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Resonance analysis of single DOF parameter-varying system of magnetic-liquid double suspension bearing①

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Abstract

Magnetic-liquid double suspension bearing (MLDSB) is mainly supported by electromagnetic suspension and supplemented by hydrostatic supporting. Its bearing capacity and stiffness can be greatly improved, and then it is suitable for the occasions of medium speed, heavy load. When the bearing system is excited by periodic force, the flow q and current i regulated by the double-closed-loop control mechanism change periodically. Then the risk of parametric resonance in MLDSB is greatly aggravated by the change of the parameter system, which seriously affects its operation stability and reliability. Therefore, this paper intends to study the resonance characteristics of the parameter system of MLDSB. Firstly, Marshall-Duffing equation of the parametric system is established by taking the flow q and the current i as variables respectively. Then, by using the asymptotic method, the occurrence condition and variation rule of the principal, 1/2 Harmonic and 1/3 Harmonic parametric resonance are solved. The results show that only the 1/2 Harmonic resonance of the flow q parameter varying system occurs accompanied by the resonance condition of high frequency. The principal, 1/2 Harmonic and 1/3 Harmonic parametric resonance of the current i occur accompanied by the resonance condition of high frequency. And the 1/2 Harmonic resonance of the current i occurs accompanied by the non-single value bifurcation and dynamic bifurcation. The paper can provide theoretical reference for the parameter design and stable operation of MLDSB.

Key words: magnetic-liquid double suspension bearing (MLDSB), resonance analysis, flow parameter varying system, current parameter varying system

0 Introduction

Due to the inherence default of active electromagnetic bearing (AMB), such as the insufficient electromagnetic attraction caused by the magnetic pole magnetic saturation, the higher temperature rise of the magnetic pole/coil caused by the copper loss and the eddy current loss, the bearing characteristics of operation stability of AMB can be limited and then it has become technical bottleneck which restricts the further development and application promotion of AMB.

The hydrostatic bearing concept is introduced into AMB to form the novel suspension bearing-magnetic-liquid double suspension bearing (MLDSB). It is supported by the electromagnetic suspension and supplemented by the hydrostatic supporting, having the advantages of electromagnetic system and hydrostatic system. The bearing capacity and stiffness can be improved drastically, which is suitable for the situation of the middle-speed and over-load, large bearing capacity and the high operation stability[1-2].

MLDSB is composed of bracket, motor, coupling, multi-diameter shaft, journal bearing unit, axial bearing unit, journal loading motor, axial loading motor and so on. The eight poles of MLDSB are evenly distributed in the circle of the stator and each pole is wound by coils of the same number of turns. Due to the

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different winding type, the eight poles are distributed in the mode of NSSNNSSN. Electromagnetic attraction is generated by forming a magnetic circuit between the two adjacent poles and the magnetic shaft sleeve on the shaft[3]. The each pole is machined with an inlet hole. The end face of the pole is used as the bearing surface of hydrostatic pressure. And there is a small gap between the bearing surface and the magnetic shaft sleeve. A large liquid resistance is formed when the flows go through the small gap. The hydrostatic pressure is established on the end face of the pole and the hydrostatic bearing force is generated[4-5].

A single degree of freedom (DOF) supporting system in the vertical direction is taken as an example. MLDSB adopts the double-closed-loop feedback control sysytem. Firstly, the displacement of the shaft is detected by the displacement sensor in real time and the signal of deviated displacement is transferred to the voltage controller and current controller. Then by adjusting the bias voltage of proportional velocity regulating valve and current of coil, the flow and current are further adjusted. In this way, the change of small gap can be suppressed and MLDSB will reach the equilibrium again. In the initial state, the bias currents of the upper and lower electromagnetic coils are i_0 and the thickness of upper and lower oil film are 30 μm. When the external load f acts on the rotor, the displacement of the rotor is x and the thicknesses of the oil films of the upper and lower supporting cavity changes, and then the hydrostatic supporting force is generated. The controlling current i_c generated by the electromagnetic system are transferred to the upper and lower coils and then the electromagnetic supporting force is generated. The rotor is adjusted by electromagnetic force and hydrostatic force together so that it can return to the balance position again[6]. However, the regulation parameters flow q and current i are forced to change periodically when MLDSB is disturbed by external periodic excitation. The risk of parametric resonance in MLDSB is greatly aggravated by the change of the parameter system, which seriously affects its operation stability and reliability.

At present, many scholars have made a deep research on parameter vibration problem, and then have achieved the fruitful results.

By studying the frequency response characteristics of the parameter system, Ref. [7] successively adopted the Sylvester theory and the Fourier series expansion to discuss influence of the system frequency response characteristics on the parameter stability, time-varying parameters and damping by taking the pair direct gear as an example.

Ref. [8] used numerical integration method to study the nonlinear dynamic response of single degree of freedom parametric system under single-frequency stiffness excitation and load excitation. It was found that the multiple frequency response can be led by the single frequency excitation, and the system has multi-frequency resonance characteristics. By studying the resonance of the Mathieu-Duiffgn equation system, Ref. [9] obtained the frequency response function under the principal parameter resonance, 1/2 Harmonic and 1/3 Harmonic parameter resonance. Ref. [10] reduced the probability of parameter vibration of the compressor by optimizing the pipeline layout and the structure of the bracket. Refs [11,12] established the coupled vibration model of a cable bridge around 2000, and they discussed the probability of parametric vibration of the cable under the excitation. Ref. [13] used Galerkin method to transform the parametric vibration equation to get the ordinary differential equation, so as to describe the parametric vibration of the cable more accurately. Ref. [14] used the multi-scale method to conduct relevant research on the vibration differential equation of cable model, and discussed the influence of nonlinear term, damping term, external excitation term and parametric excitation term in the equation on the vibration characteristics of cable.

At present, most experts and scholars pay more attention to the influence of the parameters on vibration characteristics and frequency response function of cable parameter variable system of Cable Bridge, while the research on the resonance of the parameter variable system of MLDSB has not yet appeared.

Aiming at the problems mentioned above, Mathieu-Duffing equation of single DOF parameter-varying system of MLDSB is established in the paper. Resonance characteristics and amplifier changing rules of flow varying system and current varying system are analyzed by using KBM method in order to provide the theoretical basis for the structure design and stable operation of MLDSB.

1 Resonance analysis of flow varying system

The composition of MLDSB is shown as Fig. 1. The structure of MLDSB is shown as Fig. 2 and Fig. 3.

The initial balance state of MLDSB can be broken by the external periodic load during the running process, the displacement of the rotor changes periodically. Due to the slowly response of hydrostatic system, flow q changes in cycles and is lagged behind the expected value, and then hydrostatic system can be translated into flow varying system.

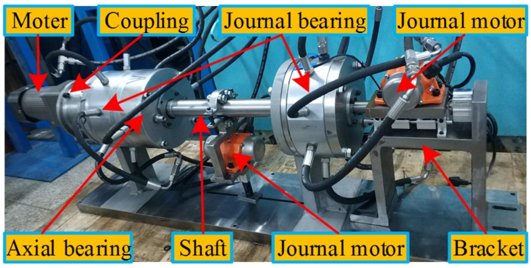

Fig. 1 Experiment table of MLDSB

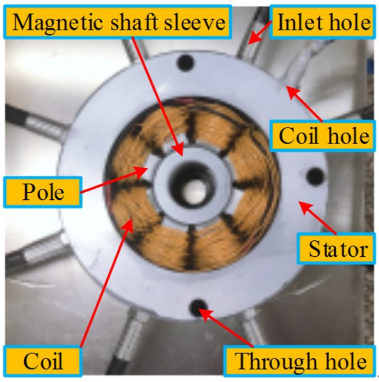

Fig. 2 Photo of journal bearing unit

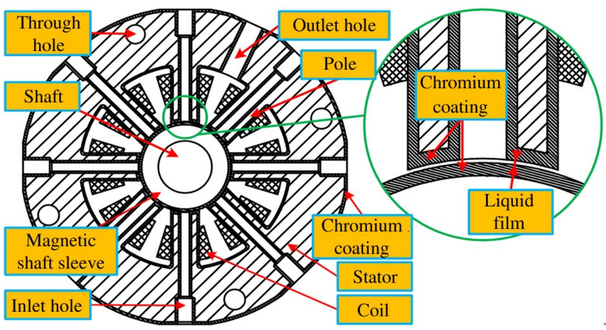

Fig. 3 Section view of journal bearing unit

It is assumed that the rotor is in the initial position (the rotation center). The flow q changes periodically with the frequency ω. The regulating principle and force diagram of MLDSB are shown as Fig. 4 and Fig. 5.

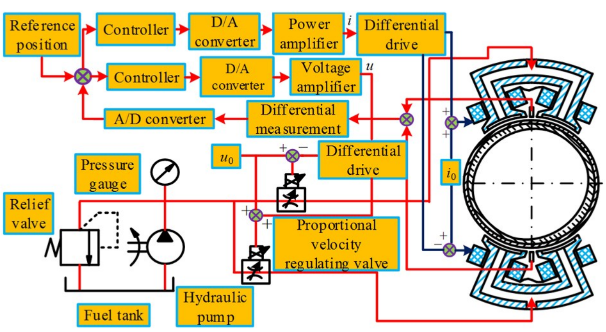

Fig. 4 Single degree of freedom bearing system

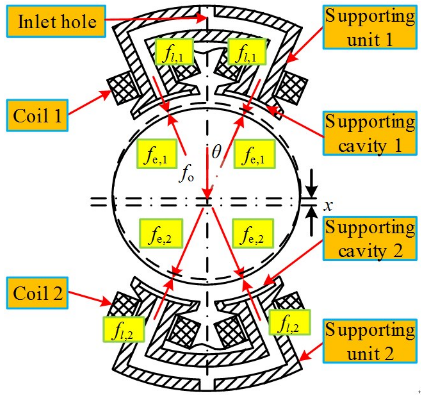

Fig. 5 Force diagram of single degree of freedom MLDSB

1.1 Mathieu-Duffing equation of flow varying system

According to Refs[15,16], the nonlinear dynamic equation of MLDSB can be established as

$$m\ddot{x} + f_k(\dot{x}, x) = 0 \tag{1}$$

where $f_k(\dot{x}, x) = f_1(\dot{x}, x) + f_2(x) + f_3(x)$;

$$f_1(\dot{x}, x) = \left[\frac{\delta_1}{(h_0 + x\cos\theta)^3} + \frac{\delta_1}{(h_0 - x\cos\theta)^3}\right]\dot{x},$$

$$f_2(x) = \frac{\delta_2}{(h_0 + x\cos\theta)^2} - \frac{\delta_2}{(h_0 - x\cos\theta)^2},$$

$$f_3(x) = \frac{\delta_3}{(h_0 - x\cos\theta)^3} - \frac{\delta_3}{(h_0 + x\cos\theta)^3},$$

$$\delta_1 = \frac{2\mu A_e A_b \cos^2\theta}{\bar{B}},\ \delta_2 = 2ki_0^2\cos\theta,$$

$$\delta_3 = \frac{2\mu q_0 A_e \cos\theta}{\bar{B}}.$$

Flow is assumed as varying parameter, and then Eq. (1) can be translated into Taylor series form to obtain Mathieu-Duffing equation as Eq. (2).

$$\ddot{x} + \omega_0^2 x = \varepsilon f(x, \dot{x}, \omega t) \tag{2}$$

where $f(x, \dot{x}, \omega t) = \xi\dot{x} + \tau x^3 + \cos\omega t(\beta x + \mu x^3)$,

$$\omega_0^2 = 8ki_0^2\cos^2\theta/m(h_0)^3,\ \xi = \frac{4\mu A_e A_b\cos^2\theta}{\bar{B}m(h_0)^3},$$

$$\tau = -16\frac{ki_0^2\cos^4\theta}{m(h_0)^5},\ \beta = 12\frac{\mu q_0 A_e\cos^2\theta}{\bar{B}m(h_0)^4},$$

$$\mu = 40\frac{\mu q_0 A_e \cos^4\theta}{\bar{B}m(h_0)^6}.$$

Due to the small parameter ε, the solution x of Eq. (2) can be obtained as

$$x = \alpha\cos\varphi + \varepsilon x_1(\alpha, \varphi) + \varepsilon^2 x_2(\alpha, \varphi) + \cdots \tag{3}$$

$$\begin{cases} \dot{\alpha} = \varepsilon A_1(\alpha) + \varepsilon^2 A_2(\alpha) + \cdots \\ \dot{\varphi} = \omega_0 + \varepsilon B_1(\alpha) + \varepsilon^2 B_2(\alpha) + \cdots \end{cases} \tag{4}$$

Substituting Eq. (3) and Eq. (4) into Eq. (2), the left part of Eq. (2) can be expressed as

$$\ddot{x} + \omega_0^2 x = \varepsilon\left[\omega_0^2\left(\frac{\partial^2 x_1}{\partial\varphi^2} + x_1\right) - 2\omega_0 A_1 \sin\varphi - 2\omega_0 a B_1 \cos\varphi\right] + \varepsilon^2\left[\begin{array}{l}\omega_0^2\left(\frac{\partial^2 x_2}{\partial\varphi^2} + x_2\right) + \left(A_1\frac{\mathrm{d}A_1}{\mathrm{d}a} - aB_1^2 - 2\omega_0 a B_2\right)\cos\varphi \\ - \left(2\omega_0 A_2 + 2A_1B_1 + aA_1\frac{\mathrm{d}B_1}{\mathrm{d}a}\right)\sin\varphi \\ + 2\omega_0 A_1\frac{\partial^2 x_1}{\partial a\partial\varphi} + 2\omega_0 B_1\frac{\partial^2 x_1}{\partial\varphi^2}\end{array}\right] \tag{5}$$

Similarly, the right part of Eq. (2) can be translated into Taylor series near $x_0 = \alpha\cos\varphi$, $\dot{x}_0 = -\alpha\omega_0\sin\varphi$ as follows.

$$\varepsilon f(x,\dot{x}) = \varepsilon f(x_0,\dot{x}_0) + \varepsilon^2\left[\begin{array}{l}x_1\frac{\partial f(x_0,\dot{x}_0)}{\partial x} \\ + (A_1\cos\varphi - aB_1\sin\varphi + \omega_0\frac{\partial x_1}{\partial\varphi})\frac{\partial f(x_0,\dot{x}_0)}{\partial\dot{x}}\end{array}\right] + \cdots \tag{6}$$

Assuming that same-power coefficients of ε of both sides of Eq. (2) are the same, and then the asymptotic equations can be expressed as

$$\omega_0^2(\frac{\partial^2 x_1}{\partial\varphi^2} + x_1) = f_0(a,\varphi) + 2\omega_0 A_1\sin\varphi + 2\omega_0 aB_1\cos\varphi \tag{7}$$

$$\omega_0^2(\frac{\partial^2 x_2}{\partial\varphi^2} + x_2) = f_1(a,\varphi) + 2\omega_0 A_2\sin\varphi + 2\omega_0 aB_2\cos\varphi \tag{8}$$

……

Resonance equation of flow varying system is given as

$$\omega_0^2 = (\lambda\omega)^2 - \varepsilon\sigma \tag{9}$$

where σ is tuning parameter.

Flow varying system can be translated into principal parameter resonance when $\lambda = 1$, Harmonic resonance when λ is fraction, super-Harmonic resonance when λ is integer.

Eq. (9) can be expressed as

$$\ddot{x} + (\lambda\omega)^2 x = \varepsilon f(x,\dot{x},\lambda\omega t) \tag{10}$$

1.2 Principal parameter resonance analysis of flow varying system

$\lambda = 1$ is substituted into Eq. (10) to eliminate secular term of x_1 as follows.

$$\omega^2(\frac{\partial^2 x_1}{\partial\varphi^2} + x_1) = \omega(2A_1 - \xi a)\sin\varphi + \frac{1}{4}a^3\tau\cos3\varphi + \frac{1}{8}a^3\mu\sin\gamma\sin4\varphi + (a\beta\cos\gamma + \mu a^3\cos\gamma)\frac{\cos2\varphi}{2} + (\frac{3}{4}a^3\tau + 2\omega aB_1)\cos\varphi + \frac{1}{4}(2a\beta + a^3\sin\gamma)\sin2\varphi + \frac{1}{8}a^3\mu\cos\gamma\cos4\varphi + \frac{1}{2}(a\beta + \frac{3}{4}a^3\mu)\cos\gamma \tag{11}$$

$$A_1 = \frac{1}{2}\xi a,\ B_1 = -\frac{3}{8\omega}\tau a^2 \tag{12}$$

Eq. (11) can be solved to obtain frequency response function as

$$4\xi\omega - 3\tau a = 0 \tag{13}$$

There are not periodic parameter term in Eq. (13), so principal parameter resonance will not occur in this case.

1.3 1/3 Harmonic parameter resonance analysis of flow varying system

$\lambda = 1/3$ is substituted into Eq. (10) to eliminate secular term of x_1 as

$$(\frac{\omega}{3})^2(\frac{\partial^2 x_1}{\partial\varphi^2} + x_1) = \frac{\omega}{3}(2A_1 - \xi a)\sin\varphi + \frac{a^3\tau}{4}\cos3\varphi + (\frac{3a^3\tau}{4} + \frac{2\omega aB_1}{3})\cos\varphi + \frac{a^3\mu\sin3\gamma}{8}\sin6\varphi + \frac{a}{2}(\beta + \frac{3a^2\mu}{4})\sin3\gamma\sin2\varphi + \frac{a^3\mu\cos3\gamma}{8}(\cos6\varphi + 1) + \frac{a}{2}(\beta + \frac{3a^2\mu}{4})\cos3\gamma\cos2\varphi + (2a\beta + \frac{3a^3\mu}{2})\sin3\gamma\sin4\varphi + \frac{a}{2}(\beta + \frac{3a^2\mu}{4})\cos3\gamma\cos4\varphi \tag{14}$$

$$A_1 = \frac{1}{2}\xi a,\ B_1 = -\frac{9\tau a^2}{8\omega} \tag{15}$$

Eq. (14) can be solved to obtain frequency response function as

$$9\tau a - 4\xi\omega = 0 \tag{16}$$

There are not periodic parameter term in Eq. (16), so 1/3 Harmonic parameter resonance will not occur in this case.

1.4 1/2 Harmonic parameter resonance analysis of flow varying system

$\lambda = 1/2$ is substituted into Eq. (10) to eliminate secular term of x_1 as

$$(\frac{\omega}{2})^2(\frac{\partial^2 x_1}{\partial\varphi^2} + x_1) = \left[\frac{\tau a^3}{4} + \frac{(8a\beta - 3a^3\mu)\cos2\gamma}{16}\right]\cos3\varphi + \left[\omega B_1 + \frac{\beta + a^2\mu}{2}\cos2\gamma + \frac{3a^2\tau}{4}\right]a\cos\varphi + \left(\omega A_1 + \frac{\beta a}{2}\sin2\gamma - \frac{a\omega}{2}\xi + \frac{11\mu a^3}{4}\sin2\gamma\right)\sin\varphi - 2a^3\mu\sin2\gamma\sin2\varphi + \frac{a\beta + 3a^3\mu}{2}\sin2\gamma\sin3\varphi + \frac{5a^3\mu}{16}\cos2\gamma\cos5\varphi \tag{17}$$

$$\begin{cases} A_1 = \frac{\xi a}{2} - \frac{\beta a \sin 2\gamma}{2\omega} - \frac{11\mu a^3 \sin 2\gamma}{4\omega} \\ B_1 = -\frac{3\tau a^2}{4\omega} - \frac{(\beta + a^2\mu)\cos 2\gamma}{2\omega} \end{cases} \quad (18)$$

According to steady-state response solution condition, amplitude-frequency and phase-frequency characteristics equations and curves of 1/2 and 1/3 Harmonic parameter resonance can be shown as Fig. 6 and Fig. 7.

$$\begin{cases} \frac{4\xi^2\omega^2}{(2\beta + 11a^2\mu)^2} + \frac{9a^4\tau^2}{4(\beta + a^2\mu)^2} = 1 \\ \tan 2\gamma = -\frac{4\omega\xi(\beta + a^2\mu)}{3\tau a^2(2\beta + 11a^2\mu)} \end{cases} \quad (19)$$

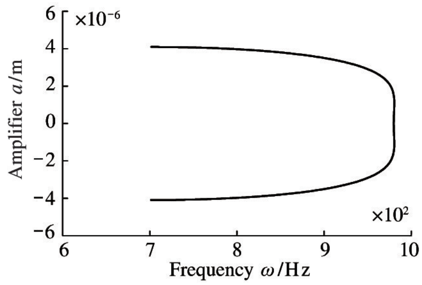

Fig. 6 Amplitude-frequency curve of 1/2 Harmonic parameter

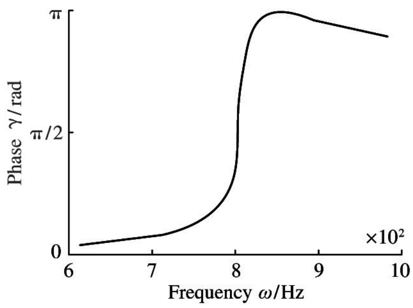

Fig. 7 Phase-frequency curve of 1/2 Harmonic parameter

When the frequency of the flow rate is twice the natural frequency of MLDSB, 1/2 Harmonic resonance of flow varying system occurs accompanied by the resonance condition of 3 times frequency and 5 times frequency. The maximum amplitude reaches 4 μm and reduces slowly with the increase of excitation frequency. In addition, 1/2 Harmonic parametric resonance is mainly related to β and μ. If and only if β and μ are both zero, 1/2 Harmonic parametric resonance will not occur in MLDSB.

2 Resonance analysis of current varying system

It is assumed that the rotor is in the initial position (the rotation center). The current i changes periodically with the frequency ω.

2.1 Mathieu-Duffing equation of current varying system

Similarly, current is assumed as varying parameter, and Eq. (1) can be translated into Taylor series form to obtain Mathieu-Duffing equation as follows.

$$\ddot{x} + \omega_0^2 x = \varepsilon f(x, \dot{x}, \kappa t) \quad (20)$$

where $\omega_0^2 = \frac{12\mu q_0 A_e \cos^2\theta}{\bar{B} m h_0^4}$, $\xi = -\frac{4\mu A_e A_b \cos^2\theta}{\bar{B} m h_0^3}$, $2\omega = \kappa$, $\beta = \frac{4ki_0^2\cos^2\theta}{mh_0^3}$, $\tau = -\frac{40\mu q_0 A_e \cos^4\theta}{\bar{B} m h_0^6}$, $f(x, \dot{x}, \omega t) = \xi\dot{x} + \beta x + (\mu + \tau)x^3 - \cos\kappa t(\beta x + \mu x^3)$

$$\mu = \frac{8ki_0^2\cos^4\theta}{mh_0^5}.$$

Similarly, resonance equation of current varying system can be given as

$$\omega_0^2 = (\lambda\kappa)^2 - \varepsilon\sigma \quad (21)$$

Similarly, Eq. (21) can be expressed as

$$\ddot{x} + (\lambda\kappa)^2 x = \varepsilon f(x, \dot{x}, \lambda\kappa t) \quad (22)$$

2.2 Principal parameter resonance analysis of current varying system

Similarly, $\lambda = 1$ is substituted into Eq. (22) to eliminate secular term of x_1 as

$$\kappa^2\left(\frac{\partial^2 x_1}{\partial\varphi^2} + x_1\right) = (2\kappa A_1 - \xi a\kappa)\sin\varphi + \frac{(\mu + \tau)\alpha^3}{4}\cos 3\varphi + \alpha\left[\beta + 2\kappa B_1 + \frac{3}{4}\alpha^2(\mu + \tau)\right]\cos\varphi + \frac{a^3\mu\cos\gamma}{8}\cos 4\varphi - \frac{2a\beta + a^3\mu\sin\gamma}{4}\sin 2\varphi + \frac{a\beta + a^3\mu}{2}\cos\gamma\cos 2\varphi + \frac{a^3\mu\sin\gamma}{8}\sin 4\varphi + \frac{4a\beta + 3a^3\mu}{8}\cos\gamma \quad (23)$$

$$A_1 = \frac{1}{2}\xi a, \; B_1 = -\frac{\beta}{2\kappa} - \frac{3\alpha^2(\mu + \tau)}{8\kappa} \quad (24)$$

Similarly, amplitude frequency and phase frequency characteristics equations and curves of principal parameter resonance can be shown as Fig. 8 and Fig. 9.

$$4\beta + 3\alpha^2(\mu + \tau) - 4a\xi\kappa = 0 \quad (25a)$$

$$3\alpha^3\beta\mu\cos\gamma - a\beta^2\cos\gamma(1 + \sin\gamma) + \frac{117a^5\mu^2}{80}\sin 2\gamma - \frac{9a^3\xi\kappa(\mu + \tau)}{2} = 0 \quad (25b)$$

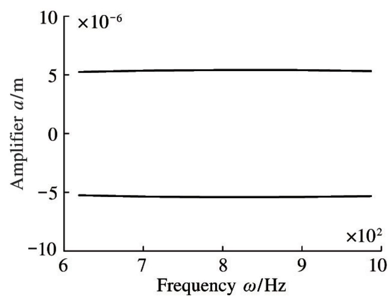

Fig. 8 Amplitude-frequency curve of principal parameter

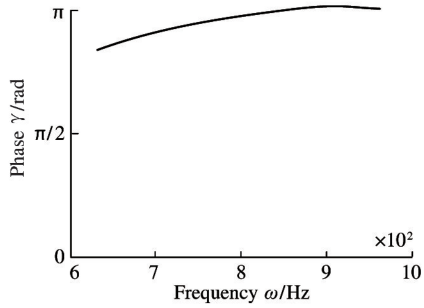

Fig. 9 Phase-frequency curve of principal parameter

When the frequency of the current rate is twice the natural frequency of MLDSB, 1/2 Harmonic resonance of current varying system occurs accompanied by the resonance condition of multi times frequency. The maximum amplitude reaches 5.5 μm and reduces slowly with the increase of excitation frequency.

2.3 1/3 Harmonic parameter resonance analysis of current varying system

Similarly, $\lambda = 1/3$ is substituted into Eq. (22) to eliminate secular term of x_1 as

$$\left(\frac{\kappa}{3}\right)^2\left(\frac{\partial^2 x_1}{\partial\varphi^2} + x_1\right) = \frac{3a^3\mu}{2}\sin3\gamma\sin4\varphi + \frac{a^3\mu}{8}\sin3\gamma\sin6\varphi + \alpha\left[\beta + \frac{2\kappa}{3}B_1 + \frac{3\alpha^2(\mu+\tau)}{4}\right]\cos\varphi + \frac{\alpha^3}{4}(\mu+\tau)\cos3\varphi + \frac{2\kappa A_1 - a\xi\kappa}{3}\sin\varphi + \frac{3\mu a^3 - 4a\beta}{8}\cos3\gamma(\cos2\varphi + \cos4\varphi) + \frac{3\mu a^3 + 8a\beta}{8}\sin3\gamma\sin2\varphi + \frac{a^3\mu\cos3\gamma}{8}(\cos6\varphi + 1) \quad (26)$$

$$A_1 = \frac{1}{2}\xi a,\ B_1 = -\frac{3\beta}{2\kappa} - \frac{9\alpha^2(\mu+\tau)}{8\kappa} \quad (27)$$

Similarly, amplitude frequency and phase frequency characteristics equations and curves of 1/3 Harmonic parameter resonance can be shown as Fig. 10 and Fig. 11.

$$\begin{cases}12\beta + 9\alpha^2(\mu+\tau) - 4\xi a\kappa = 0\\ R = T = 0\end{cases} \quad (28)$$

R and T are calculated as

$$R = \frac{\cos3\gamma\sin\gamma}{2240\kappa^3}(27a^5\mu^2 - 147a^3\mu + 140a\beta)(1-4\cos^2\gamma) + \frac{36a\beta\xi(\sin\varphi - 1) - 27\alpha^3\xi(\mu+\tau)}{16\kappa^2} - \frac{\sin3\gamma\cos\gamma}{160\kappa^3}(3a^3\mu + 10a\beta)\left[(4\beta + 6\alpha^2\mu)\sin^2\gamma - \beta + \frac{3\alpha^2\mu}{2}\right] + \frac{\sin3\gamma\cos\gamma}{2240\kappa^3}(105a^3\mu + 280a\beta - 27a^5\mu^2)(4\sin^2\gamma - 1) + \frac{\cos3\gamma\sin\gamma}{40\kappa^3}(3a^3\mu - 4a\beta)\left[\left(4\beta + \frac{15}{2}\alpha^2\mu\right)\sin^2\gamma - 3\beta - \frac{9}{2}\alpha^2\mu\right]$$

$$T = \frac{(3a^2\mu - 4\beta)}{640\kappa^3}\cos3\gamma\cos\gamma\left[(96\beta + 156\alpha^2\mu + 40)\sin^2\gamma - 24\beta - 39\alpha^2\mu - 10\right] + \frac{9a^4\mu^2(1-4\cos^2\gamma)}{560\kappa^3}\sin3\gamma\sin\gamma + \frac{(15a^2\mu + 40\beta)(1-4\cos^2\gamma) + 6a^2\mu}{320\kappa^3}\sin3\gamma\sin\gamma\left[\left(4\beta + \frac{15\alpha^2\mu}{2}\right)\sin^2\gamma - 3\beta - \frac{9\alpha^2\mu}{2}\right] + \frac{3a^2\mu + 8\beta}{32\kappa^3}\sin3\gamma\sin\gamma + \frac{9a^4\mu^2(4\sin^2\gamma - 1)\cos3\gamma\cos\gamma}{4480\kappa^3} - \frac{3[12\beta + 9a^2(\mu+\tau)]^2}{128\kappa^3} + \frac{9a^4(\mu+\tau)^2}{256\kappa^3} - \frac{3\xi^2}{8\kappa}$$

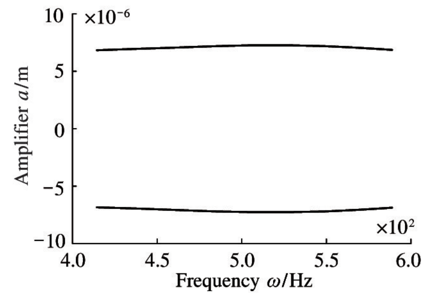

Fig. 10 Amplitude-frequency curve of 1/3 Harmonic parameter

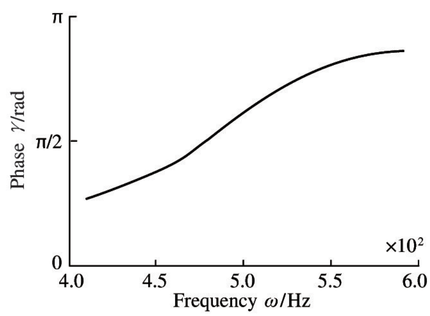

Fig. 11 Phase-frequency curve of 1/3 Harmonic parameter

When the frequency of the current rate is 3/2 times of the natural frequency of MLDSB, 1/3 Harmonic resonance of current varying system occurs accompanied by the resonance condition of multi times frequency. The maximum amplitude reaches 7 μm and reduces slowly with the increase of excitation frequency.

2.4 1/2 Harmonic parameter resonance analysis of current varying system

Similarly, $\lambda = 1/2$ is substituted into Eq. (22) to eliminate secular term of x_1 as follows.

$$(\frac{\kappa}{2})^2(\frac{\partial^2 x_1}{\partial\varphi^2}+x_1) = (\kappa A_1 - \frac{a\xi\kappa}{2} + \frac{a\beta\sin 2\gamma}{2} + \frac{11a^3\mu\sin 2\gamma}{4})\sin\varphi + \alpha[\beta + \kappa B_1 + \frac{3\alpha^2(\mu+\tau)}{4} + \frac{(\beta + a^2\mu)\cos 2\gamma}{2}]\cos\varphi - 2a^3\mu\sin 2\gamma\sin 2\varphi + \frac{3a^3\mu + a\beta}{2}\sin 2\gamma\sin 3\varphi + \frac{5a^3\mu\cos 2\gamma}{16}\cos 5\varphi + \frac{4\alpha^3(\mu+\tau) - (8a\beta + 3a^3\mu)\cos 2\gamma}{16}\cos 3\varphi \tag{29}$$

$$A_1 = \frac{\xi a}{2} - \frac{a\beta\sin 2\gamma}{2\kappa} - \frac{11a^3\mu\sin 2\gamma}{4\kappa}$$
$$B_1 = -\frac{\beta}{\kappa} - \frac{3\alpha^2(\mu+\tau)}{4\kappa} - \frac{(\beta + a^2\mu)\cos 2\gamma}{2\kappa} \tag{30}$$

Similarly, amplitude frequency and phase frequency characteristics equations and curves of 1/2 Harmonic parameter resonance can be shown as Fig. 12 and Fig. 13.

$$\begin{cases}(\frac{2\xi\kappa}{2\beta + 11a^2\mu})^2 + [\frac{4\beta + 3\alpha^2(\mu+\tau)}{2(a^2\mu + \beta)}]^2 = 1 \\ \tan 2\gamma = -\frac{4\kappa\xi(\beta + a^2\mu)}{(2\beta + 11\mu a^2)[4\beta + 3\alpha^2(\mu+\tau)]}\end{cases} \tag{31}$$

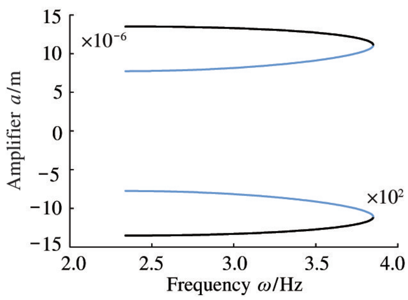

Fig. 12 Amplitude-frequency curve of 1/2 Harmonic parameter

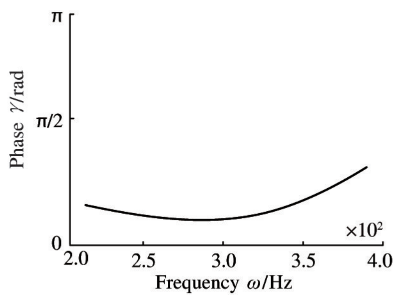

Fig. 13 Phase-frequency curve of 1/2 Harmonic parameter

When the frequency of the current rate is close to the natural frequency of MLDSB, 1/2 Harmonic resonance of current varying system occurs accompanied by the resonance condition of multi times frequency. The maximum amplitude reaches 13 μm and reduces slowly with the increase of excitation frequency.

3 Conclusions

The resonance characteristics of the parameter system of MLDSB are studied. Firstly, Marshall-Duffing equation of the parametric system is established by taking the flow q and the current i as variables respectively. Then, by using the asymptotic method, the occurrence condition and variation rule of the principal, 1/2 Harmonic and 1/3 Harmonic parametric resonance are solved. The results show that the 1/2 Harmonic parametric resonance of flow varying system occurs accompanied by the resonance condition of high frequency. The maximum amplitude reaches 4 μm and reduces slowly with the increase of excitation frequency. The principal, 1/2 Harmonic and 1/3 Harmonic parametric resonance of current varying system occur accompanied by the resonance condition of high frequency. The maximum amplitude reaches 13 μm and reduces slowly with the increase of excitation frequency.

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