HIGH TECHNOLOGY LETTERS | Vol. 28 No. 1 | Mar. 2022 | pp. 107-114

doi:10.3772/j.issn.1006-6748.2022.01.013

# A joint optimization scheme of resource allocation in downlink NOMA with statistical channel state information①

XU Lei (徐　磊)②, LI Weifeng, YAO Yijing, CHANG Jing, FANG Hongyu, LI Xiaohui
(School of Electronics and Information Engineering, Anhui University, Hefei 230039, P. R. China)

**Abstract**

In a real communication scenario, it is very difficult to obtain the real-time channel state information (CSI) accurately, so the non-orthogonal multiple access (NOMA) system with statistical CSI has been researched. Aiming at the problem that the maximization of system sum rate cannot be solved directly, a step-by-step resource allocation optimization scheme based on machine learning is proposed. First, in order to achieve a trade-off between the system sum rate and user fairness, the system throughput formula is derived. Then, according to the combinatorial characteristics of the system throughput maximization problem, the original optimization problem is divided into two sub-problems, that are power allocation and user grouping. Finally, genetic algorithm is introduced to solve the sub-problem of power allocation, and hungarian algorithm is introduced to solve the sub-problem of user grouping. By comparing the ergodic data rate of NOMA users with statistical CSI and perfect CSI, the effectiveness of the statistical CSI sorting is verified. Compared with the orthogonal multiple access (OMA) scheme, the NOMA scheme with the fixed user grouping scheme and the random user grouping scheme, the system throughput performance of the proposed scheme is significantly improved.

**Key words**: non-orthogonal multiple access (NOMA), channel state information (CSI), user grouping, power allocation, throughput

## 0 Introduction

Non-orthogonal multiple access (NOMA) technology has recently attracted tremendous attention due to its simple design and superior spectrum efficiency, which is recognized as a promising multiple access scheme in the next generation mobile communication networks[1]. By splitting multiple users via different transmission power, superposition coding (SC), adopted by NOMA, will introduce interference information, so successive interference cancellation (SIC) is required at receiver side to realize multi-user detection[2]. Much of the existing work about NOMA has assumed that the perfect channel state information (CSI) at transmitter side, which are nearly impractical for many communication scenarios. Therefore, it is of great significance to investigate the resource optimization of downlink NOMA system with statistical CSI.

Recently, some literature has investigated the resource optimization problem in NOMA system with statistic CSI. In Ref. [3], the performance of two NOMA system with partial CSI has been evaluated. The research results show that statistical CSI based on second order statistics is always better than the incomplete CSI based on feedback, and the system performance with statistical CSI is similar to that with perfect CSI, in the case of low signal to noise ratio (SNR). The power allocation scheme of the NOMA system was investigated in Ref. [4], and a sub-optimal power allocation algorithm was proposed. Genetic algorithm dynamically allocates power in a group, but the design of user grouping scheme is neglected. In Ref. [5], a dynamic power allocation scheme and a user grouping algorithm were proposed, users are divided into two sets according to their statistic CSI, and the users, in different sets and with the same sort number, were matched into one group. This grouping method is simple to implement, but it cannot guarantee the overall performance of the system. And this literature proves that the performance of the NOMA system is proportional to the difference in statistical characteristics between users within the same group. In Ref. [6], a low complexity power allocation sub-optimal solution and a user sched-

① Supported by the National Natural Science Foundation of China (No. 62001001).
② To whom correspondence should be addressed. E-mail: xulei@ahu.edu.cn.
Received on Jan. 26, 2021

uling scheme were proposed based on a hierarchical clustering algorithm. Energy efficiency is used as the evaluation index, and the near-optimal performance can be achieved.

The aforementioned literature all considers user grouping and power allocation separately. But in fact, user grouping and power allocation are intertwined with each other[7]. Since multiple users are admitted simultaneously, when the number of users is large, the complexity of the SIC receiver in the NOMA system is very high. As a result, it may not be realistic to ask all users in the system to perform NOMA jointly, a promising alternative is to construct a hybrid multiple access system, in which, NOMA is combined with orthogonal multiple access (OMA)[8]. The users in the system are divided into multiple groups, where NOMA is implemented within each group and different groups are allocated with orthogonal resources. Obviously, the performance of the NOMA system is very dependent on which users are grouped together and the power allocation between users[9]. Therefore, effective user grouping and power allocation schemes can provide feasibility for improving the performance of the downlink NOMA system with statistical CSI.

Therefore, based on the user channel difference and the correlation between power allocation and user grouping, combined the advantages of genetic algorithm and Hungarian algorithm in the field of resource optimization, a joint optimization scheme with controllable complexity is proposed in this paper, in which, it is assumed that the transmitter only knows the statistical CSI related to each user. According to the combinatorial characteristics of the system throughput maximization problem, the original optimization problem is divided into two sub-problems, that are power allocation and user grouping. Combined the advantages of genetic algorithm in solving the non-convex problems and the low-complexity characteristics of Hungarian algorithm in solving the matching problem, genetic algorithm is introduced to solve the sub-problem of power allocation, and Hungarian algorithm is introduced to solve the subproblem of user grouping.

The symbols used in this paper are defined as follows. $E(\cdot)$ represents the mean operator, $f(\cdot)$ and $F(\cdot)$ represent the probability density function (PDF) and the cumulative distribution function (CDF), respectively. And $U$ represents the uniform distribution.

## 1 System model

In this paper, a single-cell downlink multi-user system is considered, which adopts the hybrid multiple access scheme. A base station (BS) is located in the center of the cell. $N$ users are randomly distributed in the cell, and both the users and BS are equipped with a single antenna. The users are divided into $L$ groups, and each group contains $K_l$ users, which satisfies

$$\sum_{l=1}^{L} K_l = N \tag{1}$$

NOMA is implemented within each group and different groups are allocated with orthogonal resources to eliminate inter-group interference, and some intra-group interference can be eliminated by SIC.

Without loss of generality, the widely used Rayleigh fading channel for communication is adopted, which is affected by the joint effect of large-scale fading and small-scale fading. The channel model can be expressed as

$$h_n = \frac{g_n}{\sqrt{1 + d_n^{\alpha}}} \tag{2}$$

where $g_n$ denotes the small-scale fading coefficient, which is subject to $g_n \sim CN(0,1)$. $d_n$ denotes the distance between user $n$ and BS, and $\alpha$ denotes the average path loss factor. The channel coefficient $h_n$ is subject to the Rayleigh distribution, and its PDF can be expressed as

$$f(x,\ \Omega_n) = \frac{2x}{\Omega_n}\exp\left(-\frac{x^2}{\Omega_n}\right),\ x \geqslant 0 \tag{3}$$

where $\Omega_n$ denotes the large-scale fading coefficient between user $n$ and BS, expressed as $\Omega_n = \mathrm{E}(|h_n|^2)$. It is assumed that BS only has statistical CSI related to each user, i. e., BS knows the value of $\Omega$ for all users.

In order to simplify the derivation of the problem, the $l$-th group with $K_l$ users is mainly analyzed, so the signal received by the $k$-th user in the $l$-th group can be expressed as

$$y_{l,k} = h_{l,k}\sqrt{a_{l,k}P}x_{l,k} + h_{l,k}\sum_{i \neq k}\sqrt{a_{l,i}P}x_{l,i} + z_{l,k} \tag{4}$$

where $P$ denotes the transmitting power allocated by BS to each group, and it is assumed that the transmission power allocated by BS to each group is equal. $h_{l,k}$ denotes the instantaneous channel related to the $k$-th user in the $l$-th group. $x_{l,k}$ denotes the transmitted message intended for the $k$-th user in the $l$-th group. $a_{l,k}$ denotes the intra-group power allocation factor allocated to the $k$-th user in the $l$-th group. $z_{l,k}$ denotes independent and identically distributed additive white Gaussian noise, which is subject to $z_{l,k} \sim CN(0,\ \sigma^2)$. Without loss of generality, it is assumed that the users are sorted in the ascending order of statistic CSI, i. e., $\Omega_{l,1} < \Omega_{l,2} < \cdots < \Omega_{l,K}$. According to the principle of

NOMA, it can be concluded that the power allocation factor satisfies $a_{l,1} > a_{l,2} > \cdots > a_{l,K}$. The second term in Eq. (4) is caused by the user's intra-group interference, which can be partially eliminated by the SIC receiver. According to the optimal SIC decoding order under the statistical CSI proposed in Ref. [10], the decoding order of the SIC within the group is $(1, 2, \cdots, K_l)$, which is the increasing order of channel gain. The user detection model is shown in Fig. 1.

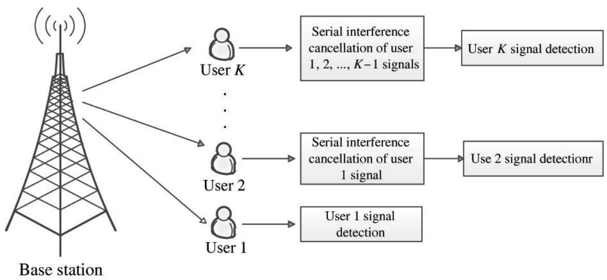


**Fig. 1** User detection model

Ideally, in the case of $j < k < m$, the $k$-th user will detect the message of $j$-th user, and then remove the message from its received in a successive manner. The message of $m$-th user will be treated as noise at the $k$-th user. Therefore, the instantaneous data rate of the $k$-th user in the $l$-th group is expressed as

$$R_{l,k} = \log_2\left(1 + \frac{\rho \mid h_{l,k} \mid^2 a_{l,k}}{\rho \mid h_{l,k} \mid^2 \sum_{m=k+1}^{K_l} a_{l,m} + 1}\right) \tag{5}$$

where $\rho = P/\sigma^2$ denotes the intra-group SNR. Note that the data rate of the $K_l$-th user is $R_{l,K_l} = \log_2(1 + \rho \mid h_{l,K_l} \mid^2 a_{l,K_l})$. Let $R_{l,\, k\to l,\, j}$ denote the instantaneous data rate of the $j$-th user's message detected by the $k$-th user in $l$-th group, which can be expressed as

$$R_{l,\, k\to l,\, j} = \log_2\left(1 + \frac{\rho \mid h_{l,k} \mid^2 a_{l,j}}{\rho \mid h_{l,k} \mid^2 \sum_{i=j+1}^{K_l} a_{l,i} + 1}\right) \tag{6}$$

## 2 Performance analysis

Since the analysis of outage probability and ergodic data rate requires the density function of channel, the widely used Rayleigh fading channel for communication is adopted. Therefore, the PDF and the CDF of unsorted channel gain are given as follows.

$$f_{|\tilde{h}_n|^2}(x;\ \Omega_n) = \frac{1}{\Omega_n} e^{-\frac{x}{\Omega_n}},\ x \geqslant 0 \tag{7}$$

$$F_{|\tilde{h}_n|^2}(x,\ \Omega_n) = 1 - e^{-\frac{x}{\Omega_n}},\ x \geqslant 0 \tag{8}$$

Since the users are sorted in the ascending order of statistic CSI, the corresponding PDF and CDF of the ordered channel gain can be written as[11]

$$f_{|h_n|^2}(x;\Omega_n) = \beta_{N,n} \sum_{t=0}^{N-n} (-1)^t \binom{N-n}{t} f_{|\tilde{h}_n|^2}(x;\Omega_n)\left[F_{|\tilde{h}_n|^2}(x;\Omega_n)\right]^{n+t-1} \tag{9}$$

$$F_{|h_n|^2}(x;\Omega_n) = \beta_{N,n} \sum_{t=0}^{N-n} \frac{(-1)^t}{n+t} \binom{N-n}{t} \left[F_{|\tilde{h}_n|^2}(x;\Omega_n)\right]^{n+t} \tag{10}$$

where $\beta_{N,n} = \dfrac{N!}{(N-n)(n-1)!}$.

### 2.1 Ergodic data rate

In order to compare the impact of statistical CSI and perfect CSI on the performance of the NOMA system, based on Eq. (5) and Eq. (9), the ergodic data rate formula of user $k$ can be derived.

$$\begin{aligned} R_k^{\mathrm{NOMA}} &= \int_0^{+\infty} R_k f_{|h|^2}(x;\Omega_k)\,\mathrm{d}x \\ &= \mathrm{E}_{x\sim f_{|h|^2}(x;\Omega_k)}\left[\log_2\left(1 + \frac{a_k x}{\rho\left(x \sum_{i=k+1}^{K} a_i + 1\right)}\right)\right] \end{aligned} \tag{11}$$

### 2.2 Outage probability

For the sake of guaranteeing the SIC decoding in Section 2 performing correctly, $R_{l,\, k\to l,\, j} \geqslant R_{l,\, j}^{th}$ must be satisfied, where $R_{l,\, j}^{th}$ denotes the target data rate of the $j$-th user in the $l$-th group. When the instantaneous data rate cannot reach the target data rate, outage occurs. Therefore, the outage probability at the $k$-th user can be expressed as[12]

$$P_{l,k}^{\mathrm{NOMA}} = 1 - \Pr\left(\bigcap_{j=1}^{k} (R_{l,k\to j} \geqslant R_{l,\, j}^{th})\right) \tag{12}$$

where $R_{l,k\to l,\, j} \geqslant R_{l,\, j}^{th}$ can be expressed as

$$\mid h_{l,k} \mid^2 \geqslant \frac{\varepsilon_{l,\, j}}{\rho(a_{l,\, j} - \varepsilon_{l,\, j} \sum_{i=j+1}^{K} a_{l,i})} \tag{13}$$

where $\varepsilon_{l,\, j} = 2^{R_{l,\, j}^{th}} - 1$. Note that Eq. (13) is obtained by assuming the following condition.

$$a_{l,k} > \varepsilon_{l,\, j} \sum_{i=k+1}^{K} a_{l,i} \tag{14}$$

Otherwise, the outage probability is always 1. Furthermore, the outage probability can be written as

$$P_{l,k}^{\mathrm{NOMA}} = \Pr\left(\mid h_{l,k} \mid^2 < \max\left\{\frac{\varepsilon_{l,\, j}}{\rho\left(a_{l,\, j} - \varepsilon_{l,\, j} \sum_{i=j+1}^{K_l} a_{l,i}\right)}\right\}\right) \tag{15}$$

let

$$\phi_{l,k} = \max\left\{\frac{\varepsilon_{l,\, j}}{\rho\left(a_{l,\, j} - \varepsilon_{l,\, j} \sum_{i=j+1}^{K_l} a_{l,i}\right)}\right\} \tag{16}$$

Based on Eq. (10) and Eq. (15), the user outage probability expression of the NOMA system with statistical CSI can be obtained as

$$P_{l,k}^{\mathrm{NOMA}} = \beta_{K_l,k} \sum_{t=0}^{K_l-k} \frac{(-1)^t}{k+t} \binom{K_l - k}{t} [1 - e^{-\frac{\Phi_{l,k}}{\Omega_{l,k}}}]^{k+t} \tag{17}$$

### 2.3 Throughput

In general, there are mainly two category of methods for optimizing the performance in communication systems. One is to maximize the achievable sum data rate, and the other is to guarantee the fairness of users[13]. On the one hand, to maximize the sum data rate, BS tends to allocate more power to users with high channel gains, which may cause users with low channel gains to be outage. On the other hand, the fairness among the users may result in the performance loss of the achievable sum data rate. In order to achieve a trade-off between achievable sum data rate and user fairness, it is considered to maximize the achievable sum data rate while guaranteeing the minimum rate requirement of each user. Therefore, throughput is adopted to characterize rate performance. Throughput combines rate performance and fairness, which is defined as the sum of target data rate of each user multiplied by its successful transmission probability[14], expressed as

$$T^{\mathrm{NOMA}} = \sum_{l=1}^{L} \sum_{k=1}^{K_l} R_{l,k}^{th} (1 - P_{l,k}^{\mathrm{NOMA}}) \tag{18}$$

the target data rate represents the rate performance, and the probability of successful transmission guarantees the fairness of users.

In order to verify the superiority of the NOMA system performance, time division multiple access (TDMA) system is selected for comparison. Assuming that the system is of equal time slots, and the transmission power allocated to each time slot is equal, the instantaneous data rate is expressed as

$$R_n^{\mathrm{OMA}} = \frac{1}{N} \log_2 (1 + \frac{P_{\mathrm{total}} | h_n |^2}{\sigma^2}) \tag{19}$$

where $P_{\mathrm{total}}$ denotes the total transmit power of BS. Similar to the derivation process of Eq. (17) and Eq. (18), the corresponding outage probability of user $n$ and system throughput can be calculated as

$$P_n^{\mathrm{OMA}} = 1 - \exp\left(- \frac{(2^{NR_n^{th}} - 1)\sigma^2}{P_{\mathrm{total}} \Omega_n}\right) \tag{20}$$

$$T^{\mathrm{OMA}} = \sum_{n=1}^{N} R_n^{th} (1 - P_n^{\mathrm{OMA}}) \tag{21}$$

where $R_n^{th}$ denotes the target data rate of user $n$.

## 3 Resource allocation optimization scheme

As analyzed in subsection 2.3, in order to realize the trade-off between system achievable sum data rate and user fairness, throughput is adopted to characterize rate performance in this paper. Therefore, the problem of maximizing system throughput can be expressed as

$$\begin{aligned} \max \quad & \sum_{l=1}^{L} \sum_{k=1}^{K_l} R_{l,k}^{th} (1 - P_{l,k}^{\mathrm{NOMA}}) \\ \text{s. t.} \quad & C_1: \sum_{l=1}^{L} P \leqslant P_{\mathrm{total}}, \ P > 0 \\ & C_2: \sum_{k=1}^{K_l} a_{l,k} = 1, \ a_{l,k} > 0 \\ & C_3: a_{l,1} > a_{l,2} > \cdots > a_{l,K_l} \\ & C_4: a_{l,k} > \varepsilon_{l.k} \sum_{i=k+1}^{K_l} a_{l,i} \end{aligned} \tag{22}$$

where the constraint condition $C_1$ is the total transmit power constraint, and the power allocated to each group should be non-negative. The constraint condition $C_2$ is the power allocation factor constraint for each group. The constraint conditions $C_3$ and $C_4$ represent the NOMA principle constraints.

The solution used to maximize system throughput in Eq. (22) is not only affected by the power allocation factor, but also depends on that, which users can be allocated to the same group. In many of the existing work, users are grouped directly according to their channel condition, and then power is allocated to users within each group. However, the inherent relationship between power allocation and user grouping is ignored. Therefore, a joint resource allocation optimization algorithm is proposed. According to the combinatorial characteristics of the system throughput maximization problem, the solution to the Eq. (22) can be divided into two stages. In the first stage, based on any pairwise grouping of users, an improved genetic algorithm is used to allocate power to maximize intra-group throughput within each group. In the second stage, based on the results of the first stage, Hungarian algorithm is used to determine the user grouping set, which can maximize the system throughput.

### 3.1 Power allocation based on improved genetic algorithm

Under the arbitrary and fixed user grouping schemes, it is assumed that transmission power allocated to each group is equal. Therefore, the problem of maximizing system throughput in Eq. (22) can be divided into multiple problems of maximizing intra-group throughput.

$$\begin{aligned} \max \quad & \sum_{k=1}^{K_l} R_k^{th}(1 - P_k^{\mathrm{NOMA}}) \\ \text{s. t.} \quad & C_1: \sum_{k=1}^{K_l} a_k = 1,\ a_k > 0 \\ & C_2: a_1 > a_2 > \cdots > a_{K_l} \\ & C_3: a_k > \varepsilon \sum_{i=k+1}^{K_l} a_i \end{aligned} \tag{23}$$

Since the objective function is non-convex, exhaustive search for the optimal solution results in heavy complexity, which is hard to accomplish in practice. Therefore, genetic algorithm is adopted for power allocation.

Genetic algorithm is based on natural selection and genetic theory, which combines the survival of the fittest in the process of biological evolution with the random information exchange mechanism of chromosomes within the population. Genetic algorithm abandons the traditional search method, simulates the biological evolution process in nature, and uses artificial evolution to search the target space randomly. Firstly, it regards the possible solution in the problem domain as an individual or chromosome of the group, and encodes each individual into a symbol string form. Then, it simulates the biological evolution process of Darwin's genetic selection and natural elimination, and iterative operations selection, crossover and mutation based on genetics are performed on the population. Finally, each individual is evaluated according to the predetermined target fitness function, and a better group can be continuously obtained according to the evolutionary rules of survival of the fittest. At the same time, the global parallel searcher is used to search for the optimal individual in the optimization group and the optimal solution that meets the requirements[15].

In the traditional genetic algorithm, parameters of the three operators of selection, crossover and mutation are fixed, which may lead to the destruction of the optimal individual, and result in the non-convergence of the evolutionary process, so the performance of genetic algorithm can be severely restricted. In addition, in the later stage of evolution, the diversity of individuals in the population is greatly reduced, and the fitness values are close to each other, which leads to the algorithm approaching the state of random search.

In order to alleviate the above problems, the mutation probability of the individual is determined dynamically according to the fitness value of the individual, and the crossover method is changed. The mutation probability of the individuals with high fitness is reduced to prevent the damage of good genes, and the mutation probability of the individuals with poor adaptability is improved by introducing new genes into the population. After determining the male parent and the female parent, multiple nodes are selected for multiple crossover, and the two best ones are selected from the results to be inherited to the next generation. The improved algorithm is described as follows.

(1) Initializes population number $S$ and the maximum generation $G_{\max}$, and randomly generates $S$ individuals expressed by a binary string of length $N_{\mathrm{BS}}$ placed in the initial population $Q(0)$.

(2) Takes the system throughput expression in Eq. (22) as the fitness function, and computes the fitness value $f$ of each individual.

(3) According to the fitness value $f$ of the individual, computes the selection probability $P_s^i = f_i / \sum_{k=1}^{K_l} f_k$ and mutation probability $P_m^i = (1 - P_s^i) P_m$ of each individual, where $P_c$ and $P_m$ represent the crossover probability and the mutation probability are initially set to the certain values.

(4) Firstly, selects and mutates the individuals in the population $Q(G)$. Then, randomly selects two individuals, crossovers them multiple times according to the crossover probability $P_c$. Finally, selects the best two individuals from the offspring individuals to obtain the next population $Q(G+1)$.

(5) If $G < G_{\max}$, goes to Step (2), else, stops and gets the individual with greatest fitness as the solution.

The optimal solution output, obtained according to the above steps, is the solution of the objective function in Eq. (22), that is the power distribution coefficient of the user.

### 3.2 User grouping based on Hungarian algorithm

It should be noted that in 3GPP LTE advanced, two users are selected for performing NOMA[16], and the number of users in each group $K_l$ cannot be too large due to the complexity of SIC. Therefore, the user grouping method for the case of $K_l = 2$ is investigated. The user grouping problem can be expressed as

$$\arg\max \sum T_{i,j}^{\mathrm{NOMA}} \tag{24}$$

where $T_{i,j}^{\mathrm{NOMA}}$ denotes the maximum intra-group throughput when user $i$ and user $j$ are allocated to the same group.

Since channel differences have a direct impact on the performance of system throughput, users are divided into two sets based on the statistical CSI, denoted as $V_1$ and $V_2$. Specific steps are as follows.

(1) Detect the value of statistical feature $\Omega$ of users in the cell, and users are sorted in the ascending

order of $\Omega$.

(2) Take user $n(n = f_{\text{floor}}(N/2))$ as the boundary and divide users into two sets $V_1$ and $V_2$, where $f_{\text{floor}}(\cdot)$ means rounding to the left.

If the total number of users $N$ is even, the numbers of users in $V_1$ and $V_2$ are equal. If $N$ is odd, the first user in $V_2$ is taken out as a user group, and then a user is selected from set $V_1$ and set $V_2$ respectively to form a two-user group. After the above processing, the channel differences between users within the same group can be enlarged, which can improve the performance gain of NOMA system. However, the arbitrary matching of users in the two sets cannot guarantee the system throughput performance. Considering the impact of power allocation on user grouping, the user grouping problem can be transformed into a one-to-one matching problem between user sets $V_1$ and $V_2$ for sake of maximizing system throughput. Taking advantage of the low complexity feature of Hungarian algorithm in solving the matching problem, Hungarian algorithm is introduced to solve Eq. (24).

Hungarian algorithm is one of the classic algorithms for bipartite graph matching in graph theory. Its application background is to solve the problem of two-dimensional task allocation, and it involves two concepts, namely bipartite graph and augmenting-path. Bipartite graph is a special model in graph theory. Let $\boldsymbol{A} = (\boldsymbol{B}, \boldsymbol{C})$ be an undirected graph. If the vertex $\boldsymbol{B}$ can be divided into two disjoint subsets $(\boldsymbol{B}_1, \boldsymbol{B}_2)$, and the two vertices $i$ and $j$ associated with each edge $(i, j)$ in the graph belong to these two different vertex sets, so $\boldsymbol{A}$ is called a bipartite graph. For augmenting-path, let $\boldsymbol{M}$ be the set of matched edges in the bipartite graph $\boldsymbol{A}$. If $X$ is a path connecting two unmatched vertices in the graph $\boldsymbol{A}$, the edges belonging to $\boldsymbol{M}$ and the edges not belonging to $\boldsymbol{M}$ appear alternately on $X$, then $X$ is an augmenting-path relative to $\boldsymbol{M}$. The basic idea of Hungarian algorithm is to exchange the matching and non-matching edges in the augmenting-path by searching for the augmenting-path, so that there will be one more matching edge until no augmenting-path is found.

Solving Eq. (24) by Hungarian algorithm can be further expressed as selecting $W$ elements from the $W \times W(W = f_{\text{floor}}(N/2))$ matrix shown in Eq. (25) for maximizing their sum, and any of these $W$ elements are not on the same row and the same column. The selected element denotes the maximum intra-group throughput obtained when users $i$ and $j$ are allocated to the same group.

$$\begin{bmatrix} T_{1,1}^{\text{NOMA}} & T_{1,2}^{\text{NOMA}} & \cdots & T_{1,W}^{\text{NOMA}} \\ T_{2,1}^{\text{NOMA}} & T_{2,2}^{\text{NOMA}} & \cdots & T_{2,W}^{\text{NOMA}} \\ \cdot & \cdot & \cdot & \cdot \\ T_{W,1}^{\text{NOMA}} & T_{W,2}^{\text{NOMA}} & \cdots & T_{W,W}^{\text{NOMA}} \end{bmatrix} \quad (25)$$

In order to prove the effectiveness of the proposed grouping method, genetic algorithm is also used in all comparison schemes. In terms of algorithm complexity, the time complexity of the exhaustive search is $O(N!)$. The time complexity of Hungarian algorithm used in this paper mainly comes from the sorting process, which is $O(W^2 + W\log_2 W)$, and the time complexity of the fixed matching scheme used in Ref. [5] is $O(W\log_2 W)$. The time complexity of the proposed scheme is slightly higher than that of the fixed matching scheme, but much lower than the exhaustive search scheme, so it is still feasible even under a huge number of users.

## 4 Simulation results and analysis

The system simulation adopts Monte Carlo method and assumes that the number of users in each group is 2, and the power allocated to each group is equal. The comparisons of ergodic data rate with perfect CSI and statistical CSI is shown in Fig. 2.

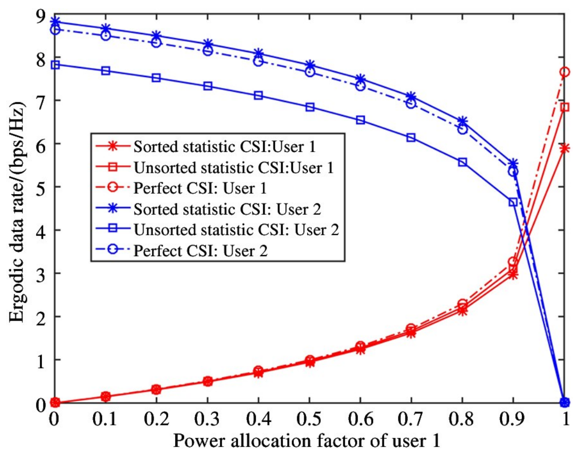


**Fig. 2** Comparison of ergodic data rate of NOMA users with statistical CSI and perfect CSI

As can be seen from Fig. 2, there is a certain gap between systems using statistical CSI and perfect CSI. In this paper, the users are sorted in the ascending order of statistic CSI, so the gap between the statistical CSI and the perfect CSI is reduced. It is proved that statistical CSI is more feasible in practical applications. In Fig. 2, user 1 and user 2 represent the user with poor channel and good channel in a group respectively. When the power allocation factor allocated to user 1 is less than 0.9, the ergodic data rate of user 2 is always much higher than that of user 1. When the power allocated to user 1 is small, user 1 may be outage due to the constraint of minimum rate requirement, so the fairness of weak users cannot be guaranteed. However, when the power allocation factor of user 1 exceeds 0.9, the ergodic data rate of user 2 will decrease rapidly, which will affect the overall system rate performance, this is consistent with the result analyzed in subsection 2.3.

The simulation parameters in Fig. 3 are as follows. SNR = 20 dB, the statistic CSI $\Omega$ and the target rate $R^{\mathrm{th}}$ of users are generated by the system, which is subject to $\Omega \sim U(0,5)$ and $R^{\mathrm{th}} \sim U(1,5)$ respectively. In genetic algorithm, the number of individuals in the population is $S = 40$, the length of binary string is $N_{\mathrm{BS}} = 20$, the maximum generation $G_{\max} = 100$, the initial set of crossover probability and the mutation probability are $P_c = 0.7$ and $P_m = 0.05$ respectively. As can be seen from Fig. 3, compared with the traditional OMA system, NOMA system can significantly improve the system throughput. For further comparison, genetic algorithm is used in all schemes to allocate power to users. It can be seen that, the gap between the random grouping scheme and the fixed user grouping scheme is always small, and the fixed grouping scheme proposed in Ref. [5] has achieved the largest channel difference. It is demonstrated that the increasing of the channel difference between users can improve the system throughput, but the advantage brought by the channel difference is reduced after the user power allocation within the group. The symtem throughput of joint optimization scheme proposed in this paper is obviously better than that of the random grouping scheme and fixed grouping scheme in Ref. [5]. As the number of user group served by BS increases, the advantages of

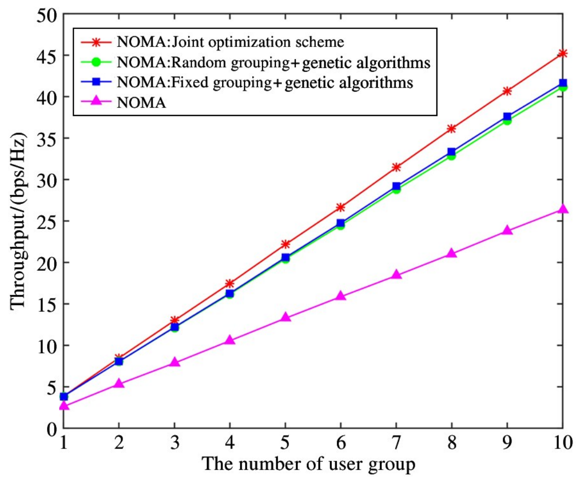


**Fig. 3** Trend of system throughput with the number of users

the proposed scheme are more obvious than other schemes. That is, the proposed optimization scheme is more suitable for the scenario with a large number of user.

The simulation parameters of Fig. 4 are as follows. the number of users is $N = 12$, the statistic CSI $\Omega$, the target rate $R^{\mathrm{th}}$ of users and the parameters of genetic algorithm are consistent with Fig. 3. It can be seen from Fig. 4 that, in low SNR region, the system throughput of these schemes are similar to each other. The reason is that the user data rate is low and outage probability is high due to the too low SNR. In high SNR region, the transmitting power is allocated to stronger users as much as possible under the constraint of NOMA principle. The system throughput of proposed grouping scheme is close to that of the fixed matching grouping scheme. However, in practical applications, the SNIR is generally in range of 10 – 20 dB. In this case, the system throughput of joint optimization schemes proposed in this paper are better than that of the other three schemes. Compared with TDMA scheme, random grouping scheme and the fixed grouping scheme, the system throughput of the proposed scheme can be improved by about 9 bps/Hz, 2.1 bps/Hz, and 2 bps/Hz, respectively.

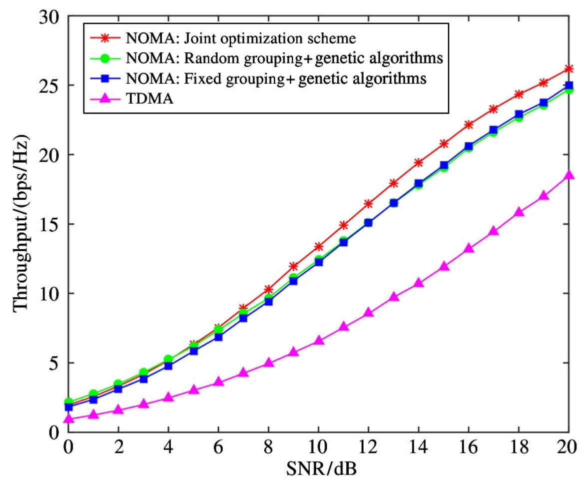


**Fig. 4** Trend of system throughput with the SNR

## 5 Conclusions

A joint optimization scheme of resource allocation is proposed in this paper based on joint genetic algorithm and Hungarian algorithm. Users are sorted in the ascending order of statistic CSI, so the performance of NOMA system with statistical CSI can further approach the NOMA system with perfect CSI. The closed-form formula of each user's outage probability is derived, and the outage performance and the achievable sum data rate are analyzed. Then throughput is adopted to characterize system performance. For the sake of maximum system throughput, a joint optimization design of genetic algorithm and Hungarian algorithm is carried out. Simulation results verify the effectiveness of the proposed scheme in improving the performance of system throughput.

**References**

[1] CAI Y, QIN Z, CUI F, et al. Modulation and multiple access for 5G networks[J]. *IEEE Communications Surveys & Tutorials*, 2018, 20(1): 629-646
[2] ISLAM S M R, ZENG M, DOBRE O A, et al. Resource allocation for downlink NOMA systems: key techniques and open issues[J]. *IEEE Wireless Communications*, 2018, 25(2): 40-47
[3] YANG Z, DING Z, FAN P, et al. On the performance of non-orthogonal multiple access systems with partial channel information[J]. *IEEE Transactions on Communications*, 2016, 64(2): 654-667
[4] WANG X, WANG J, HE L, et al. Outage analysis for downlink NOMA with statistical channel state information [J]. *IEEE Wireless Communications Letters*, 2018, 7(2): 142-145
[5] GONG M, YANG Z. The application of antenna diversity to NOMA with statistical channel state information[J]. *IEEE Transactions on Vehicular Technology*, 2019, 68(4): 3755-3765
[6] WEI Z, NG D W K, YUAN J. Power-efficient resource allocation for MC-NOMA with statistical channel state information[C]// Proceedings of 2016 IEEE Global Communications Conference, Washington, USA, 2016: 1-7
[7] ZHU L, ZHANG J, XIAO Z, et al. Optimal user pairing for downlink non-orthogonal multiple access (NOMA) [J]. *IEEE Wireless Communications Letters*, 2019, 8(2): 328-331
[8] DING Z, FAN P, POOR H V. Impact of user pairing on 5G non-orthogonal multiple access downlink transmissions [J]. *IEEE Transactions on Vehicular Technology*, 2016, 65(8): 6010-6023
[9] ZHANG X, WANG J, WANG J, et al. A novel user pairing in downlink non-orthogonal multiple access[C]// Proceedings of 2018 IEEE International Symposium on Broadband Multimedia Systems and Broadcasting, Valencia, Spain, 2018: 1-5
[10] TANG Z, WANG J, SONG J. On the achievable rate region of NOMA under outage probability constraints[J]. *IEEE Communications Letters*, 2019, 23(2): 370-373
[11] MEN J, GE J, ZHANG C. Performance analysis for downlink relaying aided non-orthogonal multiple access vetworks with imperfect CSI over nakagami-*m* fading[J]. *IEEE Access*, 2017, 5: 998-1004
[12] DING Z, YANG Z, FAN P, et al. On the performance of non-orthogonal multiple access in 5G systems with randomly deployed users[J]. *IEEE Signal Processing Letters*, 2014, 21(12): 1501-1505
[13] ZHU L, ZHANG J, XIAO Z, et al. Millimeter-wave NOMA with user grouping, power allocation and hybrid beamforming[J]. *IEEE Transactions on Wireless Communications*, 2019, 18(11): 5065-5079
[14] DO D T, NGUYEN M S V. Outage probability and ergodic capacity analysis of uplink NOMA cellular network with and without interference from D2D pair[J]. *Physical Communication*, 2019, 37: 100898
[15] LEI Y J, ZHANG S W. MATLAB Genetic Algorithm Toolbox and Application[M]. Xi'an: Xidian University Press, 2014: 11-13 (In Chinese)
[16] XIA B, WANG J, XIAO K, et al. Outage performance analysis for the advanced SIC receiver in wireless NOMA systems[J]. *IEEE Transactions on Vehicular Technology*, 2018, 67(7): 6711-6715

**XU Lei**, born in 1984. He received his Ph. D degree in electromagnetic field and microwave technology from Anhui University in 2012. He received his M. S. degree in communication and information system from Anhui University in 2009. He also received his B. S. degree in electronic science and technology from Huaibei Normal University in 2006. Since 2012, he has been a lecturer in School of Electronics and Information Engineering of Anhui University. His research interests include wireless relay communications, non-orthogonal multiple access, and mmWave massive MIMO.