

Outage performance of cognitive multisource multidestination relay networks with imperfect channel state information and interference from primary transmitter^①

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Abstract

Given imperfect channel state information (CSI) and considering the interference from the primary transmitter, an underlay cognitive multisource multidestination relay network is proposed. A closed-form exact outage probability and asymptotic outage probability are derived for the secondary system of the network. The results show that the outage probability is influenced by the source and destination number, the CSI imperfection as well as the interference from the primary transmitter, while the diversity order is independent of the CSI imperfection and the interference from the primary transmitter, yet it is equal to the minimum of the source and destination number. Moreover, extensive simulations are conducted with different system parameters to verify the theoretical analysis.

Key words: outage performance, cognitive network, channel state information (CSI), primary transmitter

0 Introduction

As the demand for wireless services has grown exponentially over last two decades, the availability of prime wireless spectrum has become severely limited^[1]. As a promising technology, cognitive radio (CR) has emerged as the solution for the spectrum scarcity in next generation wireless networks. There are mainly three CR systems: i. e., underlay, overlay as well as interweave. In the underlay CR system, the secondary system can access the licensed spectrum allocated to the primary system under the premise that the interference from the secondary system to primary system is below the interference temperature; in the overlay system, CR utilizes complicated signal processing techniques to maintain or improve performance of the primary system and interweave refers to that CR exploits the spectrum holes (unoccupied frequency bands) opportunistically to communicate without disrupting primary transmissions^[2].

Another promising technology in wireless networks is cooperative relay, which creates a virtual antenna array among cooperative nodes to combat multipath fa-

ding, and could be leveraged to achieve transmission diversity in wireless communications^[3]. There are 2 main relaying protocols in cooperative relay systems, namely, amplify-and-forward (AF) and decode-and-forward (DF). The AF relay amplifies received signals and then forwards them to the destination, while the DF relay decodes and re-encodes received signals before sending them to the destination.

Recall that in underlay CR networks, the secondary system is allowed to share the licensed spectrum of the primary system as long as the induced interference from the secondary system to the primary system is below the interference temperature. That is to say, to guarantee the quality of service (QoS) of the primary system, the transmission power of the secondary system should be confined strictly. In this case, however, the performance of the secondary system cannot be guaranteed. The cognitive relay system, i. e., the combination of CR and cooperative relaying, has been proposed as a candidate to improve the performance of the secondary system^[4]. Ref. [5] studied the performance of cognitive AF relay networks with a best relay selection strategy. The outage performance of cognitive multisource relay network with direct link (DL) for AF was

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investigated in Ref. [6], and that of cognitive multirelay multidestination with DL for DF was studied in Ref. [7]. However, the aforementioned work did not explicitly take into account the interference from the primary transmitter to secondary system, which has a great impact on the performance of the secondary system. The interference from the primary transmitter to secondary system in the cognitive relay system was considered in Ref. [8] for AF, and in Refs [9-13] for DF. In Ref. [8], the exact as well as the asymptotic outage probability of the secondary system with the interference from primary transmitter were derived. The outage performance of the cognitive DF network without and with DL were investigated in Ref. [9] and Ref. [10], respectively. The authors in Ref. [11] and Ref. [12] studied the outage performance of cognitive multirelay networks without DL, while the authors in Ref. [13] investigated that with DL. All aforementioned work assumed that the perfect channel state information (CSI) was available. In practical networks, however, due to the feedback delay, channel estimation errors as well as some other factors, the perfect CSI is hardly available. The authors in Ref. [14] stated that under the condition of imperfect CSI, the probability of guaranteeing the QoS of primary system could only reach 0.5. Therefore, the transmission power of secondary system should be adjusted with the channel variation. An alternative of the tolerate interference temperature was also studied in Ref. [14].

In this paper, given imperfect CSI and considering the interference from the primary transmitter, the outage performance for an underlay cognitive multi-source multidestination relay network (denoted as MSMD for simplicity) is studied. The closed-form expressions of both the exact and asymptotic outage probabilities for the secondary system of the MSMD are derived respectively, by exploiting the correlations among the received signal-to-interference-plus-noise ratios (SINRs) caused by multisource and multidestination. Extensive simulations are conducted with different system parameters (the source and destination number, the CSI imperfection, as well as the interference from the primary transmitter on the secondary system) to verify the theoretical analysis.

1 System model

As shown in Fig. 1, MSMD with the interference from the primary transmitter is considered. The network involves both the primary and the secondary system, and the secondary system shares the same frequency band with the primary one (composed of the

primary transmitter PT and the primary receiver PR). Specifically, the secondary system is a cooperative relay communication system where one of the source nodes S_n ($1 \leq n \leq N$) intends to communicate with one of the destination node D_m ($1 \leq m \leq M$) by the aid of relay R . It is assumed that there is no direct link between S_n and D_m due to severe channel fading. It is also assumed that each node is equipped with one single antenna and operates in the half-duplex mode. Moreover, the whole communication procedure is divided into two time slots. In the first time slot, the selected best source S_n with maximum SINR in the first hop sends its signal to R . Then, in the second time slot, R decodes, re-encodes and forwards its signal to D_m with maximum SINR in the second hop. In addition, channels between any 2 nodes are subject to independent and identically distributed (i. i. d.) zero-mean Rayleigh flat fading with unit variance. The instantaneous channel fading coefficient between any 2 nodes x and y is denoted as $h_{x,y}$, and the corresponding channel gain turns out to be $|h_{x,y}|^2$, which is exponentially distributed with parameter $\lambda_{x,y}$. The noises at receivers are zero mean Gaussian random variables with variance N_0 .

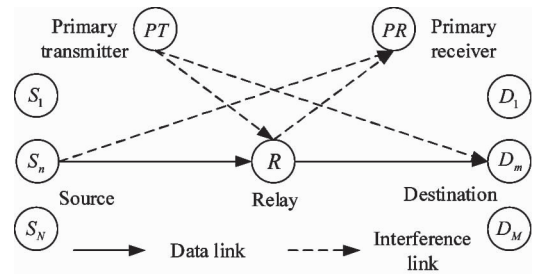


Fig. 1 System model

In real-world networks, due to the transmission delay and measurement error, perfect CSI is hardly available^[14]. As in Refs [14] and [15], a common equation used to reflect the imperfection of the CSI can be modeled as

$$\hat{h}_{x,y} = \rho h_{x,y} + \sqrt{1 - \rho^2} \varepsilon \quad (1)$$

where $h_{x,y}$ denotes the ideal channel coefficients between x and y , and $\hat{h}_{x,y}$ is the channel estimation available at the secondary system, which is independent of $h_{x,y}$. ε is a Gaussian random variable with zero mean and variance of $\frac{1}{\lambda_{x,y}}$. The correlation coefficient ρ ($0 \leq \rho \leq 1$) is a constant reflecting the degree of imperfections of CSI estimations. Note that the channel gain of $\hat{h}_{x,y}$, i. e., $|\hat{h}_{x,y}|^2$ is also exponentially distributed with parameter $\lambda_{x,y}$. Specifically, similar to Ref. [14],

CSI among the secondary system is assumed to be perfect, while that CSI between the primary and secondary systems is imperfect. Note that, in order to obtain the transmission power of the secondary source and relay, $h_{S_n, PR}$ and $h_{R, PR}$ are needed to be estimated by the secondary system. Therefore, the imperfect estimation of $h_{S_n, PR}$ and $h_{R, PR}$ can be modeled as

$$\begin{aligned}\hat{h}_{S_n, PR} &= \rho_1 h_{S_n, PR} + \sqrt{1 - \rho_1^2} \varepsilon \\ \hat{h}_{R, PR} &= \rho_2 h_{R, PR} + \sqrt{1 - \rho_2^2} \varepsilon\end{aligned}\quad (2)$$

Although the PT - R link and the PT - D link are assumed to be imperfect, the actual interference power from the primary transmitter to the secondary relay and destination are $p_P |h_{PT, R}|^2$ and $p_P |h_{PT, D_m}|^2$, respectively.

In order to guarantee the performance of the primary system, the interference power from the secondary system cannot exceed the interference temperature Q . It can be easily achieved by setting the maximum transmission power of S_n and R as $p_{S_n} = \frac{Q}{|h_{S_n, PR}|^2}$ and

$p_R = \frac{Q}{|h_{R, PR}|^2}$, respectively, assuming that perfect CSI is available. That is, the interference from S_n and R to PR are $I_{S_n, PR} = p_{S_n} |h_{S_n, PR}|^2 = \frac{Q}{|h_{S_n, PR}|^2} |h_{S_n, PR}|^2 = Q$, and $I_{R, PR} = p_R |h_{R, PR}|^2 = \frac{Q}{|h_{R, PR}|^2} |h_{R, PR}|^2 = Q$, respectively, i. e., the performance of the primary system can be guaranteed.

However, when CSI is imperfect, the interference from S_n and R to PR becomes $I_{S_n, PR} = \frac{Q}{|\hat{h}_{S_n, PR}|^2} |h_{S_n, PR}|^2$, and $I_{R, PR} = \frac{Q}{|\hat{h}_{R, PR}|^2} |h_{R, PR}|^2$. Thus, the interference from S_n and R to PR may probably exceed Q . To solve this problem, in Ref. [14], a calculation method of the new interference temperature \hat{Q}_1 and \hat{Q}_2 was proposed to confine the transmission power of S_n and R under imperfect CSI:

$$\hat{Q}_1 = \mu_1 Q, \quad \hat{Q}_2 = \mu_2 Q \quad (3)$$

where,

$$\mu_1 = \frac{1 - (2t - 1)^2 (2\rho_1^2 - 1) - 2(2t - 1) \sqrt{(1 - \rho_1^2)[1 - \rho_1^2(2t - 1)^2]}}{4t(1 - t)},$$

$$\mu_2 = \frac{1 - (2t - 1)^2 (2\rho_2^2 - 1) - 2(2t - 1) \sqrt{(1 - \rho_2^2)[1 - \rho_2^2(2t - 1)^2]}}{4t(1 - t)},$$

and t is the probability that the actual interference power from secondary source or relay to PU is less than Q .

Therefore, the transmission power at source S_n and relay R are

$$p_{S_n} = \frac{\hat{Q}_1}{|\hat{h}_{S_n, PR}|^2}, \quad p_R = \frac{\hat{Q}_2}{|\hat{h}_{R, PR}|^2} \quad (4)$$

And the corresponding received SINR at relay R and destination D_m turn out to be

$$\begin{aligned}\gamma_{S_n, R} &= \frac{\hat{Q}_1 |h_{S_n, R}|^2}{|\hat{h}_{S_n, PR}|^2 (p_P |h_{PT, R}|^2 + N_0)} \\ \gamma_{R, D_m} &= \frac{\hat{Q}_2 |h_{R, D_m}|^2}{|\hat{h}_{R, PR}|^2 (p_P |h_{PT, D_m}|^2 + N_0)}\end{aligned}\quad (5)$$

2 Outage performance analysis

The best source n^* is selected to maximize $\gamma_{S_n, R}$ and the best destination m^* is selected to maximize γ_{R, D_m} , i. e. :

$$n^* = \operatorname{argmax}_{n=1, \dots, N} \gamma_{S_n, R} \quad (6)$$

$$m^* = \operatorname{argmax}_{m=1, \dots, M} \gamma_{R, D_m} \quad (7)$$

If any of 2 hops is in outage, i. e., $\gamma_{S_n^*, R}$ or γ_{R, D_m^*} falls below the given SNR threshold γ_{th} , then the secondary system is in outage. Thus the outage probability of the secondary system can be written as

$$\begin{aligned}P_{out} &= \Pr(\min(\gamma_{S_n^*, R}, \gamma_{R, D_m^*}) < \gamma_{th}) \\ &= \Pr(\min(\max_{n=1, \dots, N} \gamma_{S_n, R}, \max_{m=1, \dots, M} \gamma_{R, D_m}) < \gamma_{th}) \\ &= 1 - \Pr(\min(\max_{n=1, \dots, N} \gamma_{S_n, R}, \max_{m=1, \dots, M} \gamma_{R, D_m}) \geq \gamma_{th})\end{aligned}\quad (8)$$

Since $\max_{n=1, \dots, N} \gamma_{S_n, R}$ and $\max_{m=1, \dots, M} \gamma_{R, D_m}$ are mutually independent, Eq. (8) can be rewritten as

$$\begin{aligned}P_{out} &= 1 - \Pr(\max_{n=1, \dots, N} \gamma_{S_n, R} \geq \gamma_{th}) \Pr(\max_{m=1, \dots, M} \gamma_{R, D_m} \geq \gamma_{th}) \\ &= 1 - (1 - F_{\max_{n=1, \dots, N} \gamma_{S_n, R}}(\gamma_{th})) \\ &\quad (1 - F_{\max_{m=1, \dots, M} \gamma_{R, D_m}}(\gamma_{th}))\end{aligned}\quad (9)$$

From Eq. (9), it follows to acquire P_{out} , CDF of $\max_{n=1, \dots, N} \gamma_{S_n, R}$ and $\max_{m=1, \dots, M} \gamma_{R, D_m}$ should be calculated firstly. Next, these two CDFs will be derived respectively in detail.

On one hand, CDF of $\max_{n=1, \dots, N} \gamma_{S_n, R}$ can be represented as

$$F_{\max_{n=1, \dots, N} \gamma_{S_n, R}}(\theta) = \Pr(\max_{n=1, \dots, N} \gamma_{S_n, R} < \theta) \quad (10)$$

From the expression of $\gamma_{S_n, R}$ in Eq. (5), it can be observed that for different n , there exists a common term $|h_{PT, R}|^2$, so that $\gamma_{S_n, R}$ with N sources are correlated with each other.

By letting $\theta_n = \gamma_{S_n, R}$, CDF of θ_n conditioned on

$|h_{PT,R}|^2$ can be calculated as

$$\begin{aligned}
 F_{\theta_n}(\theta | |h_{PT,R}|^2) &= \Pr(\gamma_{S_n,R} < \theta) \\
 &= \Pr\left(\frac{\hat{Q}_1 |h_{S_n,R}|^2}{|\hat{h}_{S_n,PR}|^2(p_P |h_{PT,R}|^2 + N_0)} < \theta\right) \\
 &= \Pr\left(|h_{S_n,R}|^2 < \frac{\theta x(p_P \gamma + N_0)}{\hat{Q}_1}\right) \\
 &= \int_0^\infty (1 - e^{-\lambda_{S_n,R} \frac{\theta x(p_P \gamma + N_0)}{\hat{Q}_1}}) \lambda_{S_n,PR} e^{-\lambda_{S_n,PR} x} dx \\
 &= 1 - \frac{\lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta (p_P \gamma + N_0) + \lambda_{S_n,PR} \hat{Q}_1} \quad (11)
 \end{aligned}$$

Then CDF of $\max_{n=1, \dots, N} \gamma_{S_n,R}$ can be represented as

$$\begin{aligned}
 F_{\max_{n=1, \dots, N} \gamma_{S_n,R}}(\theta) &= \int_0^\infty (F_{\theta_n}(\theta | |h_{PT,R}|^2))^N \lambda_{PT,R} e^{-\lambda_{PT,R} y} dy \\
 &= \int_0^\infty \left(1 - \frac{\lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P \gamma + \lambda_{S_n,R} \theta N_0 + \lambda_{S_n,PR} \hat{Q}_1}\right)^N \lambda_{PT,R} e^{-\lambda_{PT,R} y} dy \quad (12)
 \end{aligned}$$

$$\begin{aligned}
 F_{\max_{n=1, \dots, N} \gamma_{S_n,R}}(\theta) &= \int_0^\infty \sum_{i=0}^N C_N^i (-1)^i \left(\frac{\lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P}\right)^i \left(y + \frac{\lambda_{S_n,R} \theta N_0 + \lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P}\right)^{-i} \lambda_{PT,R} e^{-\lambda_{PT,R} y} dy \\
 &= \lambda_{PT,R} \sum_{i=0}^N C_N^i (-1)^i \left(\frac{\lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P}\right)^i \int_0^\infty \frac{1}{\left(y + \frac{\lambda_{S_n,R} \theta N_0 + \lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P}\right)^i} e^{-\lambda_{PT,R} y} dy \\
 &= \underbrace{\lambda_{PT,R} \int_0^\infty e^{-\lambda_{PT,R} y} dy}_{i=0} - \underbrace{\lambda_{PT,R} C_N^1 \frac{\lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P} \int_0^\infty \frac{1}{y + \frac{\lambda_{S_n,R} \theta N_0 + \lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P}} e^{-\lambda_{PT,R} y} dy}_{i=1 (J_1)} \\
 &\quad + \underbrace{\lambda_{PT,R} \sum_{i=2}^N C_N^i (-1)^i \left(\frac{\lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P}\right)^i \int_0^\infty \frac{1}{\left(y + \frac{\lambda_{S_n,R} \theta N_0 + \lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P}\right)^i} e^{-\lambda_{PT,R} y} dy}_{i \geq 2 (J_2)} \quad (14)
 \end{aligned}$$

Using Ref. [16] (Eq. 3.352.4 and Eq. 3.353.2), J_1 and J_2 can be rewritten as

$$J_1 = -\lambda_{PT,R} N \frac{\lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P} e^{\frac{\lambda_{S_n,R} \theta N_0 + \lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P} \lambda_{PT,R}} Ei\left(-\frac{\lambda_{S_n,R} \theta N_0 + \lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P} \lambda_{PT,R}\right) \quad (15)$$

and

$$J_2 = \lambda_{PT,R} \sum_{i=2}^N C_N^i (-1)^i \left(\frac{\lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P}\right)^i \left(\frac{1}{(i-1)!} \sum_{k=1}^{i-1} (k-1)! (-\lambda_{PT,R})^{i-k-1} \left(\frac{\lambda_{S_n,R} \theta N_0 + \lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P}\right)^{-k} \right. \\
 \left. - \frac{(-\lambda_{PT,R})^{i-1}}{(i-1)!} e^{\frac{\lambda_{S_n,R} \theta N_0 + \lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P} \lambda_{PT,R}} Ei\left(-\frac{\lambda_{S_n,R} \theta N_0 + \lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P} \lambda_{PT,R}\right) \right) \quad (16)$$

Next, note that in Section 1, it is stated the channels between any 2 nodes are subject to i. i. d. zero-mean Rayleigh flat fading with unit variance. Thus, the binomial theorem can be employed. This is also suitable for where the binomial theorem is employed in the following. By employing the binomial theorem, it follows that:

$$\begin{aligned}
 &\left(1 - \frac{\lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P \gamma + \lambda_{S_n,R} \theta N_0 + \lambda_{S_n,PR} \hat{Q}_1}\right)^N \\
 &= \sum_{i=0}^N C_N^i \left(-\frac{\lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P \gamma + \lambda_{S_n,R} \theta N_0 + \lambda_{S_n,PR} \hat{Q}_1}\right)^i \\
 &= \sum_{i=0}^N C_N^i (-1)^i \left(\frac{\lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P}\right)^i \left(y + \frac{\lambda_{S_n,R} \theta N_0}{\lambda_{S_n,R} \theta p_P} + \frac{\lambda_{S_n,PR} \hat{Q}_1}{\lambda_{S_n,R} \theta p_P}\right)^{-i} \quad (13)
 \end{aligned}$$

To proceed forward, substituting Eq. (13) into Eq. (12), $F_{\max_{n=1, \dots, N} \gamma_{S_n,R}}(\theta)$ turns out to be

respectively. Note here that, $Ei(\cdot)$ denotes the exponential integral function. Eventually, CDF of $\max_{n=1, \dots, N} \gamma_{S_n, R}$ can be acquired as

$$F_{\max_{n=1, \dots, N} \gamma_{S_n, R}}(\theta) = 1 + \lambda_{PT, R} N \frac{\lambda_{S_n, PR} \hat{Q}_1}{\lambda_{S_n, R} \theta p_P} e^{\frac{\lambda_{S_n, R} \theta N_0 + \lambda_{S_n, PR} \hat{Q}_1}{\lambda_{S_n, R} \theta p_P} \lambda_{PT, R}} Ei\left(-\frac{\lambda_{S_n, R} \theta N_0 + \lambda_{S_n, PR} \hat{Q}_1}{\lambda_{S_n, R} \theta p_P} \lambda_{PT, R}\right) \\ + \lambda_{PT, R} \sum_{i=2}^N C_N^i (-1)^i \left(\frac{\lambda_{S_n, PR} \hat{Q}_1}{\lambda_{S_n, R} \theta p_P}\right)^i \left(-\frac{(-\lambda_{PT, R})^{i-1}}{(i-1)!} e^{\frac{\lambda_{S_n, R} \theta N_0 + \lambda_{S_n, PR} \hat{Q}_1}{\lambda_{S_n, R} \theta p_P} \lambda_{PT, R}} Ei\left(-\frac{\lambda_{S_n, R} \theta N_0 + \lambda_{S_n, PR} \hat{Q}_1}{\lambda_{S_n, R} \theta p_P} \lambda_{PT, R}\right)\right) \quad (17)$$

On the other hand, CDF of $\max_{m=1, \dots, M} \gamma_{R, D_m}$ can be represented as

$$F_{\max_{m=1, \dots, M} \gamma_{R, D_m}}(\theta) = \Pr(\max_{m=1, \dots, M} \gamma_{R, D_m} \leq \theta) \quad (18)$$

From the expression of γ_{R, D_m} in Eq. (5), it can be observed that for different m , there exists a common term $|\hat{h}_{R, PR}|^2$. As such, γ_{R, D_m} among M destinations are correlated with each other.

By letting $\theta_m = \gamma_{R, D_m}$, CDF of θ_m conditioned on $|\hat{h}_{R, PR}|^2$ can be calculated as

$$F_{\theta_m}(\theta | |\hat{h}_{R, PR}|^2) \\ F_{\max_{m=1, \dots, M} \gamma_{R, D_m}}(\theta) = \int_0^\infty (F_{\theta_m}(\theta | |\hat{h}_{R, PR}|^2))^M \lambda_{R, PR} e^{-\lambda_{R, PR} x} dx = \int_0^\infty \left(1 - \frac{\lambda_{PT, D_m} \hat{Q}_2 e^{-\frac{\lambda_{R, D_m} \theta N_0}{\hat{Q}_2}}}{\lambda_{R, D_m} \theta p_P x + \lambda_{PT, D_m} \hat{Q}_2}\right)^M \lambda_{R, PR} e^{-\lambda_{R, PR} x} dx \\ = 1 + \lambda_{R, PR} M \frac{\lambda_{PT, D_m} \hat{Q}_2}{\lambda_{R, D_m} \theta p_P} e^{\frac{\lambda_{R, D_m} \theta N_0 + \lambda_{R, PR} \hat{Q}_2}{\lambda_{R, D_m} \theta p_P} \lambda_{PT, D_m}} Ei\left(-\left(\frac{\lambda_{R, D_m} \theta N_0 + \lambda_{R, PR} \hat{Q}_2}{\lambda_{R, D_m} \theta p_P}\right) \lambda_{PT, D_m}\right) \\ + \lambda_{R, PR} \sum_{i=2}^M C_M^i (-1)^i \left(\frac{\lambda_{PT, D_m} \hat{Q}_2}{\lambda_{R, D_m} \theta p_P}\right)^i \left(-\frac{\left(-\left(\frac{\lambda_{R, D_m} \theta N_0}{\hat{Q}_2} + \lambda_{R, PR}\right)\right)^{i-1}}{(i-1)!} e^{\left(\frac{\lambda_{R, D_m} \theta N_0 + \lambda_{R, PR} \hat{Q}_2}{\lambda_{R, D_m} \theta p_P}\right) \lambda_{PT, D_m}} Ei\left(-\left(\frac{\lambda_{R, D_m} \theta N_0 + \lambda_{R, PR} \hat{Q}_2}{\lambda_{R, D_m} \theta p_P}\right) \lambda_{PT, D_m}\right)\right) \quad (20)$$

At last, on the basis of both Eq. (17) and Eq. (20), the outage probability turns out to be

$$P_{out} = 1 - (1 - F_{\max_{n=1, \dots, N} \gamma_{S_n, R}}(\gamma_{th})) (1 - F_{\max_{m=1, \dots, M} \gamma_{R, D_m}}(\gamma_{th})) \quad (21)$$

3 Asymptotic analysis

To gain additional insights on the system performance, the asymptotic outage probability is necessary in the high SNR region. Given the Taylor's series expansion of $e^x \approx 1 + x$ for small x , Eq. (11) can be expressed as

$$= \Pr(\gamma_{R, D_m} < \theta) \\ = \Pr\left(\frac{\hat{Q}_2 |h_{R, D_m}|^2}{|\hat{h}_{R, PR}|^2 (p_P |h_{PT, D_m}|^2 + N_0)} < \theta\right) \\ = \int_0^\infty (1 - e^{-\lambda_{R, D_m} \frac{\theta x (p_P y + N_0)}{\hat{Q}_2}}) \lambda_{PT, D_m} e^{-\lambda_{PT, D_m} y} dy \\ = 1 - \frac{\lambda_{PT, D_m} \hat{Q}_2}{\lambda_{R, D_m} \theta p_P x + \lambda_{PT, D_m} \hat{Q}_2} e^{-\frac{\lambda_{R, D_m} \theta N_0 x}{\hat{Q}_2}} \quad (19)$$

Then, after some manipulations similar to obtain Eq. (17), the CDF of $\max_{m=1, \dots, M} \gamma_{R, D_m}$ can be represented as

$$F_{\theta_m}(\theta | |h_{PT, R}|^2) \\ = \int_0^\infty (1 - e^{-\lambda_{S_n, R} \frac{\theta x (p_P y + N_0)}{\hat{Q}_1}}) \lambda_{S_n, PR} e^{-\lambda_{S_n, PR} x} dx \\ \approx \int_0^\infty \lambda_{S_n, R} \frac{\theta x (p_P y + N_0)}{\hat{Q}_1} \lambda_{S_n, PR} e^{-\lambda_{S_n, PR} x} dx \\ = -\lambda_{S_n, R} \frac{\theta (p_P y + N_0)}{\hat{Q}_1} \left(\int_0^\infty x e^{-\lambda_{S_n, PR} x} d(-\lambda_{S_n, PR} x)\right) \\ = -\lambda_{S_n, R} \frac{\theta (p_P y + N_0)}{\hat{Q}_1} \left(\int_0^\infty x d e^{-\lambda_{S_n, PR} x}\right) \\ = \frac{\lambda_{S_n, R}}{\lambda_{S_n, PR}} \frac{\theta (p_P y + N_0)}{\hat{Q}_1} \quad (22)$$

Then, the CDF of $F_{n=1, \dots, N}^{\max} \gamma_{S_n, R}(\theta)$, i. e., Eq. (12)

can be represented as

$$\begin{aligned} F_{n=1, \dots, N}^{\max} \gamma_{S_n, R}(\theta) &= \int_0^\infty \left(\frac{\lambda_{S_n, R}}{\lambda_{S_n, PR}} \frac{\theta(p_P y + N_0)}{\hat{Q}_1} \right)^N \lambda_{PT, R} e^{-\lambda_{PT, R} y} dy \\ &= \lambda_{PT, R} \left(\frac{\lambda_{S_n, R}}{\lambda_{S_n, PR}} \frac{p_P \theta}{\hat{Q}_1} \right)^N \int_0^\infty \left(y + \frac{N_0}{p_P} \right)^N e^{-\lambda_{PT, R} y} dy \end{aligned} \quad (23)$$

Next, by employing the binomial theorem, it leads to that:

$$\left(y + \frac{N_0}{p_P} \right)^N = \sum_{i=0}^N C_N^i y^i \left(\frac{N_0}{p_P} \right)^{N-i} \quad (24)$$

To proceed forward, substituting Eq. (24) into Eq. (23), it follows that:

$$\begin{aligned} F_{n=1, \dots, N}^{\max} \gamma_{S_n, R}(\theta) &= \lambda_{PT, R} \left(\frac{\lambda_{S_n, R}}{\lambda_{S_n, PR}} \frac{p_P \theta}{\hat{Q}_1} \right)^N \int_0^\infty \sum_{i=0}^N C_N^i y^i \left(\frac{N_0}{p_P} \right)^{N-i} e^{-\lambda_{PT, R} y} dy \\ &= \lambda_{PT, R} \left(\frac{\lambda_{S_n, R}}{\lambda_{S_n, PR}} \frac{p_P \theta}{\hat{Q}_1} \right)^N \sum_{i=0}^N C_N^i \left(\frac{N_0}{p_P} \right)^{N-i} \underbrace{\int_0^\infty y^i e^{-\lambda_{PT, R} y} dy}_{J_3} \end{aligned} \quad (25)$$

By using Ref. [16] (Eq. 3.351.3), J_3 can be calculated as

$$J_3 = i! (\lambda_{PT, R})^{-i-1} \quad (26)$$

Eventually, the CDF of $F_{n=1, \dots, N}^{\max} \gamma_{S_n, R}(\theta)$ turns out to be

$$\begin{aligned} F_{n=1, \dots, N}^{\max} \gamma_{S_n, R}(\theta) &= \lambda_{PT, R} \left(\frac{\lambda_{S_n, R}}{\lambda_{S_n, PR}} \frac{p_P \theta}{\hat{Q}_1} \right)^N \sum_{i=0}^N C_N^i \left(\frac{N_0}{p_P} \right)^{N-i} i! (\lambda_{PT, R})^{-i-1} \\ &= \left(\frac{1}{\hat{Q}_1} \right)^N \left(\frac{\lambda_{S_n, R} p_P \theta}{\lambda_{S_n, PR}} \right)^N \sum_{i=0}^N C_N^i \left(\frac{N_0}{p_P} \right)^{N-i} i! (\lambda_{PT, R})^{-i} \\ &= \left(\frac{1}{\mu_1 Q} \right)^N \left(\frac{\lambda_{S_n, R} p_P \theta}{\lambda_{S_n, PR}} \right)^N \sum_{i=0}^N C_N^i \left(\frac{N_0}{p_P} \right)^{N-i} i! (\lambda_{PT, R})^{-i} \\ &= \left(\frac{1}{Q} \right)^N \left(\frac{\lambda_{S_n, R} p_P \theta}{\mu_1 \lambda_{S_n, PR}} \right)^N \sum_{i=0}^N C_N^i \left(\frac{N_0}{p_P} \right)^{N-i} i! (\lambda_{PT, R})^{-i} \end{aligned} \quad (27)$$

Similarly, by means of the Taylor's series expansion of $e^x \approx 1 + x$ for small x , Eq. (19) can be expressed as

$$\begin{aligned} F_{\theta_m}(\theta | | \hat{h}_{R, PR} |^2) &= \int_0^\infty \left(1 - e^{-\lambda_{R, D_m} \frac{\theta x(p_P y + N_0)}{\hat{Q}_2}} \right) \lambda_{PT, D_m} e^{-\lambda_{PT, D_m} y} dy \\ &\approx \int_0^\infty \lambda_{R, D_m} \frac{\theta x(p_P y + N_0)}{\hat{Q}_2} \lambda_{PT, D_m} e^{-\lambda_{PT, D_m} y} dy \end{aligned}$$

$$= \frac{\theta}{\hat{Q}_2} \left(\frac{\lambda_{R, D_m} p_P}{\lambda_{PT, D_m}} + \lambda_{R, D_m} N_0 \right) x \quad (28)$$

Then, the CDF of $\max_{m=1, \dots, M} \gamma_{R, D_m}$, i. e., Eq. (20)

can be represented as

$$\begin{aligned} F_{m=1, \dots, M}^{\max} \gamma_{R, D_m}(\theta) &= \int_0^\infty \left(\frac{\theta}{\hat{Q}_2} \left(\frac{\lambda_{R, D_m} p_P}{\lambda_{PT, D_m}} + \lambda_{R, D_m} N_0 \right) x \right)^M \lambda_{R, PR} e^{-\lambda_{R, PR} x} dx \\ &= \lambda_{R, PR} \left(\frac{\theta}{\hat{Q}_2} \left(\frac{\lambda_{R, D_m} p_P}{\lambda_{PT, D_m}} + \lambda_{R, D_m} N_0 \right) \right)^M \underbrace{\int_0^\infty x^M e^{-\lambda_{R, PR} x} dx}_{J_4} \end{aligned} \quad (29)$$

Next, according to Ref. [16] (Eq. 3.351.3), J_4 can be calculated as

$$J_4 = M! (\lambda_{R, PR})^{-M-1} \quad (30)$$

CDF of $F_{n=1, \dots, N}^{\max} \gamma_{R, D_n}(\theta)$ can be eventually represented as

$$\begin{aligned} F_{n=1, \dots, N}^{\max} \gamma_{R, D_n}(\theta) &= \lambda_{R, PR} \left(\frac{\theta}{\hat{Q}_2} \left(\frac{\lambda_{R, D_m} p_P}{\lambda_{PT, D_m}} + \lambda_{R, D_m} N_0 \right) \right)^M \int_0^\infty x^M e^{-\lambda_{R, PR} x} dx \\ &= \lambda_{R, PR} \left(\frac{\theta}{\hat{Q}_2} \left(\frac{\lambda_{R, D_m} p_P}{\lambda_{PT, D_m}} + \lambda_{R, D_m} N_0 \right) \right)^M M! (\lambda_{R, PR})^{-M-1} \\ &= \left(\frac{1}{\hat{Q}_2} \right)^M \left(\frac{\lambda_{R, D_m} p_P \theta}{\lambda_{PT, D_m} \lambda_{R, PR}} + \frac{\lambda_{R, D_m} N_0 \theta}{\lambda_{R, PR}} \right)^M M! \\ &= \left(\frac{1}{Q} \right)^M \left(\frac{\lambda_{R, D_m} p_P \theta}{\mu_2 \lambda_{PT, D_m} \lambda_{R, PR}} + \frac{\lambda_{R, D_m} N_0 \theta}{\mu_2 \lambda_{R, PR}} \right)^M M! \end{aligned} \quad (31)$$

At last, from Eq. (27) and Eq. (31), it follows that the outage probability of the secondary system, i. e., Eq. (21) can be asymptotically calculated as

$$\begin{aligned} P_{out} &= 1 - (1 - F_{n=1, \dots, N}^{\max} \gamma_{S_n, R}(\gamma_{th})) (1 - F_{m=1, \dots, M}^{\max} \gamma_{R, D_m}(\gamma_{th})) \\ &= 1 - (1 - J_5) (1 - J_6) = J_5 + J_6 - J_5 J_6 \\ &\approx \begin{cases} J_5 & N < M \\ J_6 & N > M \\ J_5 + J_6 & N = M \end{cases} \end{aligned} \quad (32)$$

where,

$$\begin{aligned} J_5 &= \left(\frac{1}{Q} \right)^N \left(\frac{\lambda_{S_n, R} p_P \theta}{\mu_1 \lambda_{S_n, PR}} \right)^N \sum_{i=0}^N C_N^i \left(\frac{N_0}{p_P} \right)^{N-i} i! (\lambda_{PT, R})^{-i} \text{ and} \\ J_6 &= \left(\frac{1}{Q} \right)^M \left(\frac{\lambda_{R, D_m} p_P \theta}{\mu_2 \lambda_{PT, D_m} \lambda_{R, PR}} + \frac{\lambda_{R, D_m} N_0 \theta}{\mu_2 \lambda_{R, PR}} \right)^M M! \end{aligned}$$

From Eq. (32), it can be observed that the system diversity order is $\min(N, M)$, which indicates that whether $N = 1$ or $M = 1$, the diversity gain is always equal to 1, although the outage probability decreases with the increase of M or N . The diversity order can be increased by increasing the numbers of N and M simultaneously. Moreover, the system diversity order is independent of both the imperfection degree of the CSI

and the interference from the primary transmitter.

4 Simulation results and analysis

In this section, simulations are performed to verify the analytical results for MSMD. The parameters used for numerical analysis and simulations are

$$\lambda_{PT,R} = \lambda_{S_n,PR} = \lambda_{S_n,R} = \lambda_{R,PR} = \lambda_{PT,D_m} = \lambda_{R,D_m} = 1 \text{ and } \gamma_{th} = 3 \text{ bps/Hz}, p_p = 0 \text{ dB}, N_0 = 1, p = 0.95.$$

Fig. 2 shows the outage probability of MSMD under different channel correlation coefficient ρ ($\rho_1 = \rho_2 = \rho$), for $N = M = 1$ and $N = M = 2$, respectively. It can be observed that with the increase of Q , the outage probability will reduce. It can also be observed that the outage probability is greatly influenced by ρ : the smaller ρ is, the worse the outage probability gets, and the changing rate of the outage probability gets smaller as ρ decreases. Moreover, for fixed ρ , the outage performance of $N = M = 2$ is significantly improved compared to that of $N = M = 1$. In addition, the simulation results are in well agreement with the analytical results.

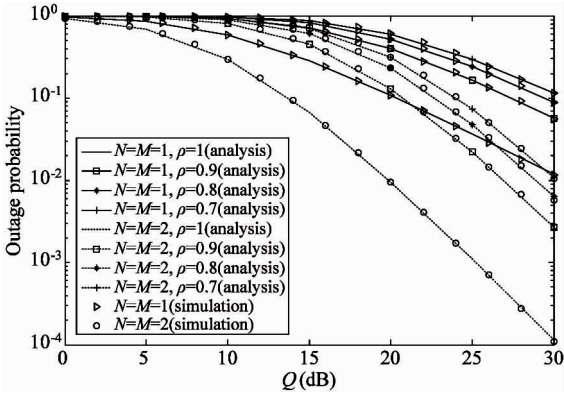


Fig. 2 Outage probability of MSMD versus Q with different ρ ($\rho_1 = \rho_2 = \rho$) for $N = M = 1$ and $N = M = 2$

Fig. 3 demonstrates the outage probability of MSMD under different channel correlation coefficient ρ ($\rho_1 \neq \rho_2$), for $N = M = 1$ and $N = M = 2$, respectively. It can be seen from the curves of $N = M = 1$ that, the outage probability with perfect CSI only available on the relay primary receiver link (i.e., $\rho_1 = 0.8, \rho_2 = 1$) is the same as that with perfect CSI only available on the source primary receiver link ($\rho_1 = 1, \rho_2 = 0.8$). It reveals that the CSI imperfection of the relay primary receiver link has the same impact on the outage probability as the source primary receiver link. Furthermore, for the curves of $N = M = 2$, the curve of $\rho_1 = 1, \rho_2 = 0.8$ approaches more closely to that of $\rho_1 = 0.8, \rho_2 = 0.8$, compared with the curve of $\rho_1 = 0$.

8, $\rho_2 = 1$, which illustrates that the CSI imperfection of the relay primary receiver link has a greater impact on the outage probability than that of the source primary receiver link. More efforts should be put on the channel estimation for the link with a greater impact on the outage probability. In addition, it can also be observed that the outage probability decreases with the increase of both Q and ρ , and increases with the increase of both N and M . The simulation results are also in well agreement with the analytical results likewise.

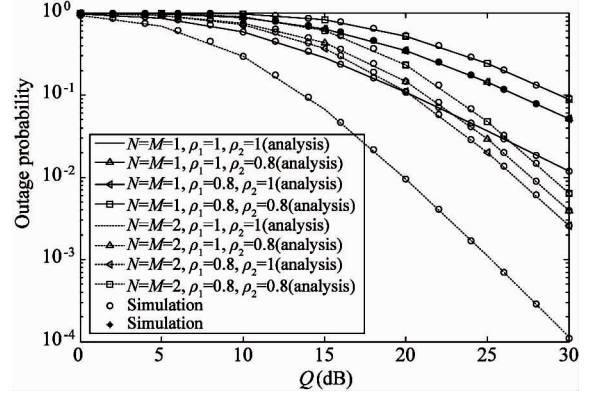


Fig. 3 Outage probability of MSMD versus Q with different ρ ($\rho_1 \neq \rho_2$), for $N = M = 1$ and $N = M = 2$

Fig. 4 shows the outage probability of MSMD versus Q with different ρ and p_p for $N = M = 1$ and $N = M = 2$. It can be observed that the bigger the p_p is, the worse the outage probability gets. It can be also observed that with the increase of ρ , the outage probability will reduce. Moreover, with the increase of Q , the outage probability will reduce.

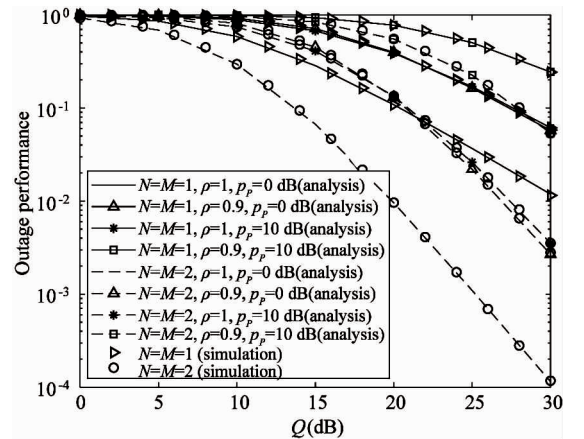


Fig. 4 Outage probability of MSMD versus Q with different ρ and p_p for $N = M = 1$ and $N = M = 2$

Fig. 5 shows the outage probability of MSMD versus p_p with different Q for different N and M ($N = 2, 3, M = 2, 3$). It can be observed that the outage probabil-

ity increases as p_p increases. It can be seen that when $Q = 20$ dB, the outage probability of $N = 2, M = 3$ and $N = 3, M = 2$ is smaller than that of $N = 2, M = 2$. And when the numbers of N and M are changed into 3, i. e., $N = 3, M = 3$, the outage probability decreases very significantly. The same results are also applicable to the case with $Q = 15$ dB. Meanwhile, given fixed N and M , the outage performance of $Q = 20$ dB is better than that of $Q = 15$ dB. Generally, the outage probability can be reduced through increasing Q and the number of either sources or destinations when there exists the interference from the primary transmitter. The effect is more distinct by increasing both the sources and destinations than only increasing one of them.

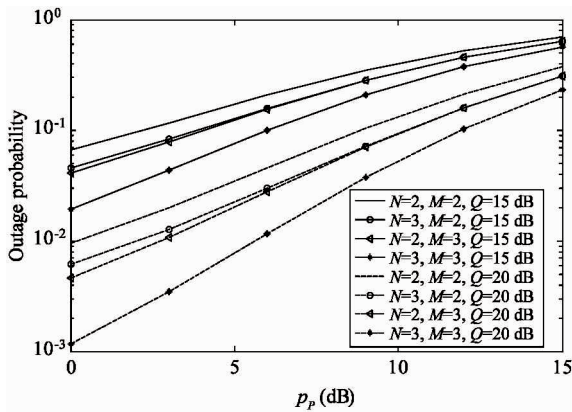


Fig. 5 Outage probability of MSMD versus p_p with different Q for different N and M ($N = 2, 3, M = 2, 3$)

Fig. 6 presents the outage probability of MSMD versus p_p with different Q for different N, M ($N = 1, 2, M = 1, 2$). It can be seen that with the increase of p_p , the outage probability will increase. It also can be seen that when $Q = 20$ dB, the outage probability of $N = 2, M = 1$ and $N = 1, M = 2$ is smaller than that of $N = 1, M = 1$. When $N = 2, M = 2$, i. e., the numbers of N and M are changed into 2, the outage probability

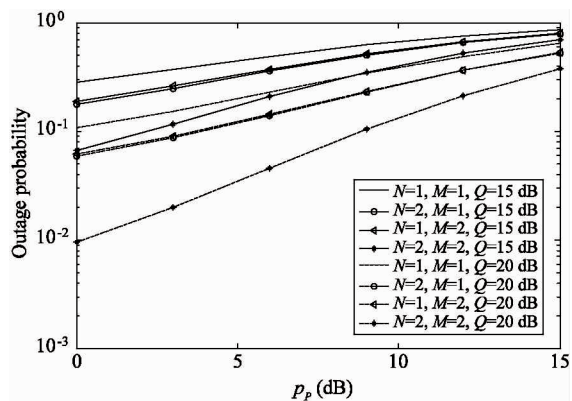


Fig. 6 Outage probability of MSMD versus p_p with different Q for different N and M ($N = 1, 2, M = 1, 2$)

decreases very significantly. The situations of $Q = 15$ dB are in agreement with $Q = 20$ dB. Meanwhile, given fixed N and M , the outage performance of $Q = 20$ dB is better than that of $Q = 15$ dB. Generally, the outage probability can be reduced through increasing the number of either sources or destinations given the interference from the primary transmitter. The effect is more distinct by increasing both the sources and destinations than only increasing one of them.

Fig. 7 gives the exact and asymptotic outage probability of MSMD versus Q with different ρ for different N and M . It can be seen that when $\rho = 1$, the exact outage probability of $N = 1, M = 1$ is higher than that of both $N = 2, M = 1$ and $N = 1, M = 2$, and the exact outage probability of $N = 2, M = 2$ is lower than that of both $N = 2, M = 1$ and $N = 1, M = 2$. Similarly, the exact outage probability of both $N = 3, M = 2$ and $N = 2, M = 3$ is lower than that of $N = 2, M = 2$, and the exact outage probability of $N = 3, M = 3$ is lower than that of both $N = 3, M = 2$ and $N = 2, M = 3$, which indicates that by increasing either N or M , the outage probability could be reduced, and the outage performance will be better improved through increasing N and M simultaneously. Meanwhile, it can also be observed that the asymptotic outage probability of $N = 1, M = 1$ and $N = 2, M = 1$ as well as $N = 1, M = 2$ are in parallel with each other, implying that the diversity order of these 3 cases are identical, yet the case of $N = 2, M = 2$ has a bigger diversity order. Similarly, the asymptotic outage probabilities of $N = 2, M = 2$ and $N = 3, M = 2$ as well as $N = 2, M = 3$ are in parallel with each other, which indicates that the diversity order of these 3 cases are identical, and the case of $N = 3, M = 3$ has a bigger diversity order. This is consistent with the asymptotic Eq. (32), that is to say, the system diversity order is equal to $\min(N, M)$. Moreover, given fixed N and M , the exact outage probability with $\rho = 1$

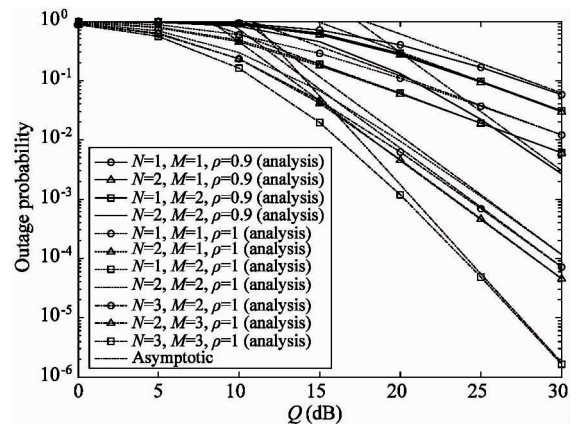


Fig. 7 Outage probability of MSMD versus Q with different ρ for different N and M ($N = 1, 2, 3, M = 1, 2, 3$)

is lower than that with $\rho = 0.9$, but their asymptotic outage probabilities are in parallel with each other, revealing that the system diversity order is independent of it. In addition, the asymptotic results are very tight with the analytical curves in the large SINR region.

Fig. 8 presents the exact and asymptotic outage probability of MSMD versus Q with different p_p for different N and M . When both N and M are fixed, the exact outage probability with $p_p = 0$ dB is smaller than that with $p_p = 10$ dB, yet the asymptotic outage probabilities of these 2 cases are in parallel with each other. This implies that the outage probability also increases as the interference from the primary transmitter increases, but the system diversity order is independent of the interference. The asymptotic results are matched well with the analytical curves in the large SINR region.

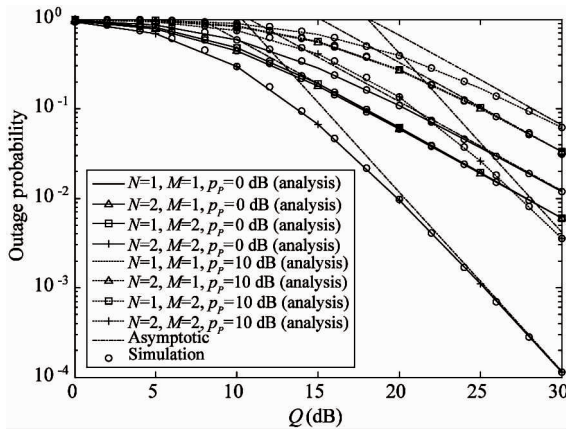


Fig. 8 Outage probability of MSMD versus Q with different p_p for different N and M ($N = 1, 2$, $M = 1, 2$)

5 Conclusion

An underlay cognitive multisource multidestination relay network is studied given the imperfect CSI and considering the interference from the primary transmitter. Both the exact and asymptotic outage probabilities are derived for the network, which provides an efficient approach to investigate impact of the number of sources and destinations, CSI imperfection, as well as interference from the primary transmitter, on the outage performance of considered networks. Eventually, simulation results verifies the correctness of theoretical analyses.

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