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Online prediction of EEG based on KRLST algorithm①

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Abstract

Kernel adaptive algorithm is an extension of adaptive algorithm in nonlinear, and widely used in the field of non-stationary signal processing. But the distribution of classic data sets seems relatively regular and simple in time series. The distribution of the electroencephalograph (EEG) signal is more randomness and non-stationarity, so online prediction of EEG signal can further verify the robustness and applicability of kernel adaptive algorithms. What's more, the purpose of modeling and analyzing the time series of EEG signals is to discover and extract valuable information, and to reveal the internal relations of EEG signals. The time series prediction of EEG plays an important role in EEG time series analysis. In this paper, kernel RLS tracker (KRLST) is presented to online predict the EEG signals of motor imagery and compared with other 13 kernel adaptive algorithms. The experimental results show that KRLST algorithm has the best effect on the brain computer interface (BCI) dataset.

Key words: brain computer interface (BCI), kernel adaptive algorithm, online prediction of electroencephalograph (EEG)

0 Introduction

Time series online prediction is widely used in a variety of fields, such as stock trend prediction[1], real-time traffic flow prediction[2], and online monitoring of medical devices[3] and so on. A lot of articles have been published on common datasets, such as Lorenz[4], chaotic time-series prediction[5], respiratory motion[6] and traffic flow prediction[7]. However the distribution of the signals on those datasets seems relatively regular and simple in time series.

The electroencephalograph (EEG) signals are randomness and non-stationarity[8], which can better test the robustness and applicability of the kernel adaptive algorithms on processing time-varying signals and non-stationarity signals. In addition, the purpose of modeling and analyzing the time series of EEG signals[9] is to discover and extract valuable information contained in the data, and to reveal the internal relations of EEG signals. The time series prediction of EEG plays an important role in EEG time series analysis.

In Ref. [10], the bag-of-wave features were used to learn EEG synchronization patterns for seizure prediction. In Ref. [11], the machine learning approaches were used for seizure prediction from EEG signals. In Ref. [12], the classical deep learning methods such as convolutional neural network (CNN) were used for seizure prediction from EEG signals. In Ref. [13], the DenseNet was used for epileptic seizure prediction from EEG signals. In Ref. [14], a novel method was proposed for seizure prediction from EEG signals by common spatial pattern (CSP) and CNN. Seizure prediction of EEG signals can predict the impending epileptic seizures according to the scalp EEG signals, so as to improve the quality of life. But the EEG prediction in these articles is based on classification of inter-ictal and pre-ictal state, which is the prediction of disease rather than online prediction of EEG

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signal itself. Online prediction of EEG signal itself in time series can reconstruct the missing signal, make the signal smoother and eliminate the abnormal points of the EEG signal.

The basic idea of the online prediction in time series is to use the current and historical sequence for mathematical modeling to find the dynamic dependency relation contained in the time series[15]. In the practical prediction application, most time series are nonlinear.

On the one hand, the kernel adaptive algorithm[16] inherits the advantages of the adaptive algorithm, which can automatically adjust the parameters of the digital filter according to the input signal[17]. On the other hand, it also extends the ability of the adaptive algorithm to solve the nonlinear and non-stationarity signals.

In this paper, the kernel RLS tracker (KRLST) adaptive algorithm is presented to online predict the EEG signal, and compared with the other 13 kernel adaptive algorithms on the brain computer interface (BCI) dataset about motor imagery. It is found that the KRLST algorithm has the best online prediction performance on BCI dataset, that is, the root mean square error (RMSE) of the KRLST algorithm on the all 6 electrodes is the minimum.

1 Online prediction of EEG based on kernel adaptive algorithm

The EEG online prediction in this paper is based on EEG signals at old time points to online predict EEG signals at new time points by using kernel adaptive algorithm. The framework of the algorithm is shown in Fig. 1. According to the different ways of minimizing objective function, kernel adaptive algorithms can be roughly divided into two categories, i. e.

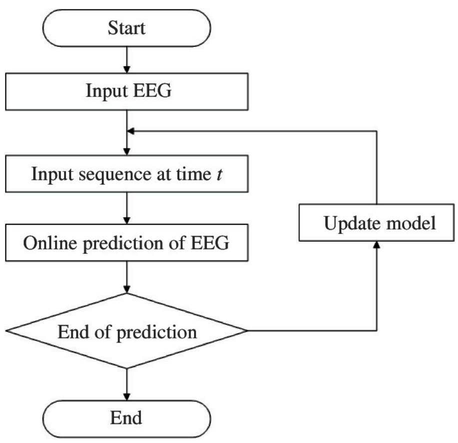


Fig. 1 The framework of online prediction of EEG

the improved least mean squares (LMS) and recursive least squares (RLS) algorithms.

Naive online regularized risk minimization algorithm (NORMA) is a version of kernel-based LMS algorithm which includes regularization[18]. Leaky kernel affine projection algorithm (LKAPA) adds expansion coefficient in each iteration to avoid cost function posed in the conventional empirical risk minimization (ERM)[19]. As the KAPA-3 has a scaling factor, the far past data is scaled down exponentially.

Kernel affine projection (KAP) algorithm proposes a new model simplification standard, which introduces the coherence criterion into sparse dictionary[20]. Kernel-based normalized LMS algorithm (KNLMS) uses a new reduction criterion to replace the sparse process[20]. The increase of the variables is controlled by several parameters, which is a basic quantitative standard of the dictionary in sparse approximation problem. In the time series prediction problem, KNLMS introduces correlation criterion to a new kernel reflection projection algorithm. Kernel affine projection sub-gradient method (KAPSM) generalizes the kernel-based normalized LMS algorithm (KNLMS) and affine projection (AP) algorithm[21]. It has strong convergence under mild conditions. Quantized kernel least mean square (QKLMS) uses quantization instead of sparse method to curb the radial growth of adaptive filtering[22]. The input space is quantized and compressed by updating the nearest center coefficient and using redundant data. Random Fourier feature kernel LMS (RFF-KLMS) uses inner products in finite dimensions to approximate the kernel function[23]. It solves the problem that the computation complexity increases linearly with time. The computational complexity is reduced while maintaining performance.

Multi-kernel normalized LMS algorithm with coherence sparsification (MKNLMS-CS) is an effective adaptive algorithm for nonlinear systems with multi-kernel[24]. It adopts adaptive proximal forward-backward splitting method and introduces the L1 form penalty item. Thus the sparsity of the block adaptive algorithm is improved and effective for non-stationary data. Fixed budget quantized KLMS (QKLMS-FB) is a fixed memory budget QKLMS algorithm and uses significance measure to prune[25]. The least significant center in the dictionary which is the least influence on the whole system is discarded. Probabilistic LMS (PROBLMS) introduces a probability method to improve LMS algorithm which provides a adaptable step-size LMS algorithm based on the different estimation values[26]. In addition, the algorithm also maintains the complexity of standard LMS approximately.

Sliding window kernel RLS (SWKRLS) only selects M samples to model and keeps the latest M samples at each iteration[27]. Extended kernel RLS (E-KRLS) only needs to do inner product operation for input vectors for reproducing kernel Hilbert spaces (RKHS)[28]. The method is effective in nonlinear observation and state models. Fixed budget KRLS (FB-KRLS) is fixed memory budget KRLS algorithm[29]. The algorithm uses a combination strategy that supports merging and pruning, which can learn nonlinear mapping recursively. Compared with sliding windows, the algorithm is not pruning the oldest data points, but pruning the most unimportance data points. It also add the label, with time tracking ability.

2 Online prediction of EEG based on KRLST algorithm

The distribution of the EEG signal is more complex, non-stationary, randomness and the EEG signal may contain noise disturbance. KRLST is a kernel RLS algorithm that can track nonlinear time-varying data, which derived the KRLS from Bayesian[30]. KRLST includes a forgetting factor λ and a budget dependent dictionary size D_{dict} to enhance the ability of tracking complex EEG signals. In order to track non-stationary EEG signals, it provides confidence intervals and add uncertainty module γ_t^2 per each iteration. The EEG signal may contain noise disturbance. In order to improve the stability and generalization ability of the algorithm in tracking EEG signals, the concept of regularization σ_n^2 is strictly introduced into KRLST.

Specific process of EEG online prediction based on KRLST is as follows.

(1) Set the initial parameters of the model, such as μ_1, Σ_1, Q_1, M, λ, and so on.

$$\begin{cases} \mu_1 = \dfrac{y_1 k(x_1, x_1)}{\sigma_n^2 + k(x_1, x_1)} \\ \Sigma_1 = k(x_1, x_1) - \dfrac{k(x_1, x_1)^2}{\sigma_n^2 + k(x_1, x_1)} \\ Q_1 = \dfrac{1}{k(x_1, x_1)} \end{cases} \tag{1}$$

(2) Choose the back to the prior (B2P) forgetting and the B2P forgetting is as follows.

$$\begin{cases} \Sigma_t = \lambda \Sigma_t + (1 - \lambda) K_{t-1} \\ \mu_t = \sqrt{\lambda}\mu_{t-1} \end{cases} \tag{2}$$

(3) Input signal sequence and output signal. Generating input sequence at time t based on the input EEG signal x_t.

$$x_t = [EEG(t), EEG(t+1), \cdots, EEG(t+\tau)] \tag{3}$$

where, $EEG(t)$ is electric value of EEG at time t, τ is delay time. Output label y_t and estimated value $\hat{y}_t$ of EEG at time t are as follows.

$$\begin{cases} y_t = EEG(t + \tau + 1) \\ \widehat{EEG}(t + \tau + 1) = \hat{y}_t \end{cases} \tag{4}$$

(4) Online prediction. According to the input signal x_t at time t, the input signal is mapped to the feature space by kernel function, and the prediction value is calculated through the kernel adaptive algorithm model.

The uncertainty γ_t^2 is as follows.

$$\begin{cases} q_t = Q_{t-1} k_t \\ \gamma_t^2 = k_t - k_t^{\mathrm{T}} q_t \end{cases} \tag{5}$$

The noiseless variance $\hat{\sigma}_{f_t}^2$, predictive mean $\hat{y}_t$, and predictive variance $\hat{\sigma}_{y_t}^2$ are as follows.

$$\begin{cases} \hat{\sigma}_{f_t}^2 = \gamma_t^2 + q_t^{\mathrm{T}} \Sigma_{t-1} q_t \\ \hat{y}_t = q_t^{\mathrm{T}} u_{t-1} \\ \hat{\sigma}_{y_t}^2 = \sigma_n^2 + \hat{\sigma}_{f_t}^2 \end{cases} \tag{6}$$

(5) Update prediction model. If $t < N - \tau$, that is to say, the prediction is not finished, the model is updated according to the deviation between the predicted value and the real value. First, evaluate the sample sequence, and then update the sample dictionary and algorithm subject according to the relevant rules.

$$\begin{cases} \mu_t = \begin{bmatrix} \mu_{t-1} \\ \hat{y} \end{bmatrix} + \dfrac{y_t - \hat{y}_t}{\hat{\sigma}_{y_t}^2} \begin{bmatrix} h_t \\ \hat{\sigma}_{f_t}^2 \end{bmatrix} \\ \Sigma_t = \begin{bmatrix} \Sigma_{t-1} & h_t \\ h_t^T & \hat{\sigma}_{f_t}^2 \end{bmatrix} - \dfrac{1}{\hat{\sigma}_{y_t}^2} \begin{bmatrix} h_t \\ \hat{\sigma}_{f_t}^2 \end{bmatrix} \begin{bmatrix} h_t \\ \hat{\sigma}_{f_t}^2 \end{bmatrix}^{\mathrm{T}} \\ Q_t = \begin{bmatrix} Q_{t-1} & 0 \\ 0^T & 0 \end{bmatrix} - \dfrac{1}{\hat{\sigma}_{y_t}^2} \begin{bmatrix} q_t \\ -1 \end{bmatrix} \begin{bmatrix} q_t \\ -1 \end{bmatrix}^{\mathrm{T}} \end{cases} \tag{7}$$

$\widehat{EEG}$ is online prediction EEG signal estimated by KRLST. The $t + \tau + 1$ in parentheses is the index of $\widehat{EEG}$. Q_t is the value of Q at iteration t. Subscript means at iteration t.

The KRLST algorithm update process is shown in Algorithm 1.

Algorithm 1 The KRLST algorithm update process

Input: The EEG signals x_t;

Output: The online prediction signals $\widehat{EEG}$

1. Initialize μ_1, Σ_1, Q_1 as Eq. (1)
2. Add x_1 as Eq. (2) to dictionary D_{dict}
3. For each $t \in [2, 3, \cdots]$ Do
4. Choose B2P as Eq. (2);
5. Input sequence x_t as Eq. (3) at time t;
6. Update q_t and uncertainty γ_t^2 as Eq. (5);
7. Update $\hat{\sigma}_{f_t}^2$, $\hat{y}_t$, $\hat{\sigma}_{y_t}^2$ as Eq. (6) and $\widehat{EEG}$ as Eq. (4);

```
    Update μ_t, Σ_t as Eq. (7);
    If γ_t^2 < ε Then
        Remove basis t from u_t, Σ_t;
    Else
        Update Q_t as Eq. (7);
        Add basis x_t to dictionary D_dict
        If Number of dictionary > M Then
            Remove basis i from μ_t, Σ_t, Q_t;
            Remove basis x_t from dictionary D_dict;
        End if
    End if
End for
Return EEG^
```

3 Experimental verification

3.1 Evaluation criterion

The RMSE between the predicted signal and the real signal can reflect the deviation degree of the predicted signal and the real signal relatively effectively. It is an important method to evaluate the online prediction performance, and also the comprehensive embodiment of stability and tracking sensitivity. If the algorithm pursues tracking sensitivity unilaterally, the algorithm is easy to over fit, and its stability and generalization performance will be slightly poor. RMSE may be higher before and lower after, and the overall RMSE is slightly higher. If the algorithm pursues stability unilaterally, the tracking sensitivity may be lost and it will take a long time to reach a smaller RMSE value. Although the RMSE value will not vibrate obviously, the overall deviation between the predicted value and the actual signal will be slightly larger, and the overall RMSE is also slightly higher. Only when the stability of the algorithm is relatively good and the tracking sensitivity is fast, the overall RMSE will be relatively low. The RMSE is defined as

$$RMSE = \sqrt{\frac{1}{N}\sum_{n=1}^{N}[\widehat{EEG}(i) - EEG(i)]^2} \quad (8)$$

where, $\widehat{EEG}$ is the predicted value evaluated by various kernel adaptive algorithms. EEG is true value of EEG. N is the number of samples, which is 896 in this paper.

3.2 Dataset description

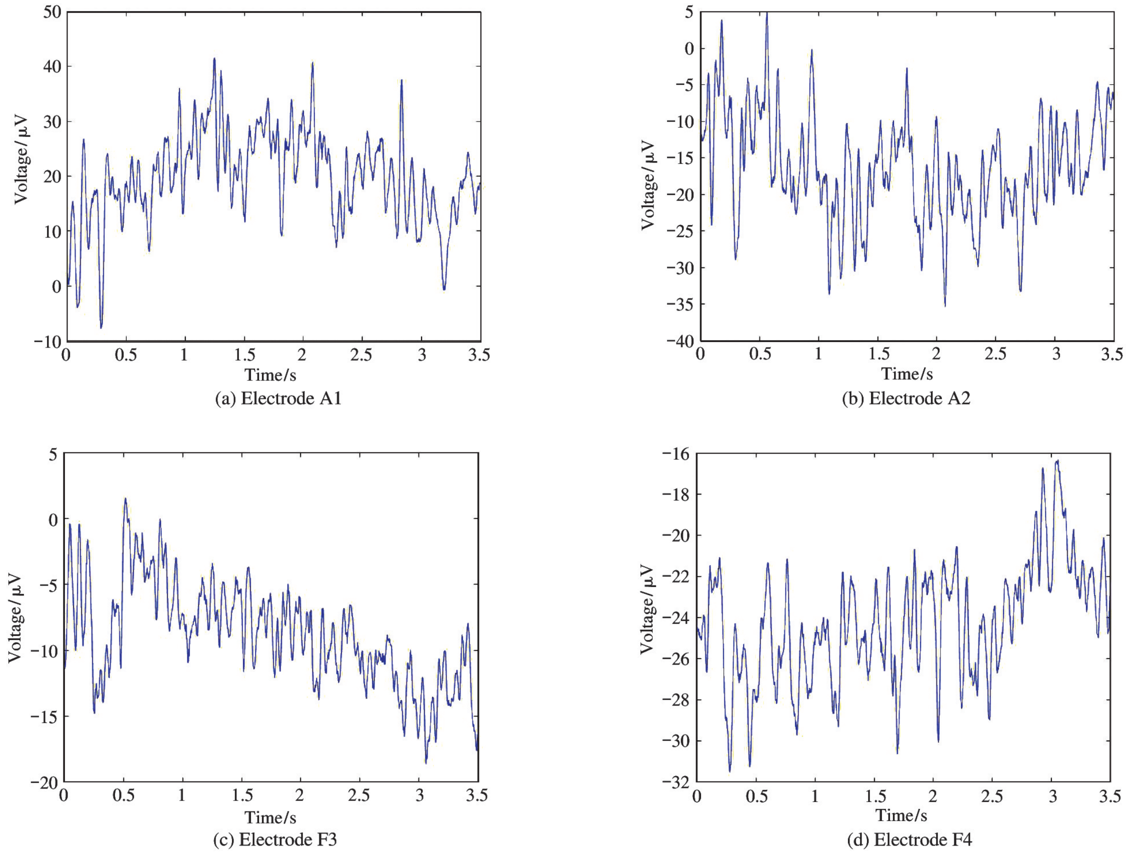
Adopting the second session of the BCI competition II Ia data set[31], and the subjects of the dataset are healthy. The data is collected in the cerebral cortex of the subjects. The task of motor imagery is to control the motion of the computer screen cursor up and down and record the potential value of the corresponding electrode in the cerebral cortex. The duration of each sample is 6 s. In these 6 s, the first 1 s time is rest, the 1.5 s time in the middle is a reminder of the motor imagery, and the post 3.5 s time is the information feedback. Among them, the post 3.5 s is recorded by 256 Hz with 6 electrodes as samples. There are 561 samples, and each sample contains 6 electrode sample segments and 896 sample points per segment.

Fig. 2 contains 4 visual sub figures of each electrode which is randomly extracted in 561 samples. The subtitle is composed of electrode number and sample number. Fig. 3 is the visualization of some common online prediction datasets, such as Lorenz attractor, Mackey-Glass chaotic time series, respiratory motion and Santa Fe laser time series. It can be seen from the figure that the distribution of 4 commonly used datasets contains some regularity and seems relatively simple on the time series. But the randomness of the data distribution is strong in the time series between inside the single electrode of single sample and the different electrodes of different samples in BCI dataset. It can better test the ability of the kernel adaptive algorithm on processing time varying signals and unstable signals.

Because of the difference of brain function, gray matter, white matter and neurotransmitter, the EEG signals at different electrodes will be significantly different, so the prediction results of EEG signals at different electrodes are compared respectively. The competition dataset used in this paper provides a total of six electrode signals, the results of six electrodes are compared. In order to eliminate the influence of individual differences on the results of the algorithm, the RMSE of all 561 samples is chosen in the dataset to do the average again.

3.3 Contrast experiment

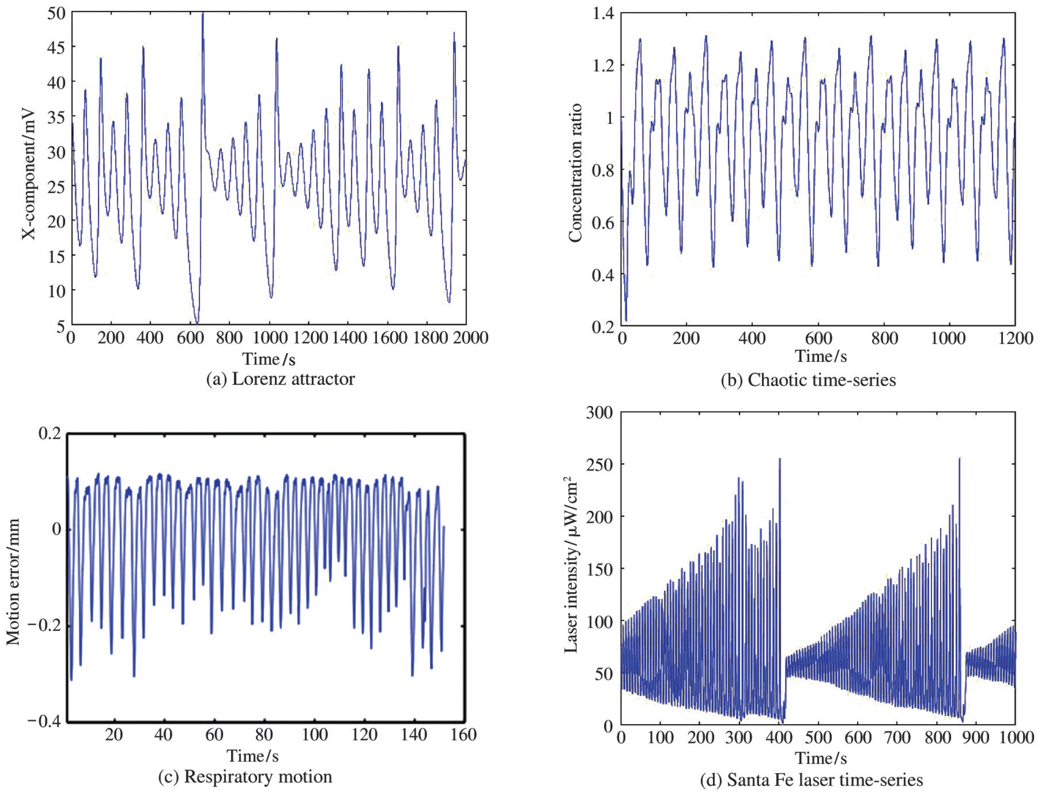
In this paper, 561 samples are randomly disordered to generate dimensions data, and then 14 kernel adaptive algorithms are applied to online predict the EEG signals. The parameters of different algorithms are not exactly the same, but the variation ranges of some key parameters with similar function are set as consistent as possible, such as M, λ, and η in Table 1. Selecting the optimal key parameters of each algorithm in the reference range is similar to the parameter optimization. Other secondary parameters refer to the default optimized parameters in their citations, which is similar to the transfer learning module that directly introduces optimized models and parameters from other fields in deep learning. The key parameters of each algorithm in



(a) Electrode A1 (b) Electrode A2

(c) Electrode F3 (d) Electrode F4

Fig. 2 The visualization of different electrodes on BCI dataset



(a) Lorenz attractor (b) Chaotic time-series

(c) Respiratory motion (d) Santa Fe laser time-series

Fig. 3 The visualization of some common online prediction datasets this paper have been given in Table 1, and have been adjusted adaptively within the range of parameters according to their algorithms. The settings of other secondary parameters can be found in the citations, so as to facilitate other researchers to follow-up experimental reproduction and algorithm improvement. The average RMSE value of 561 samples of each electrode is calculated, and the predictive performance of the adaptive algorithm on each electrode is obtained.

Fig. 4 is the line diagram of mean RMSE values by different kernel adaptive algorithms on EEG online prediction of each electrode. Fig. 5 is histogram of mean RMSE values by different kernel adaptive algorithms on EEG online prediction of all electrodes. The vertical coordinate in Fig. 4 and Fig. 5 is voltage value. In Table 2, the horizontal direction represents different kernel adaptive algorithms, and the vertical direction represents different electrodes in the motor imagery dataset. The value of Table 2 is the corresponding average RMSE value. As shown in Table 2, compared with the other 13 algorithms, the KRLST algorithm has the lowest average RMSE value of all the samples on the 6 electrodes of the motor imagery dataset. So the estimated value of KRLST is the closest to the real EEG signal, and its performance is best.

Table 1 The parameters setting of different algorithms

Algorithms	Parameters setting
SWKRLS	$c = 0.01$, $\lambda = 0.999$, $M = [3,5,7,10,20,30,50,100, 200,400]$
FBKRLS	$\lambda = 0.999$, $M = [3,5,7,10,20,30,50,100,200,400]$
KRLST	$sn_2 = 0.01$, $\lambda = 0.999$, $M = [3,5,7,10,20,30,50,100,200,400]$
EXKRLS	$\beta = 0.995$, $M = [3,5,7,10,20,30,50,100,200,400]$
NORMA	$\lambda = [0.001,0.01,0.1]$
PROBLMS	$\lambda = [0.001,0.01,0.1]$
QKLMS _ FB	$\eta = 0.999$, $M = [3,5,7,10,20,30,50,100,200,400]$
RFFKLMS	$\eta = 0.999$, $\mu = 0.9$, $D = [3,5,7,10,20,30,50,100,200,400]$
KAP	$\eta = 0.999$, $\mu = 0.9$, $P = [3,5,7,10,20,30,50,100,200,400]$
KNLMS	$\eta = 0.999$, $\mu_0 = [0.1,0.3,0.5,0.7,0.8,0.85,0.9,0.95]$
KAPSM	$M = 200$, $\mu = [0.1,0.3,0.5,0.7,0.8,0.9,2,3,5,10]$
QKLMS	$\eta = 0.999$, $\varepsilon = [0.1,0.3,0.5,0.7,0.8,0.9,2,3,5,10]$
MKNLMS _ CS	$\eta = 0.999$, $rho = 0.01$, $\delta = [0.1,0.3,0.5,0.7,0.8,0.85,0.9,0.95]$
LKAPA	$P = [10,20,25,30,35,40,45,50,55,60]$

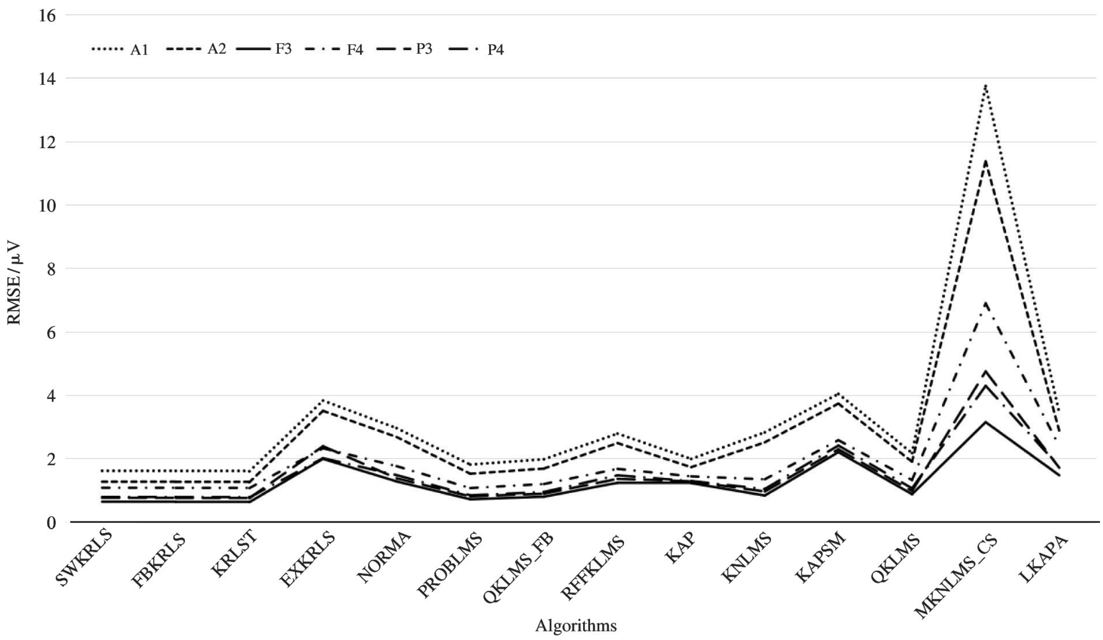


Fig. 4 Line diagram of mean RMSE values by different kernel adaptive algorithms on EEG online prediction of each electrode

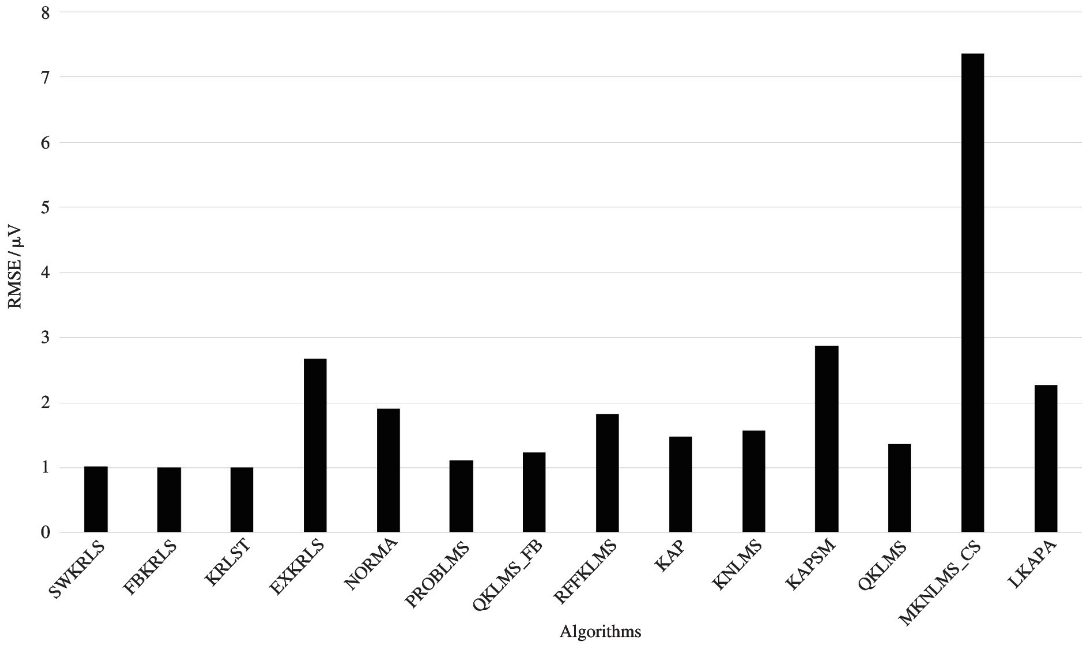


Fig. 5 Histogram of mean RMSE values by different kernel adaptive algorithms on EEG online prediction of all electrodes

Table 2 Comparison of mean RMSE values of different kernel adaptive algorithms on EEG online prediction (unit: μV)

Algorithms	A1	A2	F3	F4	P3	P4
SWKRLS	1.6058	1.2610	0.6298	1.0696	0.7527	0.7783
FBKRLS	1.6036	1.2569	0.6246	1.0666	0.7482	0.7757
KRLST	**1.5897**	**1.2429**	**0.6157**	**1.0587**	**0.7396**	**0.7659**
EXKRLS	3.8193	3.4968	1.9819	2.3146	2.3817	2.0037
NORMA	2.9507	2.6714	1.2697	1.7505	1.3686	1.4689
PROBLMS	1.8014	1.5128	0.7070	1.0611	0.7973	0.8313
QKLMS_FB	1.9698	1.6757	0.7883	1.1889	0.8832	0.9363
RFFKLMS	2.7778	2.4799	1.2269	1.6659	1.3495	1.4635
KAP	1.9803	1.7183	1.2179	1.4293	1.2731	1.2802
KNLMS	2.8184	2.5031	0.8251	1.3371	0.9520	1.0308
KAPSM	4.0301	3.7246	2.1936	2.5704	2.2815	2.4021
QKLMS	2.1627	1.8911	0.8683	1.3138	0.9675	1.0527
MKNLMS_CS	13.7392	11.3692	3.1393	6.8847	4.7397	4.2866
LKAPA	3.4341	2.8664	1.4639	2.3794	1.6996	1.7374

4 Conclusion

The data distribution in some common online prediction datasets, such as Lorenz, Chaotic time-series, Respiratory motion, and so on, seems relatively regular and simpler. However, the data distribution of EEG is more random, which can better test the ability to process time-varying signals and unstable signals of the kernel adaptive algorithm. What's more, the time series prediction of EEG is necessary and important to discover and extract valuable information, and to reveal the internal relations of EEG signals. KRLST algorithm is presented to online predict the EEG, and compared with the other 13 kinds of kernel adaptive algorithms. The experimental results show that KRLST algorithm has the best effect of online prediction on the BCI dataset, and its average RMSE value is the smallest.

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