

# Decision-feedback subset aided multiple-symbol differential detection<sup>①</sup>

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## Abstract

In view of the inaccuracy of the estimated symbols on the edge of the observation window, a decision-feedback subset aided multiple-symbol differential detection (MSDD) framework, dubbed DF-S-MSDD, is proposed in ultra-wideband impulse radio (UWB-IR) system with differential space-time block-code (DSTBC) modulation. Specifically, motivated by the decision-feedback aided MSDD (DF-MSDD), a subset of the decision-feedback symbols is selected, and the optimal symbols are preserved, and then all the remaining symbols are optimized. Furthermore, the simulations validate that the proposed DF-S-MSDD provides solid bit error-rate performance with a low complexity in UWB-IR system with DSTBC modulation.

**Key words:** multiple-symbol differential detection (MSDD), decision-feedback (DF), decision-feedback subset, ultra-wideband impulse radio (UWB-IR), differential space-time block-code (DSTBC)

## 0 Introduction

Coherent detection is capable of achieving performance improvement with accurate channel state information (CSI)<sup>[1]</sup>. However, collecting the perfect CSI is a demanding task. By contrast, the noncoherent receivers mitigate the problem of estimating CSI. Therefore, transmitted reference and differential detection get attention as noncoherent schemes in the single antenna system<sup>[2,4]</sup>. Unfortunately, there is a wastage of the transmit power with the transmitted-reference, and the differential detection leads to bit error rate (BER) performance loss because the current symbol is detected using a noisy template of the received signal<sup>[5,6]</sup>. Furthermore, multiple-symbol differential detection (MSDD) receives much research attention<sup>[7-9]</sup>.

As a milestone for noncoherent detection, differential space time block-codes (DSTBC) becomes attractive<sup>[10,11]</sup>, which is capable of improving the BER performance significantly compared to the single-antenna system. Furthermore, generalized likelihood ratio test based MSDD (GLRT-MSDD) and sphere decoding based MSDD have been conceived for the system<sup>[12]</sup>. However, the aforementioned MSDD algorithms impose exponentially increasing complexity with the number of the observation window. Thus, the GLRT-MSDD and the sphere decoding based MSDD are impractical when

the number of the observation window increases. Decision-feedback based MSDD (DF-MSDD) represents the computationally efficient algorithm with a low complexity<sup>[13]</sup>. On the other hand, due to the inaccuracy of the estimated symbols on the edge of the observation window, there leaves a gap in terms of the BER performance between DF-MSDD and GLRT-MSDD.

Aiming to solve the problem of the performance loss caused by inaccurate detection on the edge of the DF-MSDD, in this work, a decision-feedback subset aided MSDD (DF-S-MSDD) is developed in the system with DSTBC modulation. More specifically, based on the DF-MSDD, a subset of decision-feedback symbols is employed, and the remaining symbols are optimized with GLRT-MSDD. The proposed DF-S-MSDD can simplify the classic exhaustive search as a detection with a lower computational complexity. Another benefit of the DF-S-MSDD is that, by getting rid of the inaccurate decision-feedback symbols on the edge of the observation window, it also enjoys a better BER performance when compared to the DF-MSDD. In general, the proposed DF-S-MSDD strikes an appealing performance-versus-complexity trade off.

It is worth emphasizing that the proposed DF-S-MSDD can be generalized in many noncoherent transmission with DSTBC modulation. Particularly, the ultra-wideband impulse radio (UWB-IR) signal contains lots of dense multi-paths components<sup>[14]</sup>. Therefore,

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using the optimal coherent detector will increase the difficulty and cost of system implementation. Thus, noncoherent schemes become inevitable in UWB-IR system. Relying on this technique, the system description and simulation experiments are discussed in the UWB-IR system with DSTBC modulation.

The structure of this paper is formulated as follows. The system description is offered in Section 1. In Section 2, the GLRT-MSDD and DF-MSDD are introduced. The proposed DF-S-MSDD is described in details in Section 3. Considering the practicability, the detection performance of the DF-S-MSDD is discussed by simulations in Section 4. Finally, conclusion is shown in Section 5.

Notations: lower-case ( upper-case ) boldface symbols represent vectors (matrices);  $()^T$  and  $\text{Tr}()$  denote the transpose and the trace of a matrix, respectively;  $*$  stands for convolution;  $\delta(t)$  represents the Dirac delta function.

## 1 System description

In this section, the multiple-input multiple output (MIMO) system description will be introduced for point-to-point UWB system with DSTBC modulation. The transmitter is equipped with  $T(T > 1)$  transmit antennas, the receiver is equipped with  $R(R > 1)$  receive antennas. At the transmitter,  $S$  denotes the data rate,  $TS$  bits map to  $T \times T$  unitary matrices  $\mathbf{C}_i$ , which is the  $i$ -th information symbol. With the aid of differential encoding, the transmission symbol is given as

$$\mathbf{G}_{i+1} = \mathbf{G}_i \mathbf{C}_{i+1} \quad (1)$$

where,  $i = 0, 1, \dots, M-1$ , and  $M$  denotes the total number of transmission symbols. The DSTBC information symbols are selected from a codeword set as  $\mathbf{C}_i \in \Omega$ .  $T \times T$  matrices  $\mathbf{G}_i$  are transmitted from  $T$  transmit antennas in  $T$  successive intervals. It is noted that each codeword symbol employed has to be a unitary matrix. When the unitary matrices are designed according to the DSTBC codex of Refs[15-17], the proposed MSDDs are applicable to the system where the number of antennas is more than 2. In order to facilitate the encoding scheme, the transmit antenna is set to  $T = 2$  in the following. Each DSTBC symbol maps onto the information-bearing DSTBC symbol<sup>[18]</sup>, and it belongs to the set  $\Omega = \{\mathbf{C}^0, \mathbf{C}^1, \mathbf{C}^2, \mathbf{C}^3\}$ . The corresponding rule for bits information and DSTBC symbols are given as follows.  $00 \rightarrow \mathbf{C}^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ ,  $01 \rightarrow \mathbf{C}^1 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ ,  $10 \rightarrow \mathbf{C}^2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ ,  $11 \rightarrow \mathbf{C}^3 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ . The ref-

erence symbol is  $\mathbf{G}_0 = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$ . The  $m$ -th row,  $n$ -th column entry of  $\mathbf{G}_i$  is represented as  $g_{m, 2i+n-1}$  with  $m = 1, 2$ ;  $n = 1, 2$ . For the  $m$ -th antenna, the transmitted signal is given by

$$s_m(t) = \sqrt{\frac{E_b}{2}} \sum_{i=0}^{M-1} \sum_{n=1}^2 g_{m, 2i+n-1} \omega(t - (n-1)T_f - iT_s) \quad (2)$$

where,  $t$  is time parameter,  $\omega(t)$  represents the mono-cycle pulse with duration  $T_\omega$ ,  $T_f$  is the frame duration, the duration for transmitting a symbol is  $T_s = 2T_f$ , which indicates that 2 frames are needed to transmit one information symbol,  $E_b$  is the energy for transmitting one bit. To facilitate the demonstration,  $j = 2i + n - 1$  is brought, and  $g_{m, 2i+n-1}$  is simplified as  $g_{m, j}$  correspondingly. Then

$$\begin{aligned} s_m(t) &= \sqrt{\frac{E_b}{2}} \sum_{i=0}^{M-1} \sum_{n=1}^2 g_{m, 2i+n-1} \omega(t - (2i + n - 1)T_f) \\ &= \sqrt{\frac{E_b}{2}} \sum_{j=0}^{2M-1} g_{m, j} \omega(t - jT_f) \end{aligned} \quad (3)$$

The channel impulse response between the  $m$ -th transmit antenna and the  $r$ -th ( $1 \leq r \leq R$ ) receive antenna is given by

$$h_{m,r}(t) = \sum_{l=1}^{L_{m,r}} \alpha_l^{m,r} \delta(t - \tau_l^{m,r}) \quad (4)$$

where,  $L_{m,r}$  represents the total number of propagation paths,  $\alpha_l^{m,r}$  and  $\tau_l^{m,r}$  denote the path-gain and the delay of the  $l$ -th path, respectively. Correspondingly, the overall channel response between the  $m$ -th transmit antenna and the  $r$ -th receive antenna is formulated as

$$\rho_{m,r}(t) = \omega(t) \times h_{m,r}(t) = \sum_{l=1}^{L_{m,r}} \alpha_l^{m,r} \omega(t - \tau_l^{m,r}) \quad (5)$$

As a result, the received signal at the  $r$ -th receive antenna can be expressed as

$$\begin{aligned} y_r(t) &= \sum_{m=1}^2 s_m(t) \times h_{m,r}(t) + n_r(t) \\ &= \sqrt{\frac{E_b}{2}} \sum_{m=1}^2 \sum_{j=0}^{2M-1} g_{m, j} \rho_{m,r}(t - jT_f) + n_r(t) \end{aligned} \quad (6)$$

where  $n_r(t)$  is the additive white Gaussian noise (AWGN), its mean is zero and power spectral density is  $N_0/2$  with two-sided. To help understand and implement the proposed algorithms, the received signal from  $R$  receive antenna can be rewritten with matrix as

$$\mathbf{y}_i(t) = \begin{pmatrix} y_1(t + 2iT) & y_1(t + 2iT + T) \\ \vdots & \vdots \\ y_R(t + 2iT) & y_R(t + 2iT + T) \end{pmatrix} \quad (7)$$

The noncoherent transmission is designed under

the the assumption, that the channel is constant during  $T$  symbols intervals<sup>[12]</sup>. Based on the assumption and the received signal in Eq. (7), MSDDs will be formulated in the UWB system with DSTBC modulation.

## 2 GLRT-MSDD and DF-MSDD

It is assumed that the number of the observation window is  $N$  DSTBC symbols. The target is how to detect  $N - 1$  information-bearing symbols jointly from the  $N$  received symbols. As such, the relationship is investigated between the information symbols  $C_i$  and the received symbols. Firstly,  $N - 1$  information symbols are expressed as the set  $\mathcal{C} = [C_1, C_2, \dots, C_{N-1}]$ . Furthermore,  $\mathcal{C}$  will be detected from the received signal  $\{y_r(t)\}$  of  $N$  observation window, where  $0 < t \leq NT_s$ . When  $y_r(t)$  is obtained, the signal matrices will be combined with  $R$  received signal as Eq. (7). According to the GLRT criterion<sup>[8]</sup>,  $\mathcal{C}$  can be determined with Ref. [12].

$$\hat{\mathcal{C}} = \underset{\tilde{\mathcal{C}} \in \Omega^{N-1}}{\operatorname{argmax}} \left\{ \sum_{\beta=1}^{N-1} \sum_{\gamma=0}^{\beta-1} \operatorname{Tr} \left[ \left( \prod_{i=\gamma+1}^{\beta} \tilde{C}_i \right) \mathcal{Q}_{\beta,\gamma} \right] \right\} \quad (8)$$

where  $\mathcal{Q}_{\beta,\gamma}$  is the correlation matrix received from  $R$  receive antenna, its entries are the correlation function of the  $\beta$ -th and the  $\gamma$ -th received signal expressed as

$$\mathcal{Q}_{\beta,\gamma} = \sum_{\tau=1}^R \left( \int_0^{T_i} y_{\beta}^T(t) y_{\gamma}(t) dt \right) \quad (9)$$

where  $T_i$  is the integration interval. By GLRT-MSDD in Eq. (8),  $N - 1$  consecutive symbols will be estimated according to the  $N$  received symbols. It can be seen that GLRT-MSDD in Eq. (8) is an exhaustive search algorithm. Inspired by the decision-feedback algorithm, instead of optimizing all the symbols over a observation window, the previous decision symbols are feedback and implemented into the GLRT-MSDD metric. For example, in the observation window of  $[\tilde{C}_{\beta-N+1}, \dots, \tilde{C}_{\beta-1}, \tilde{C}_{\beta}]$ , the previous decision-feedback symbols, i. e.,  $[\tilde{C}_{\beta-N+1}, \tilde{C}_{\beta-N+2}, \dots, \tilde{C}_{\beta-1}]$  are substituted into Eq. (8), then DF-MSDD is given by Ref. [11].

$$\hat{C}_{\beta} = \arg \max_{C_{\beta} \in \Omega} \left\{ \sum_{\gamma=\beta-N+1}^{\beta-1} \operatorname{Tr} \left[ \left( \prod_{i=\gamma+1}^{\beta-1} \tilde{C}_i \right) \tilde{C}_{\beta} \mathcal{Q}_{\beta,\gamma} \right] \right\} \quad (10)$$

According to the DF-MSDD given in Eq. (10), it is obvious that only a codeword symbol is detected in a observation window. Thus, when compared to the GLRT-MSDD, the computational complexity of DF-MSDD will be reduced significantly. On the other hand, it is pointed out that<sup>[19]</sup>, the reliability of the estimated symbols in the middle of the observation window is higher than that at the edge. Thus, for the DF-MSDD, the error detection on the edge of the observation window results in performance loss. Therefore, DF-MSDD is mod-

ified, and DF-S-MSDD is proposed in the next section.

## 3 DF-S-MSDD in UWB system

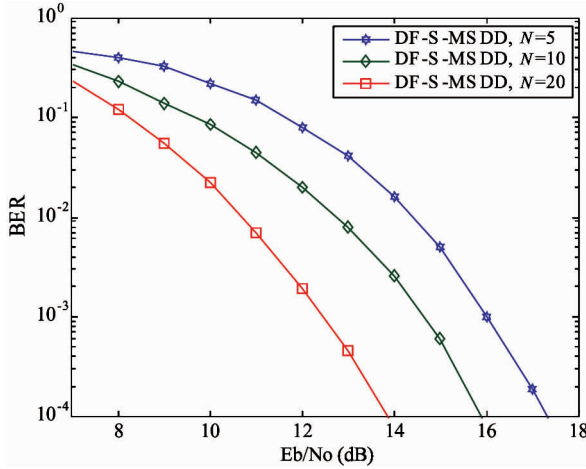
In the GLRT-MSDD,  $N - 1$  information symbols can be estimated from the observation of  $N$  symbols; for the DF-MSDD, the previous  $N - 2$  estimated symbols are feedback, and then the last symbol is detected. In the following, the DF-S-MSDD will be formulated. For the DF-S-MSDD, the previous  $N - 2 - \iota$  symbols are feedback, and the optimization is employed over the last  $\iota + 1$  symbols with GLRT-MSDD. Moreover, in order to reduce the performance loss caused by the inaccuracy at the edge of the observation, instead of returning all the  $\iota + 1$  decision symbols but only  $\iota + 1 - \zeta$  symbols, and the remaining  $\zeta$  detection symbols are discarded at the edge of the observation. Accordingly, the sampling architecture for the DF-S-MSDD is designed. Note that the observation window slides forward  $\iota + 1 - \zeta$  symbol durations each time. The proposed sliding mechanism is particularly suitable for extracting the cross-correlation information among different blocks.

## 4 Simulations and analysis

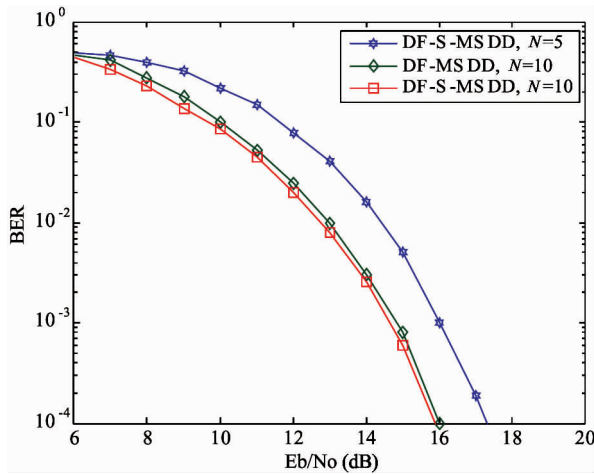
In this section, Monte-Carlo simulations are carried out to validate the advantages of the proposed non-coherent transmission in the UWB system with DSTBC. The channel is given as the IEEE 802.15.3a CM2 model<sup>[16]</sup>. The monocycle waveform  $\omega(t)$  is  $\omega(t) = [1 - 4\pi(t/T_{\omega})^2] \exp[-2\pi(t/T_{\omega})^2]$ , where the pulse duration is  $T_{\omega} = 0.287$  ns. The frame duration is  $T_f = 80$  ns, the maximum excess delay of the channel is chosen as  $T_n = 40$  ns, where  $T_f > T_n$  eliminates the inter-symbol interference. The transmit antenna is set to  $T = 2$ . Furthermore, when the receive antenna is  $R = 1$ ,  $\iota = 1$ ,  $\zeta = 1$ , the BER performance of the DF-S-MSDD is demonstrated. Specifically, as shown in Fig. 1, as the observation window increases from 5 to 10 and 20, the BER performance of the proposed DF-S-MSDD has been improving. When the size of observation window is  $M = 10$ , the SNR is 9 - 10, as shown in Fig. 2, the BER performance of the DF-MSDD is less than DF-S-MSDD about 0.3 dB. This is due to the accurate detection at the edge of the observation window for the DF-S-MSDD.

## 5 Conclusion

In contrast to the existing DF-MSDD in UWB system with DSTBC, the proposed DF-S-MSDD generates the performance gain by discarding the symbols at the



**Fig. 1** BER performance of the proposed DF-S-MSDD in the UWB system with DSTBC



**Fig. 2** BER performance comparison of the proposed DF-S-MSDD and DF-MSDD in the UWB system

edge of the observation window. On the other hand, DF-S-MSDD requires a low computational complexity by exploiting the decision-feedback. Finally, Monte-Carlo simulations confirm that DF-S-MSDD provides the BER performance better than the DF-MSDD benchmark regardless of the number of the observation window, whereas with a reasonable complexity.

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