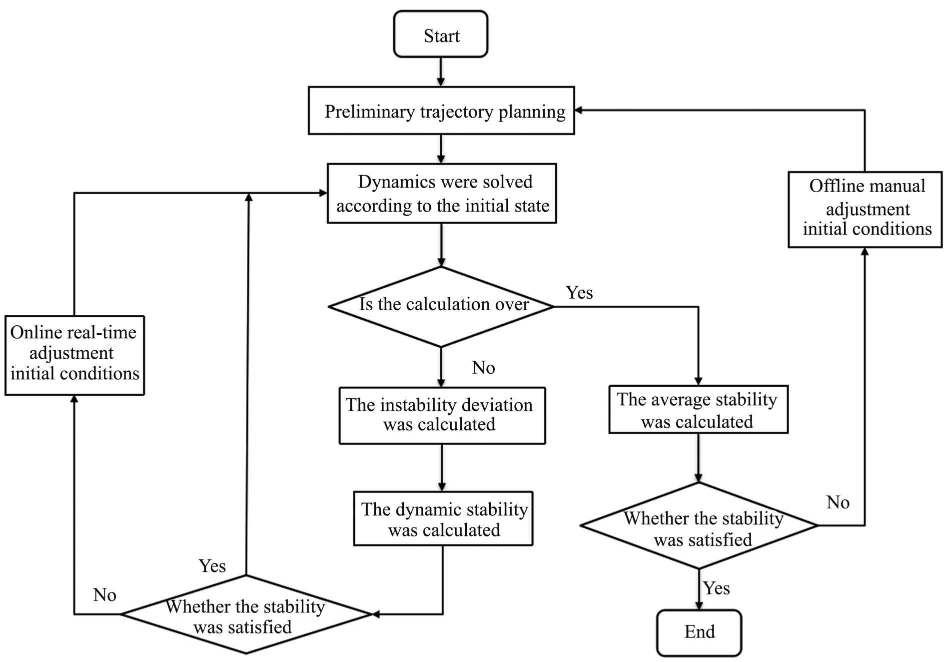


Fig. 2 Flowchart of the trajectory planning

First, the expected trajectory of the suspended object is determined based on the given towing task, the instability deviation Δ is calculated, and then the dynamic stability σ is calculated. The dynamic stability is compared with the set stability margin. If the dynamic stability σ is less than the stability margin, the instability deviation Δ of the suspended object is forced to zero. If the dynamic stability is greater than the stability margin, the dynamic solution of the next cycle is directly entered. When the process of dynamic solution is completed, the average stability of the whole trajectory is calculated. If the average stability of the whole trajectory does not meet the requirements, the initial length of the rope and the end position of the robot are adjusted, after which the instability deviation is calculated again. If the average stability of the whole trajectory meets the requirements, then the trajectory planning of the suspended object is finished.

4 Simulation analysis

To verify the effectiveness of the aforementioned trajectory planning method, numerical simulation analysis was performed on the towing system. The three robots are arranged in an equilateral triangle in space, and the ends of the three robots remain at the same height. The parameters of the towing system are shown in Table 1. Assuming that the three connecting points between the suspended object and the ropes form a positive triangle, the distance between each connection point and the center of the suspended object is 0.5 m, and the pose of the suspended object is (0,0,0). The projection of the towing system in its initial state is shown in Fig. 3.

Table 1 Parameters of the towing system

Parameter	Value
Size of link 1 /m	5
Size of link 2 /m	4
Size of link 3 /m	3
Range of joint angles	[0, π/2]
Mass of the suspended object / kg	231.156
X-axis inertia of the suspended object / (kg · m^2)	14.478
Y-axis inertia of the suspended object / (kg · m^2)	14.478
Z-axis inertia of the suspended object / (kg · m^2)	28.895
Length of the rope /m	5
Number of the cycles	60 000
Stability margin	0.96

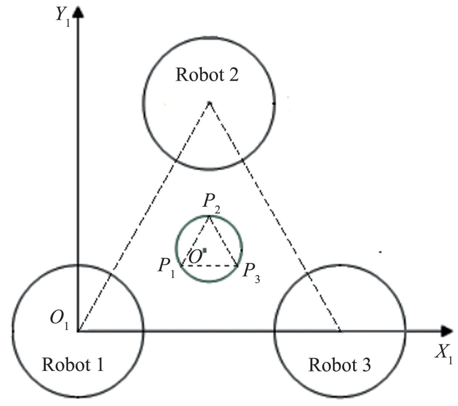


Fig. 3 Projection of the towing system on the *XOY* plane

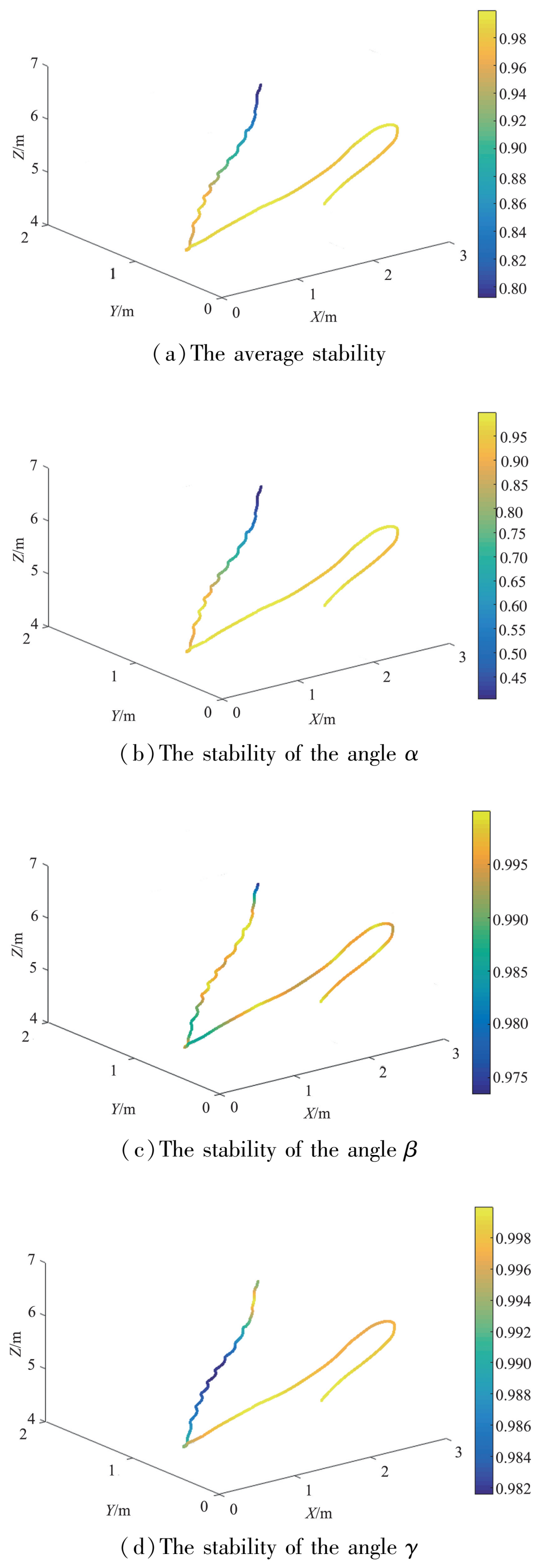
The desired trajectory of the suspended object is

$$\begin{cases} x = 1.5 + \sin(11.5t) \\ y = 0.075\sqrt{3} + \dfrac{2}{300}t^{1.5} \\ z = 10 - 1.5\sqrt{3} + \sin(11.5t) \end{cases} \tag{11}$$

4.1 Online real-time planning

First, the trajectory at the end of the robot is planned online based on real-time dynamic stability. The instability deviation is the real-time pose angle of the suspended object. When the dynamic stability does not meet the requirements, the system will take all pose angles as zero, the suspended object and the current position of the robot end as the input, and conduct real-time adjustment on the length of the rope according to the calculated results, so as to achieve the purpose of increasing the stability. The motion trajectories of the suspended object obtained by the trajectory planning are compared with the trajectories obtained from the direct solution of the dynamics. The change in stability with the motion trajectory is shown in Figs 4 and 5 before and after trajectory planning. The different colors in the figures represent different degrees of stability, and the colors on the right side of the figures from warm to cool represent the degree of stability from large to small. Fig. 4 (a) represents the change of the average stability with the motion trajectory of the suspended object; Fig. 4 (b), (c) and (d) represent the change of the stability of the pose angle α, β and γ with the motion trajectory, respectively.

Figs 4 and 5 show that the stability of the suspended object decreases with the motion of the suspended object, but the stability of the after trajectory planning decreases relatively slowly, and the smoothness is also improved. Moreover, the average stability fluctuates around the stability margin of 0.96 after trajectory planning, which proves that the motion of the suspended object is controllable. It can be seen from Fig. 5 that



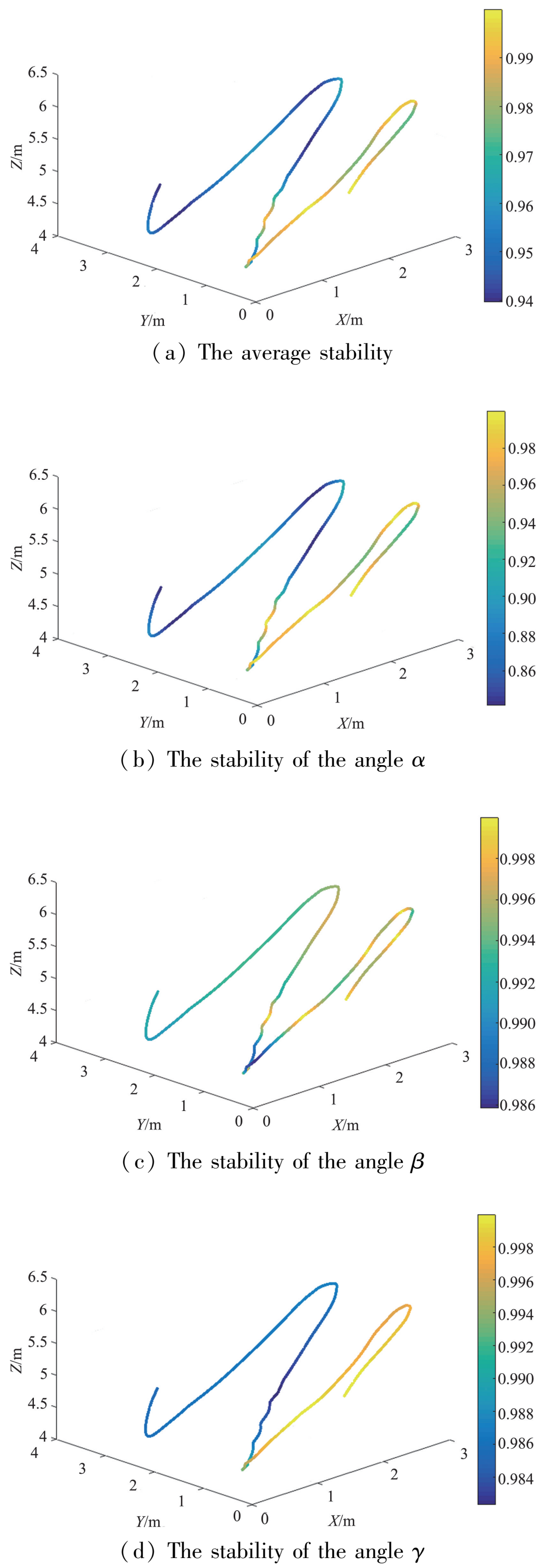
(a) The average stability

(b) The stability of the angle α

(c) The stability of the angle β

(d) The stability of the angle γ

Fig. 4 Stability of the suspended object with the motion trajectory before trajectory planning



(a) The average stability

(b) The stability of the angle α

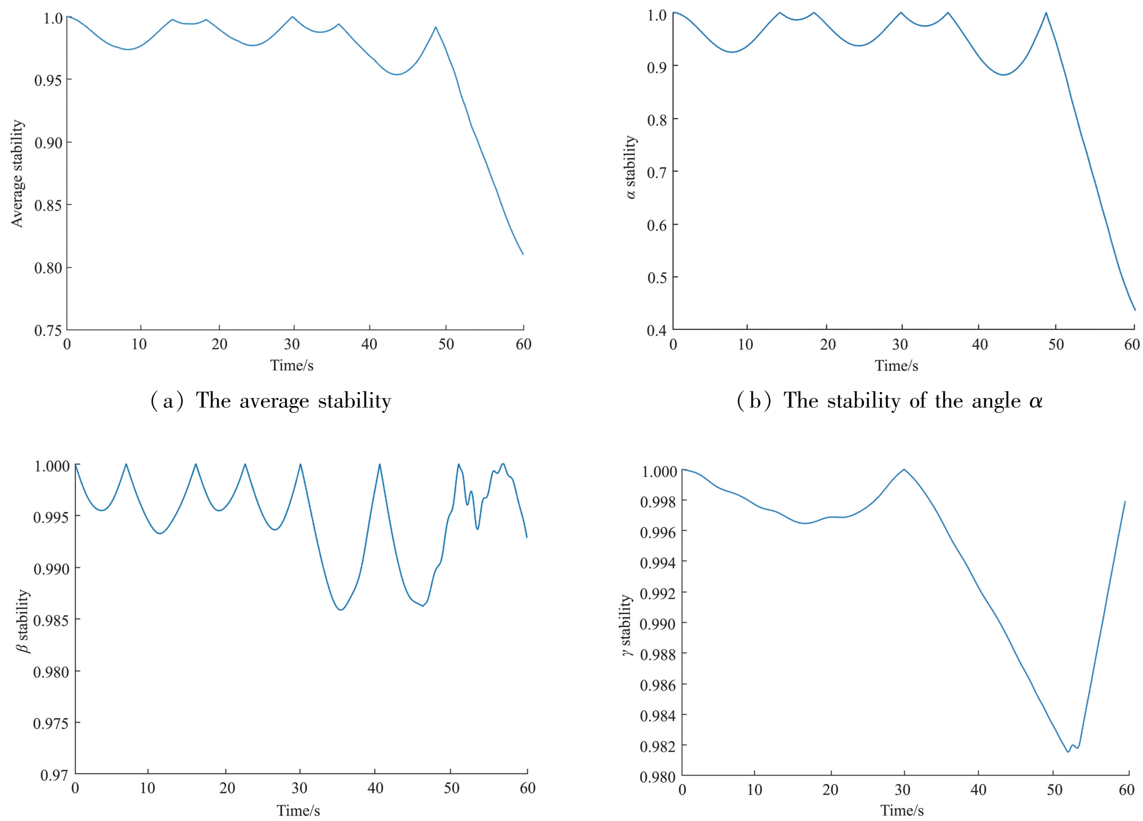
(c) The stability of the angle β

(d) The stability of the angle γ

Fig. 5 Stability of the suspended object with the motion trajectory after trajectory planning

the stability of the after trajectory planning has a 'trough period' with low stability, while there is always a 'stable period' with high stability between the two adjacent 'trough periods'. As the trajectory goes further and further, the average stability of the 'stable period' decreases continuously.

Figs 6 and 7 show the changes in pose stability of the suspended object over time before and after trajectory planning. It can be seen from the figures that the stability of the pose after planning increases significantly compared with before planning, especially after 30 s, the average stability before planning drops sharply, while the average stability after planning fluctuates around 0.94.



(a) The average stability (b) The stability of the angle α

(c) The stability of the angle β (d) The stability of the angle γ

Fig. 6 Stability of the suspended object over time before trajectory planning

It can be seen from Figs 6 and 7 that the average stability and the stability of pose angle α have very similar changing trends, which indicates that pose angle α plays a leading role in the influence of stability. Combined with the initial conditions of the simulation, the acceleration of the robot end in the Y-axis direction is much greater than that in other directions, and the change of pose angle α is caused by the swing of the suspended object in the Y-axis direction. It can be inferred that the greater the acceleration of the robot in a certain direction, the greater the swing of the suspended object in the corresponding direction.

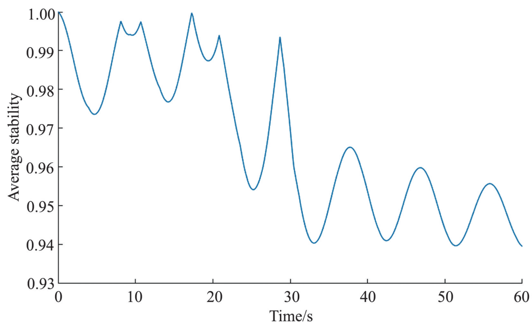
4.2 Offline manual adjustment

During the planning stage of the offline manual adjustment, the length of the rope was manually adjusted to 3 m, and the other initial parameters were kept unchanged. The stability of the suspended object with the actual trajectory and the expected trajectory changes as shown in Fig. 8, Fig. 8 (a) represents the change of average stability with the trajectory of the suspended object, and Fig. 8 (b), (c) and (d) are three-dimensional projection maps, respectively. The very smooth curves in the figures are the expected trajectory calculated when the length of the rope is 3 m.

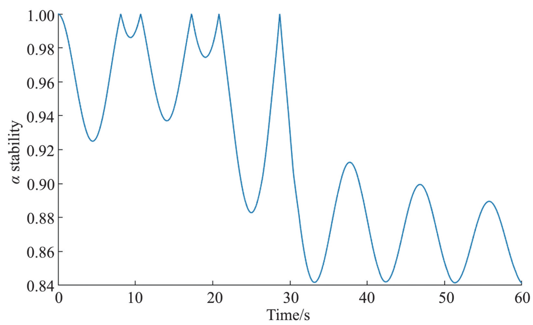
When the length of the rope is 3 m and 5 m respectively, the maximum deviations between the actual trajectory and the expected trajectory of the suspended object on each axis are shown in Table 2.

The maximum instability deviation Δ_{max} can be calculated for each trajectory, and it can be seen from Table 2 that the maximum deviation between the actual trajectory of the suspended object and the expected trajectory at l_i = 3 m is larger than at l_i = 5 m, indicating that the motion of the suspended object at l_i = 3 m is more stable from the definition of dynamic stability.

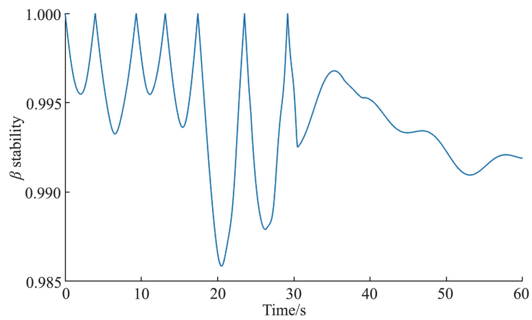
The existing studies only consider the trajectory planning under dynamic analysis, but do not consider



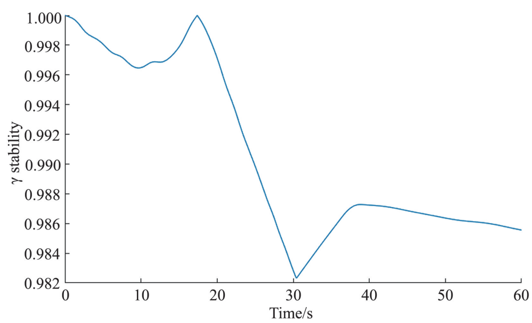
(a) The average stability



(b) The stability of the angle α

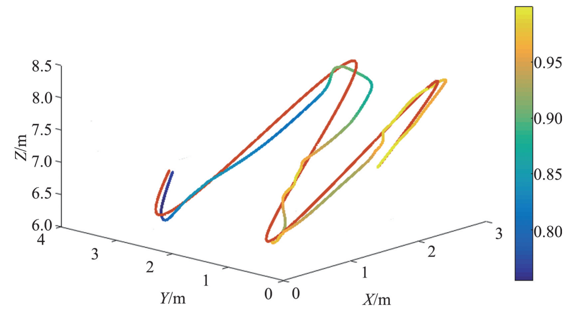


(c) The stability of the angle β

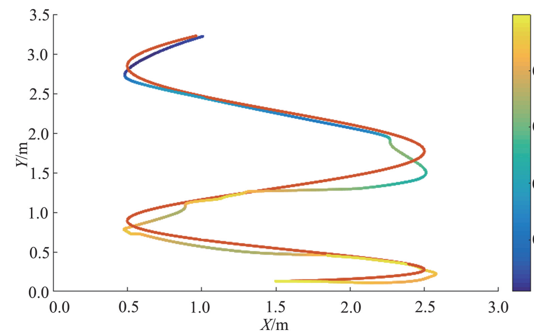


(d) The stability of the angle γ

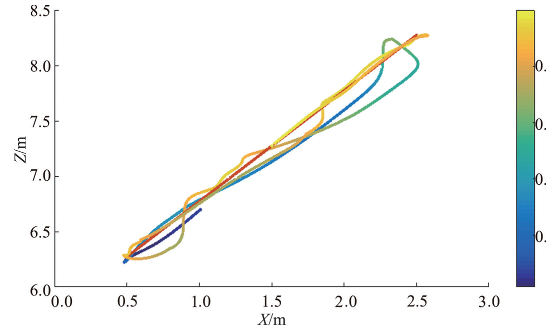
Fig. 7 Stability of the suspended object over time after trajectory planning



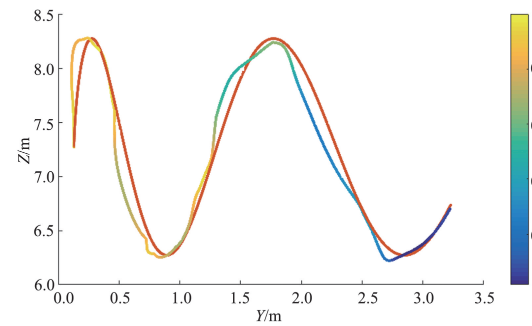
(a) The average stability



(b) The average stability on the *XOY* plane



(c) The average stability on the *XOZ* plane



(d) The average stability on the *YOZ* plane

Fig. 8 Stability of the suspended object before and after trajectory planning at $l_i = 3$ m

the influence of the stability of the rope traction parallel system on the movement trajectory. In this paper, the concept of dynamic stability of movement trajectory is proposed by using the instability offset and maximum instability offset between the actual trajectory and the expected trajectory. Based on the dynamic stability of the movement trajectory, the movement trajectory whose dynamic stability is less than the stability margin is adjusted in real time. Based on the average dynamic stability of the whole trajectory, the trajectory that does not meet the requirements is artificially adjusted to improve the stability of the movement trajectory. The research results are helpful for the multi-robot system to complete all kinds of towing tasks safely, and expand the related planning and control theory.

Table 2 Maximum deviation of each axis

Parameter	l_i = 3 m	l_i = 5 m
Positive deviation in X-axis /m	0.2967	0.0791
Negative deviation in X-axis /m	0.2020	0.1225
Positive deviation in Y-axis /m	0.1087	0.0085
Negative deviation in Y-axis /m	1.1068	0.9870
Positive deviation in Z-axis /m	0.0588	0.0537
Negative deviation in Z-axis /m	0.0925	0.0138

5 Conclusions

Based on the UK equation, the dynamic model of the multi-robot coordinated towing system with fixed base is established. Combined with the dynamics and stability of the towing system, the trajectory planning method for the suspended object is proposed based on the real-time dynamic stability of the motion trajectory. When the dynamic stability is less than the stability margin, the method of forcing back to zero is used to improve the stability of the next point. Then, based on the average stability of the entire trajectory, the offline manual adjustment is performed on the trajectory that does not meet the requirements. The effectiveness of this planning method is verified by comparing the motion trajectories of the suspended object before and after the planning, and the results provide a basis for subsequent studies of the towing system.

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