

## Analysis of fluid vibration transfer path and parameter sensitivity of swash plate axial piston pump<sup>①</sup>

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### Abstract

Taking swash plate axial piston pump as the research object, the mechanism of fluid vibration and transfer rule are analyzed. The pump shell can be assumed as the ultimate recipient of vibration transmission, the path model and differential equations from the fluid to the shell are established. The parameters of the path model are determined by the simulation software, and the mathematical model is solved by the simulation software. And time/frequency domain analysis of vibration acceleration of shell is presented. Based on the different influence of various parameters in the transfer path model on transfer characteristics and vibrational recipients, the time-varying parameters are studied by using sensitivity analysis theory, and the influence of the structural parameters on the vibration characteristics of vibration subject is quantitatively analyzed. The research in the paper provides theoretical basis for vibration analysis and structure parameter optimization of axial piston pump.

**Key words:** axial plunger pump, fluid vibration, transfer path, shell vibration, fault analysis

## 0 Introduction

Axial piston pump is the core element of hydraulic system, which provides continuous power for the whole hydraulic system and ensures the security and stability of the safe and stable operation of hydraulic system. Therefore, it is very important to monitor the flow rate, pressure, vibration and temperature of the piston pump, and it is necessary to study on generation of vibration, transfer law and the key parameter of performance indexes and health status in order to reveal the occurrence mechanism and influence law of the malfunction of the piston pump and improve the operation stability and longer working life of piston pump<sup>[1]</sup>.

The vibration of axial piston pump mainly includes fluid vibration and mechanical vibration<sup>[2]</sup>, in which fluid vibration is caused by the inherent flow pulsation of piston pump and the vibration pressure shock, as well as the vibration is caused by flow backward in oil trapped area of port plate and pressure shock<sup>[3]</sup>.

Many scholars have done a lot of researches on the fluid vibration of piston pump. Ref. [4] established

the mathematical model of the pressure in the piston chamber of the axial piston pump firstly. Ref. [5] calculated the pressure of plunger cavity by using the differential method. Ref. [6] analyzed the influence of the oil characteristics of the buffer trough on the flow pulsation of the axial piston pump and obtained the pressure, flow rate and flow pulsation of the piston cavity of the piston chamber of the axial piston pump under the condition of gas-liquid two-phase flow. Ref. [7] established a simulation model of pressure in the plunger cavity of axial piston pump and the system identification method was used to determine the values of the parameters in the model to improve the accuracy of the plunger cavity pressure simulation model. Refs[8-10] conducted vibration experiments on the axial piston pump, which proved that the main source of vibration of the plunger pump was the swash plate-variable mechanism, as well as flow reverse irrigation and pressure impact in oil-trapped areas of the oil distribution pan.

Ref. [11] solved the problem of uncertain vibration transmission paths by analyzing the random response of vibration transmission path systems in the time domain. The new concept of path transmission

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was proposed and the problem of measuring the probability of vibration and noise transmission paths in the time/frequency domain was solved. Based on dynamic sensitivity analysis, an effective method to evaluate the effect of changes in parameters and nonlinear stiffness on the dynamic response of vibration receptor at each transfer path was proposed<sup>[12-14]</sup>.

These researches are of great significance in revealing the vibration mechanism of axial piston pumps.

This paper takes front, middle and rear shell of axial piston pump as the final vibration receptors, and establishes the path model of pump fluid vibration transmission with the vibration transmission path method. The numerical simulation and finite element analysis are used to determine the model parameters and to solve the vibration mathematical model, and then the result shows that the main vibration receptor of the piston pump shell is the rear shell. The contribution of time-varying parameters to rear shell vibration is analyzed, and the first-order sensitivity time domain curve and sensitivity index of each parameter can be obtained. The research provides a new idea of revealing the vibration transmission regularity of axial piston pump and the vibration control of axial piston pump.

## 1 Fluid vibration model of swash plate axial plunger pump

### 1.1 Boundary conditions of fluid vibration transfer path

A swash plate axial plunger pump is taken as the research object of the fluid vibration of the plunger pump, and some assumptions can be shown as follows.

(1) The unbalance and eccentricity of the rotor are ignored.

(2) The source of fluid vibration is the flow impact, pressure pulsation and flow backflow of the front shell.

(3) The source of excitation force is the plunger cavity pressure, and the front, middle and rear shell are the final acceptors.

(4) Cylinder and shaft are assumed as the whole rotating components.

(5) The ball-hinge movement between plunger and slipper is ignored.

(6) The rotation and swing motion of the plunger are ignored.

(7) The quality of the connection element is equivalent to the vibration body, and the oil film between the contact surfaces is considered as rigid elastomer.

(8) The axial damping of the bearing is ignored.

### 1.2 Physical model of fluid vibration transfer path

There are three transfer paths in physical model of fluid vibration transfer path (Fig. 1).

**Path 1** Plunger cavity oil → Plunger slipper assembly → Swash plate → Cylindrical roller bearing → Rear shell.

**Path 2** Plunger cavity oil → Plunger slipper assembly → Swash plate → Variable mechanism → Middle shell.

**Path 3** Plunger cavity oil → Cylinder block and drive shaft → Front shell.

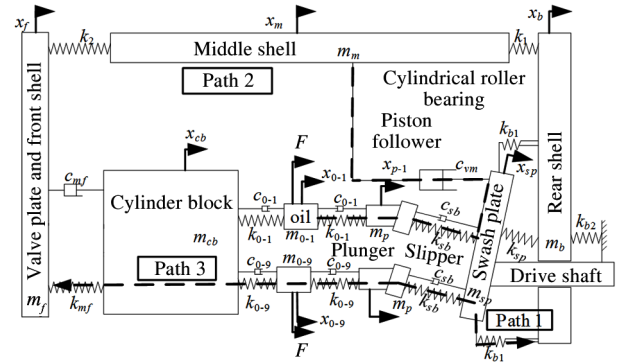


Fig. 1 Physical model of transfer path of fluid vibration shell of axial piston pump

As shown in Fig. 1,  $F$  is the exciting force on the oil in the plunger cavity;  $x_b$ ,  $x_m$ ,  $x_f$ ,  $x_{vm}$ ,  $x_{sp}$ ,  $x_p$ ,  $x_o$  and  $x_{cb}$  are respectively the vibration displacement of the rear shell, middle shell, front shell, variable mechanism, swash plate, plunger slipper component, plunger cavity oil, cylinder block, drive shaft and valve plate under the action of excitation force  $F$ ;  $m_b$ ,  $m_m$ ,  $m_f$ ,  $m_{vm}$ ,  $m_{sp}$ ,  $m_p$ ,  $m_o$  and  $m_{cb}$  are the actual mass of the rear shell, middle shell, front shell, variable mechanism, swash plate, slipper, plunger, plunger cavity oil, cylinder block and drive shaft respectively;  $k_1$  is axial stiffness between the rear shell and the middle shell;  $k_2$  is stiffness between the middle shell and the front shell;  $k_3$  is axial stiffness of cylindrical roller bearing;  $k_4$  is axial stiffness of deep groove ball bearing;  $k_{b1}$  is axial stiffness of cylindrical roller bearing;  $k_{b2}$  is axial stiffness of deep groove ball bearing;  $k_{sp}$  is axial stiffness between the rear shell and the variable mechanism;  $k_{sb}$  and  $c_{sb}$  are stiffness and damping of oil film supported by slipper;  $k_{ow}$  and  $c_{ow}$  are stiffness and damping of the oil film of with plunger cavity;  $c_{vm}$  is damping of oil film of variable mechanism;  $k_{mf}$  and  $c_{mf}$  are stiffness and damping of oil film of port plate pair support. There are nine plungers in the axial plunger pump in the paper.

### 1.3 Mathematical model of fluid vibration

The model of the fluid vibration transmission path of the axial piston pump is established by using analytical mechanics to determine the functional relationship between the components in the paper, and Lagrange function is shown as Eq. (1) <sup>[15]</sup>.

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_j} \right) - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} + \frac{\partial D}{\partial \dot{q}_j} = \Omega \quad (1)$$

$$\begin{cases} m_b \ddot{x}_b - k_{sp}(x_{sp} - x_b) + k_1(x_b - x_m) - k_{b1}(x_{sp} - x_b) - k_{b2}(x_{cb} - x_b) = 0 \\ m_m \ddot{x}_m - k_1(x_b - x_m) + k_2(x_m - x_f) + c_{vm}(\dot{x}_m - \dot{x}_{vm}) = 0 \\ m_f \ddot{x}_f - k_2(x_m - x_f) - c_{mf}(\dot{x}_{cb} - \dot{x}_f) - k_{mf}(x_{cb} - x_f) = 0 \\ m_{vm} \ddot{x}_{vm} - c_{vm}(\dot{x}_m - \dot{x}_{vm}) - k_{sp} \cos \beta (x_{sp} \cos \beta - x_{vm}) = 0 \\ m_{sp} \ddot{x}_{sp} - \sum_{w=1}^9 c_{sb}(\dot{x}_{pw} \cos \beta - \dot{x}_{sp}) + k_{sp}(x_{sp} - x_{sp} \cos \beta) \\ \quad - \sum_{w=1}^9 k_{sb}(x_{pw} \cos \beta - x_{sp}) + k_{b1}(x_{sp} - x_b \cos \beta) - c_{vm}(\dot{x}_{vm} \cos \beta - \dot{x}_{sp}) = 0 \\ m_{p1} \ddot{x}_{p1} - c_{ow}(\dot{x}_{o1} - \dot{x}_{p1}) - k_{o1}(x_{o1} - x_{p1}) + c_{sb} \cos \beta (\dot{x}_{p1} - \dot{x}_{sp} \cos \beta) + k_{sb} \cos \beta (x_{p1} - x_{sp} \cos \beta) = 0 \\ \vdots \\ m_{p9} \ddot{x}_{p9} - c_{ow}(\dot{x}_{o9} - \dot{x}_{p9}) - k_{o1}(x_{o9} - x_{p9}) + c_{sb} \cos \beta (\dot{x}_{p9} - \dot{x}_{sp} \cos \beta) + k_{sb} \cos \beta (x_{p9} - x_{sp} \cos \beta) = 0 \\ m_{o1} \ddot{x}_{o1} + c_{o1}(\dot{x}_{o1} - \dot{x}_{p1}) + c_{o1}(\dot{x}_{o1} - \dot{x}_{cb}) + k_{o1}(x_{o1} - x_{cb}) + k_{o1}(x_{o1} - x_{p1}) = F_n \\ \vdots \\ m_{o9} \ddot{x}_{o9} + c_{o9}(\dot{x}_{o9} - \dot{x}_{p9}) + c_{o9}(\dot{x}_{o9} - \dot{x}_{cb}) + k_{o9}(x_{o9} - x_{cb}) + k_{o9}(x_{o9} - x_{p9}) = F_n \\ m_{cb} \ddot{x}_{cb} - \sum_{w=1}^9 c_{ow}(\dot{x}_{ow} - \dot{x}_{cb}) + c_{mf}(\dot{x}_{cb} - \dot{x}_f) - \sum_{w=1}^9 k_{ow}(x_{ow} - x_{cb}) + k_{mf}(x_{cb} - x_f) = 0 \end{cases} \quad (2)$$

where,  $F_n = F_0 \cdot \sin(2\pi n/60)t$ ,  $n$  is rotate speed of axial plunger pump, when the pump speed is not the same, the frequency of excitation force will be different.

## 2 Axial plunger pump determination of model parameter

The assumptions are shown as follows.

(1) Stiffness and damping are constant, nonlinear

where,  $T$  is work done by inertial forces,  $U$  is work done by elastic forces,  $D$  is work done by damping forces,  $q_j$  is generalized coordinates,  $\Omega$  is generalized force.

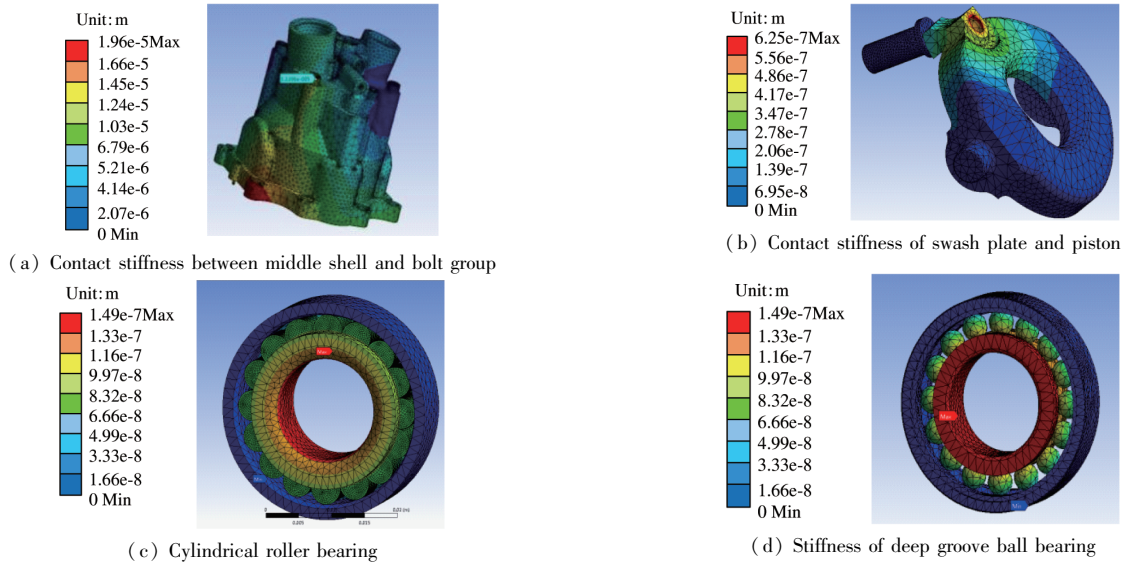
The fluid vibration transfer path of axial plunger pump is established and the functional relationship between components is determined in order to get Lagrange equation of plunger pump as Eq. (2).

and time-varying characteristics are ignored.

(2) The effects of roughness and friction force are ignored in the finite element modeling process.

Finite element modeling analysis is shown as Fig. 2.

Referencing to Refs [15-17], through the above finite element analysis in Fig. 2, combined with theoretical calculation and field measurement methods, the parameters are determined as shown in Table 1.



**Fig. 2** Finite element analysis of stiffness of deep groove ball bearing

Table 1 Parameters of axial piston pump

Parameter	Meaning	Values
$F/N$	Excitation force of oil	—
$x_b/m$	Displacement of rear shell	—
$x_m/m$	Displacement of middle shell	—
$x_f/m$	Displacement of front shell	—
$x_{vmf}/m$	Displacement of variable mechanism	—
$x_{spf}/m$	Displacement of swash plate	—
$x_{pf}/m$	Displacement of plunger slipper kit	—
$x_{of}/m$	Displacement of plunger cavity oil	—
$x_{cbf}/m$	Displacement of cylinder block, drive shaft, valve plate	—
$m_b/kg$	Mass of rear shell	0.4695
$m_m/kg$	Mass of middle shell	0.6076
$m_f/kg$	Mass of front shell	1.0554
$m_{vm}/kg$	Mass of variable mechanism	0.0224
$m_{sp}/kg$	Mass of swash plate	0.5821
$m_p/kg$	Mass of plunger slipper kit	0.0855
$m_o/kg$	Mass of plunger cavity oil	0.1854
$m_{cb}/kg$	Mass of cylinder block, drive shaft, valve plate	0.8954
$k_1/(N/m)$	Axial stiffness between rear shell and middle shell	$2.2 \times 10^{10}$
$k_2/(N/m)$	Stiffness between middle shell and front shell	$2.2 \times 10^8$
$k_3/(N/m)$	Axial stiffness of cylindrical roller bearing	—
$k_4/(N/m)$	Axial stiffness of deep groove ball bearing	—
$k_{b1}/(N/m)$	Axial stiffness of cylindrical roller bearing	$3.0577 \times 10^8$
$k_{b2}/(N/m)$	Axial stiffness of deep groove ball bearing	$6.9117 \times 10^8$
$k_{sp}/(N/m)$	Axial stiffness between rear shell and variable mechanism	$7.85 \times 10^8$
$k_{sb}/(N/m)$	Stiffness of oil film supported by slipper	$1.267 \times 10^5$
$c_{sb}/(N \cdot S/m)$	Damping of oil film supported by slipper	51719
$k_{ow}/(N/m)$	Stiffness of oil in plunger cavity	$15.1 \times 10^7$
$c_{ow}/(N \cdot S/m)$	Damping of oil film of plunger pair	102.63
$c_{vm}/(N \cdot S/m)$	Damping of oil film of variable mechanism	102.63
$k_{mf}/(N/m)$	Stiffness of oil film supported by distribution pair	$1.1 \times 10^7$
$c_{mf}/(N \cdot S/m)$	Damping of the oil film supported by distribution pair	1694.9

### 3 Solution and analysis on transfer path model of vibration

Due to the periodic and time-varying characteristics of the fluid vibration source, Runge-Kutta method<sup>[16]</sup> can be used to solve the vibration model.

The vibration acceleration time/frequency domain diagram of the final receptor (front shell, middle shell, rear shell) of the transfer path system are presented as follows. The response curve in the time domain/frequency domain of accelerated vibration of the front shell under 5900 r/min and 19.6 MPa is shown in Fig. 3.

Accelerated vibration time/frequency domain response curve of middle shell under 5900 r/min and 19.6 MPa is shown in Fig. 4.

The accelerated vibration time domain/frequency domain response curve of the rear shell under 5900 r/min and 19.6 MPa is shown in Fig. 5.

According to Fig. 3 – Fig. 5, the time domain response of shell includes the transient state and the steady state. In the transient state stage, the amplitude of the shell is large and it can be arranged as rear shell > front shell > middle shell.

The initial maximum amplitude of front shell is about  $5.7 \text{ m/s}^2$ , after that, the vibration amplitude decreases to about  $0.2 \text{ m/s}^2$ ; the initial maximum amplitude of middle shell is about  $4.2 \text{ m/s}^2$ , after a period

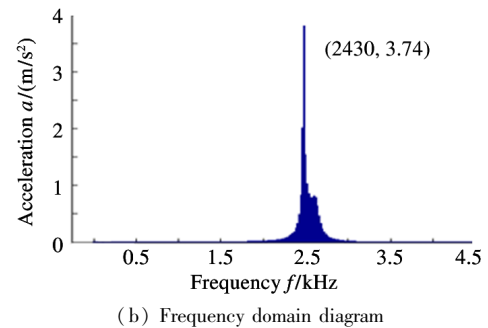
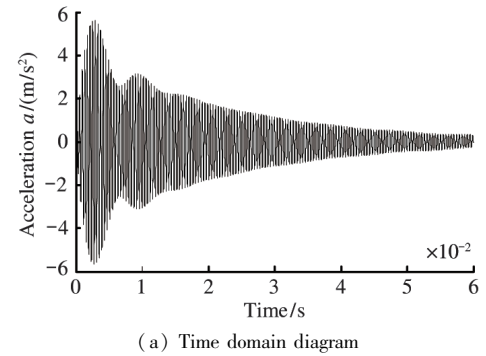
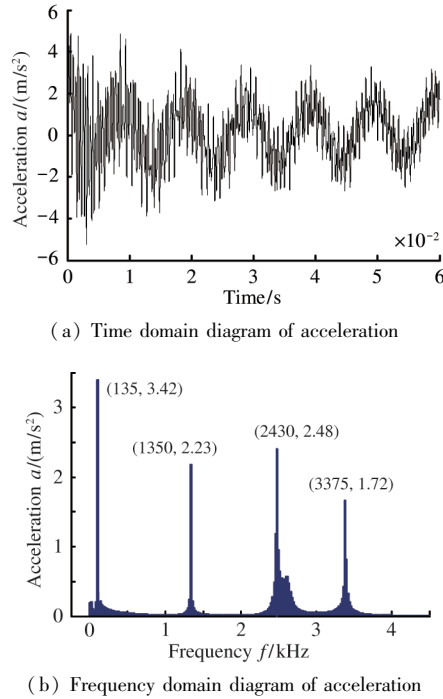
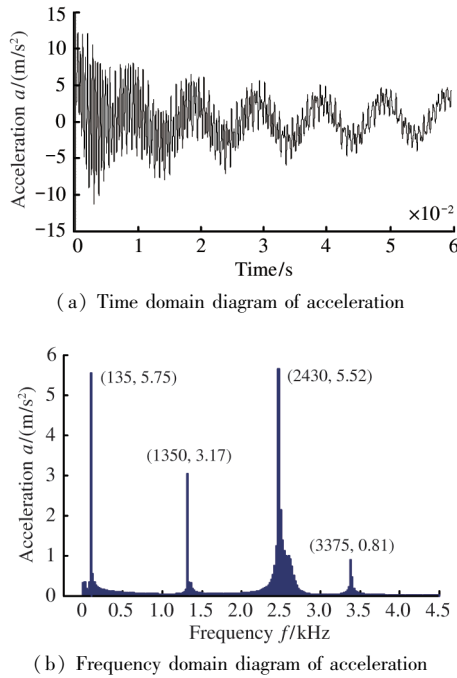


Fig. 3 Vibration response curve of front shell



**Fig. 4** Vibration response curve of middle shell



**Fig. 5** Vibration response curve of rear shell

of stability, the vibration amplitude decreases to about  $2.3 \text{ m/s}^2$ ; the initial maximum amplitude of rear shell is about  $12 \text{ m/s}^2$ , after a period of stability, the vibration amplitude decreases to about  $5.3 \text{ m/s}^2$ ; due to the influence of oil damping at the sliding shoe pair, plunger cavity and swash plate variable mechanism on the system, the vibration of the shell will gradually decrease and finally enter the steady-state response stage. At this time, the vibration is periodic.

The fundamental frequency of the shell is 135 Hz, and all order harmonics are integral times of the fundamental frequency, which is consistent with the exciting frequency caused by fluid vibration of axial piston pump. The first order response frequency of vibration acceleration is 1350 Hz, the second order is 2430 Hz and the third order is 3375 Hz. The shell is easy to incur the strong resonance in the vicinity of 2430 Hz.

Compared with Fig. 3 – Fig. 5, the front shell and bell cover are fixed, the vibration structure of the whole pump is cantilever beam structure, and its main vibration mode is vertical to the axial swing, and the vibration of the front shell and the middle shell are finally be superimposed on the rear shell. Moreover, when the pump is working, the rotating part around the main shaft rotates at high speed, and the vibration is transmitted from inside to outside along the pump shaft direction, which leads to the largest vibration amplitude of the rear shell compared with the front shell and the middle shell.

#### 4 Time domain response sensitivity analysis of fluid vibration system

The vibration transfer path is the hub connecting the excitation source and causing the vibration of the receptor. However, the influence degree of the parameters such as damping and stiffness in the system is different. In order to reveal the influence factors of the parameters on the vibration response of the whole system more intuitively, it is necessary to analyze the sensitivity of the parameters in the current system<sup>[17]</sup>. In this section, the sensitivity analysis method is used to analyze the parameters in the system, and the influence of the change on the vibration response of each vibration receptor is studied, which provides guidance for the optimization of structural parameters of axial piston pump<sup>[18]</sup>.

##### 4.1 State space description of vibration system

The state space expression of fluid vibration system of axial piston pump is

$$\dot{\mathbf{y}} = \mathbf{f}(\mathbf{y}, \mathbf{u}, \mathbf{a}, t) \quad (3)$$

In which,  $\mathbf{a}$  is  $p$  dimensional state variable,  $\mathbf{y}$  is  $m$  dimension parameter vector,  $\mathbf{u}$  is the  $r$  input vector independent of  $\mathbf{a}$ ,  $t$  is time.

The 16 state variables (velocity and displacement), 1 input (excitation force) and 12 structural parameters (stiffness and damping) are selected as the research objects, and each vector in the equation of state can be expressed as

$$\begin{cases} \dot{\mathbf{y}} = f(\mathbf{y}, \mathbf{u}, \mathbf{a}, t) \\ \mathbf{y} = [y_1 \ y_2 \ y_3 \ \cdots \ y_{16}]^T \\ \mathbf{u} = [u_1]^T \\ \mathbf{a} = [a_1 \ a_2 \ a_3 \ \cdots \ a_{12}]^T \end{cases} \quad (4)$$

According to the above statement, the 16 state variables in state variable  $\mathbf{y}$  are as follows.

$$\begin{aligned} y_1 &= y_b, y_2 = \dot{y}_b, y_3 = y_m, y_4 = \dot{y}_m, y_5 = y_f, y_6 = \dot{y}_f, \\ y_7 &= y_{vm}, y_8 = \dot{y}_{vm}, y_9 = y_{sp}, y_{10} = \dot{y}_{sp}, y_{11} = y_p, \\ y_{12} &= \dot{y}_p, y_{13} = y_o, y_{14} = \dot{y}_o, y_{15} = y_{cb}, y_{16} = \dot{y}_{cb} \end{aligned}$$

Input vector  $\mathbf{u}$  is  $\mathbf{u} = F$ , 12 parameters in the parameter vector  $\mathbf{a}$  are defined as follows.

$$\begin{aligned} a_1 &= k_1, a_2 = k_2, a_3 = k_{mf}, a_4 = k_{sp}, a_5 = k_{sb}, \\ a_6 &= k_o, a_7 = k_{b1}, a_8 = k_{b2}, a_9 = c_{sb}, a_{10} = c_{ow}, \\ a_{11} &= c_{mf}, a_{12} = c_{vm} \end{aligned}$$

## 4.2 First order trajectory sensitivity equation

Eq. (3) can be solved as follows.

$$\psi_n(t) = y(t, \mathbf{a})_n \quad (5)$$

where,  $n$  ( $n = 1, 2, \dots, m$ ) represents the  $n$ th state variable.

The mathematical expression of first-order trajectory sensitivity of the state variable  $\mathbf{y}$  to the parameter vector  $\mathbf{a}$  is

$$\lambda_n^i = \left( \frac{\partial y}{\partial a_i} \right)_n \quad (6)$$

where,  $\lambda_n^i$  is  $i$ th parameter vector in the matrix of  $m \times p$ ,  $i = 1, 2, \dots, p$ .

Since the state variable  $\mathbf{y}$  is a function related to the parameter vector  $\mathbf{a}$  and the input vector  $\mathbf{u}$ , Eq. (7) can be obtained by partial derivatives of the parameter vector  $\mathbf{a}$  on both side of Eq. (3).

$$\left( \frac{\partial \dot{\mathbf{y}}}{\partial a_i} \right)_n = \frac{\partial f}{\partial a_i} \quad (7)$$

First-order trajectory sensitivity equation can be obtained by substituting Eq. (7) into Eq. (6).

$$\dot{\lambda}_n^i = \left( \frac{\partial f}{\partial \mathbf{y}} \right)_n \times \lambda_n^i + \left( \frac{\partial f}{\partial a_i} \right)_n \quad (8)$$

where,  $\left( \frac{\partial f}{\partial \mathbf{y}} \right)_n$  and  $\left( \frac{\partial f}{\partial a_i} \right)_n$  are coefficient term matrix and free term matrix.

The coefficient term matrix can be obtained as follows by taking the partial derivative of function  $f(\mathbf{y}, \mathbf{u}, \mathbf{a}, t)$  with respect to state variable  $\mathbf{y}$ .

$$\left( \frac{\partial f}{\partial \mathbf{y}} \right)_n = \begin{bmatrix} \frac{\partial f}{\partial y_1} & \cdots & \frac{\partial f}{\partial y_{16}} \end{bmatrix} = \begin{bmatrix} a_{1,1} & \cdots & a_{1,16} \\ \vdots & \ddots & \vdots \\ a_{16,1} & \cdots & a_{16,16} \end{bmatrix} \quad (9)$$

Taking the partial derivative of the function  $f(\mathbf{y}, \mathbf{u}, \mathbf{a}, t)$  with respect to the parameter vector  $\mathbf{a}$ , the free term matrix is got.

$$\left( \frac{\partial f}{\partial a_i} \right)_n = \begin{bmatrix} \frac{\partial f}{\partial a_1} & \cdots & \frac{\partial f}{\partial a_{12}} \end{bmatrix} = \begin{bmatrix} b_{1,1} & \cdots & b_{1,12} \\ \vdots & \ddots & \vdots \\ b_{16,1} & \cdots & b_{16,12} \end{bmatrix} \quad (10)$$

Assuming that the initial value of each state variable in the model is  $y_0 = 0$ , the initial value of the corresponding first-order trajectory sensitivity function is

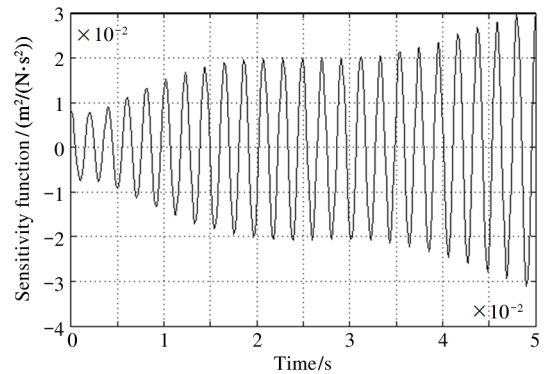
$$\lambda_{n0}^i = 0_{m \times p} \quad (11)$$

## 4.3 Solution of first order trajectory sensitivity

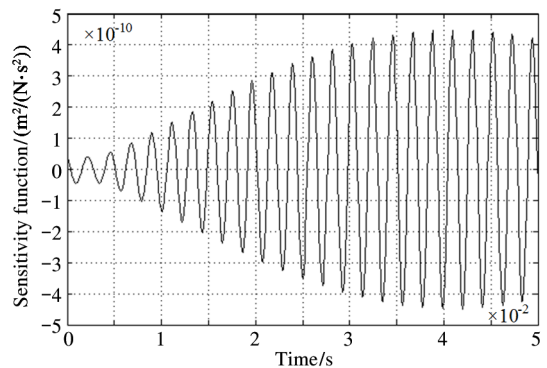
According to the established first-order trajectory sensitivity model of the vibration system, the time-domain curve of first-order trajectory sensitivity of structural parameters to state variables can be obtained by using the simulation software. Owing to spatial confine, the rear shell with the most violent vibration is selected to analyze the vibration velocity sensitivity<sup>[19]</sup>.

Fig. 6 shows the time-domain curve of the first-order trajectory sensitivity function of the vibration velocity  $y_2$  of the rear shell to the parameter  $a_i$ .

It can be seen that the sensitivity curve oscillates around zero within the set sampling time range. According to the first-order sensitivity curve of each parameter, the order of magnitude of amplitude is  $(\lambda_2^4, \lambda_2^5) < \lambda_2^8 < (\lambda_2^1, \lambda_2^2, \lambda_2^3, \lambda_2^6, \lambda_2^7) < \lambda_2^9 < \lambda_2^{11} < \lambda_2^{10} < \lambda_2^{12}$ .

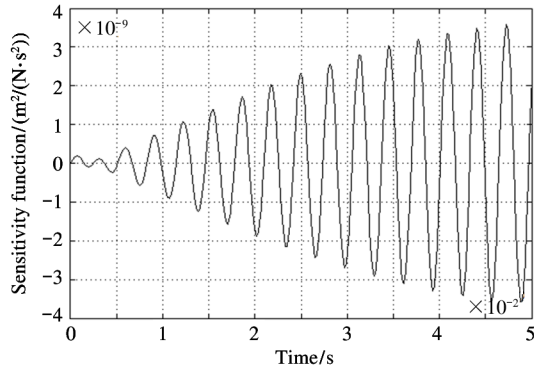
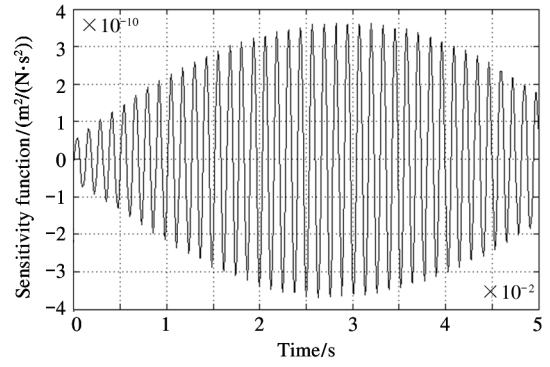
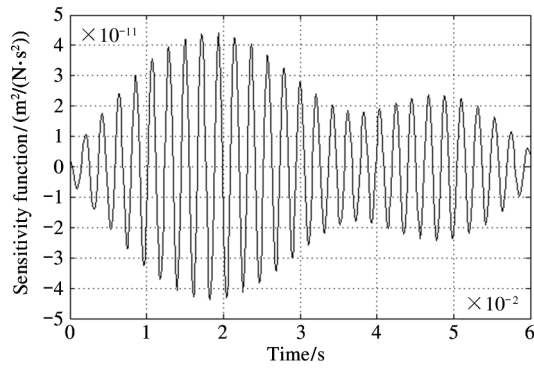
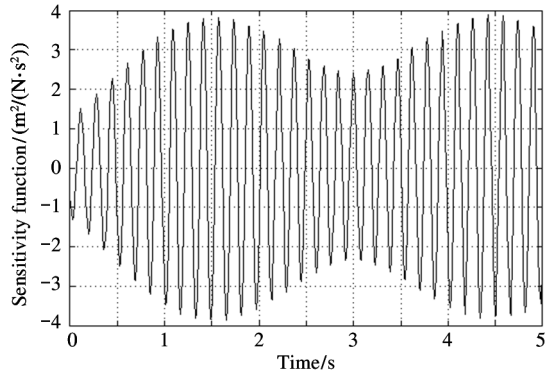
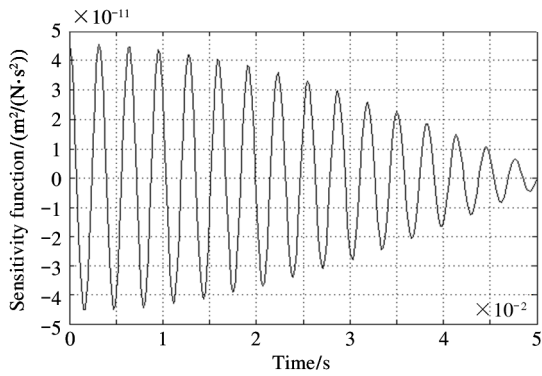
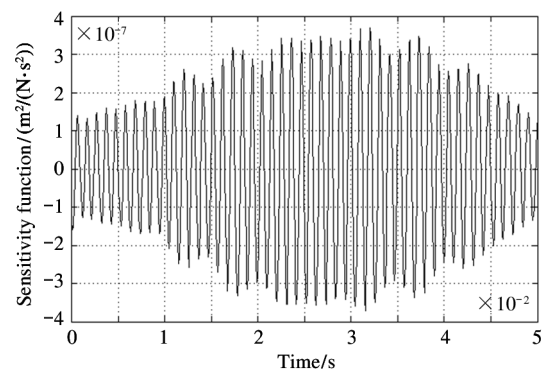
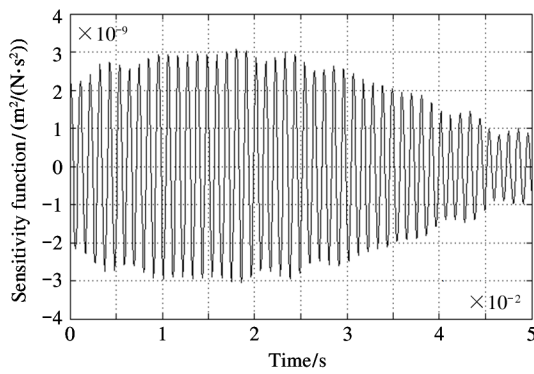
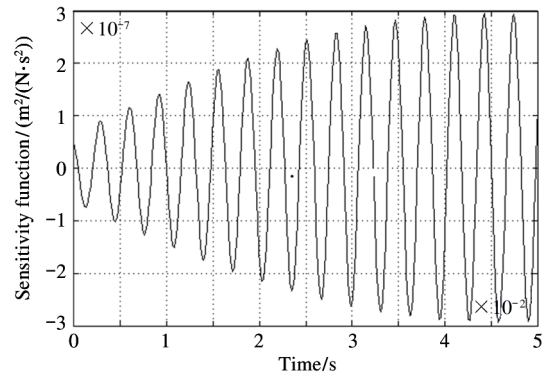


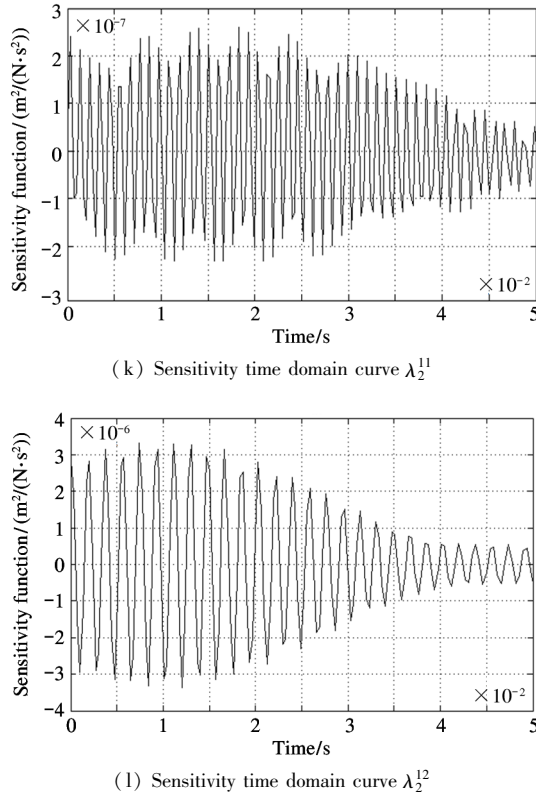
(a) Sensitivity time domain curve  $\lambda_2^1$



(b) Sensitivity time domain curve  $\lambda_2^2$



(c) Sensitivity time domain curve  $\lambda_2^3$ (g) Sensitivity time domain curve  $\lambda_2^7$ (d) Sensitivity time domain curve  $\lambda_2^4$ (h) Sensitivity time domain curve  $\lambda_2^8$ (e) Sensitivity time domain curve  $\lambda_2^5$ (i) Sensitivity time domain curve  $\lambda_2^9$ (f) Sensitivity time domain curve  $\lambda_2^6$ (j) Sensitivity time domain curve  $\lambda_2^{10}$



**Fig. 6** Time domain curve of first-order trajectory sensitivity function of vibration velocity

It can be seen that the contribution of structural parameters to the vibration velocity response of the pump rear shell is obviously different,  $\lambda_2^9$ ,  $\lambda_2^{11}$ ,  $\lambda_2^{10}$ ,  $\lambda_2^{12}$  are the sensitivity functions of the vibration velocity of the rear shell to the damping parameters, which are greater than other (stiffness parameters) sensitivity functions.

When the parameters change with the same value, the influence of damping parameter is more significant than that of stiffness parameter. Due to the great difference between the damping parameters and stiffness parameters, if only comparing the amplitude of the above sensitivity function, one can not accurately judge the influence degree of each parameter on the vibration velocity of the shell, therefore it needs further analysis.

#### 4.4 Measurement index and solution of first order trajectory sensitivity

The first order trajectory sensitivity curve describes the change process of the influence degree of each structural parameter on the state variables, but it can not accurately determine the change trend of the vibration velocity of the shell when the parameters change by percentage. Therefore, based on the two sensitivity evaluation indexes defined in Ref. [20], this section obtains the first-order trajectory sensitivity

measurement index for the changes of various structural parameters according to the vibration response of the pump rear shell.

The relationship between the change of parameter vector  $\Delta a$  and the change of state variable  $\Delta y$  is

$$\Delta y = \lambda_n^i \times \Delta a + T_h \quad (12)$$

where,  $T_h$  is higher order term.

According to Eq. (12), if the influence of higher-order term on the change of state variable  $\Delta y$  is ignored, then the change of state variable  $\Delta y$  is linearly related to the change of parameter vector  $\Delta a$ . Therefore, only the first order trajectory sensitivity function  $\lambda_n^i$  of each parameter requires to be calculated, then the change size of state variable caused by structural parameter change can be obtained.

**Sensitivity index 1** The percentage of the ratio of the state variable variation  $\Delta y$  to its maximum value  $y_{\max}$ , and the expression of the percentage is shown in Eq. (13).

This percentage varies over time and its maximum value is expressed as

$$s_1 = \frac{|\Delta y|}{y_{\max}} \Big|_{\max} \times 100\% \quad (13)$$

where,  $|\Delta y| = |\lambda_n^i| \times \Delta a_i$ .

**Sensitivity index 2** Integral of  $|\lambda_n^i| \times \Delta a_i$  to sampling time  $t$ . Its expression is shown as

$$s_2 = \int_0^t |\lambda_n^i| \times \Delta a_i dt \quad (14)$$

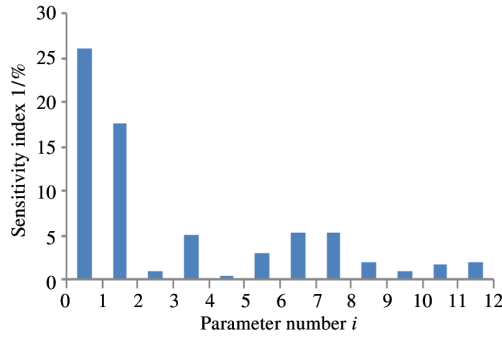
According to Eq. (13) and Eq. (14), two bar charts of sensitivity measures are obtained when each parameter changes 1%, 2% and 3% respectively, as shown in Fig. 7.

The figure shows that the two sensitivity indexes change linearly with the change of each parameter. The influence of the parameters on the vibration velocity  $Y_2$  of the rear shell is as follows.

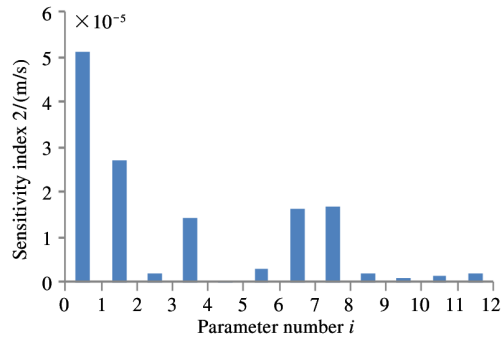
The two sensitivity indexes of stiffness  $a_3$  and damping  $a_{11}$  of oil film of supporting oil film of port plate pair, stiffness  $a_5$  and damping  $a_9$  of oil film of supporting oil film of slippers, oil damping  $a_{10}$  of plunger chamber and damping  $a_{12}$  between the rear shell and the variable mechanism account for a small proportion compared with other parameters, among which, the influence of the stiffness  $a_5$  of the slippers support oil film on the vibration speed of the pump rear housing has minimal impact. The two sensitivity indexes of the axial stiffness  $a_1$  of the bolt group connection between the rear shell and the middle shell and the axial stiffness  $a_2$  between the middle shell and the front shell are obviously larger than other parameters, this is because the parameter value is relatively higher, among which, the



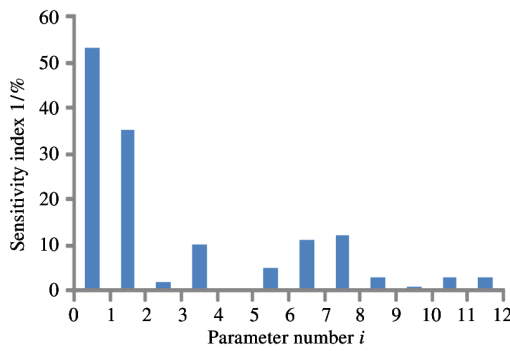
sensitivity index of parameter  $a_1$  is the largest. In addition to these two parameters, the sensitivity indexes of the axial stiffness  $a_4$  between the rear shell and the variable mechanism, the stiffness  $a_6$  of the oil in the plunger chamber, the axial stiffness  $a_7$  of the cylindrical roller bearing and the axial stiffness  $a_8$  of the deep groove



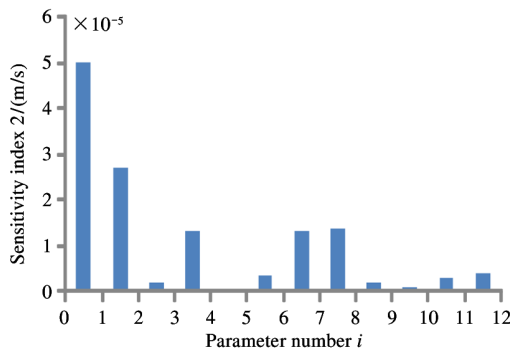
(a) Sensitivity index 1 when each parameter changes 1%



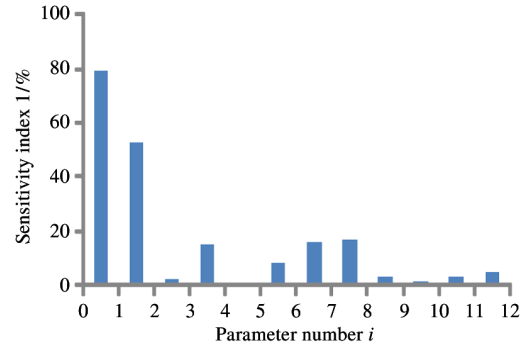
(b) Sensitivity index 2 when each parameter changes 1%



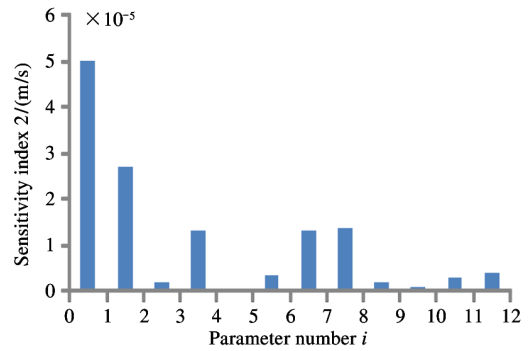
(c) Sensitivity index 1 when each parameter changes 2%



(d) Sensitivity index 2 when each parameter changes 2%



(e) Sensitivity index 1 when each parameter changes 3%



(f) Sensitivity index 2 when each parameter changes 3%

**Fig. 7** Histogram of two sensitivity measures when each parameter changes  $\Delta a$

ball bearing are also larger. Therefore, in the process of fault diagnosis for a piston pump, the influence of these parameters on the system characteristics should be considered.

## 5 Conclusions

The physical model of fluid vibration transmission path is abstracted for axial piston pump, and the corresponding mathematical model of vibration are deduced by analytical mechanics, which can completely reflects the characteristics of fluid vibration of the pump.

The model parameters are determined by the finite element method, and the mathematical model is programmed by the Runge-Kutta method to obtain the vibration response of the pump shell structure in the time/frequency domain, which lays a foundation for improving the accurate solution of the axial piston pump fluid vibration transmission path model.

The parameter sensitivity analysis is used to analyze the contribution of time-varying parameters to the vibration of the rear shell, which has the strongest vibration. The first-order sensitivity time-domain curve of each parameter and the sensitivity index of each parameter varying by 1% are obtained, which reflects the influence of each parameter on the response of fluid vibration in the backshell.

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