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Rate-compatible spatially coupled repeat-accumulate codes via successive extension①

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Abstract

A rate-compatible spatially coupled repeat-accumulate (RC-SC-RA) code is proposed. Its protograph is obtained by extending a given (J, K, L) SC-RA coupled chain (denoted as the mother chain) with extra check nodes and parity bit nodes T times. At each time, the extension is realized via coupling the message bits in the same way as that in the mother chain. Rate-compatibility is achieved by adjusting the extension parameters and applying random puncturing technique. Density evolution analysis shows that the iterative decoding thresholds of all the member codes in the proposed RC-SC-RA code family are very close to Shannon limits over the binary erasure channel. Finite length simulation results are consistent with the thresholds well. Moreover, the proposed RC-SC-RA codes perform better than spatially coupled low density parity check (SC-LDPC) codes in decoding performance especially in lower-rate region.

Key words: spatially coupled LDPC codes, spatially coupled RA codes, rate compatibility, density evolution, iterative decoding

0 Introduction

A rate-compatible code family is a set of codes with different rates, where the higher rate codes are embedded in the lower rate codes[1]. The rate-compatible codes can be constructed via two ways: puncturing and graph extending. Generally, higher rate code is obtained by puncturing a low-rate mother code[2-4], while lower rate code is designed by extending the parity matrix of a high-rate code[4,5]. In these work, the irregular degree distribution of each member code must be optimized to guarantee the belief propagation (BP) threshold approaches Shannon limit, which complicates both design and implementation.

Recently, a class of rate-compatible codes based on the spatial coupling technique attracts much attention due to the threshold saturation of spatially coupled low density parity check (SC-LDPC) codes[6,7]. A feasible rate-compatible SC-LDPC code is constructed by coupling the previously designed rate-compatible LDPC codes[8]. All member codes in this family are proved to be capacity achieving over the binary erasure channel (BEC). Based on a raptor-like code structure, another kind of protograph-based rate-compatible LDPC convolutional codes is proposed in Ref. [9]. In these two schemes, the rate-compatible SC-LDPC codes are all constructed by first designing the rate-compatible codes and then applying the spatial coupling technique to them.

Another alternative is designing spatially coupled code first and then puncturing and extending the mother code to obtain rate compatible codes. This method can reduce the implement complexity to some extent. Considering that the spatially coupled repeat-accumulate (SC-RA) codes have several advantages over SC-LDPC codes, such as simpler encoders and slightly better thresholds[10], a class of rate-compatible SC-RA (RC-SC-RA) codes is proposed in this paper using this alternative designing method.

The proposed RC-SC-RA codes are constructed by extending a given (J, K, L) SC-RA coupled chain (denoted as the mother chain) with adding extra check nodes and parity bit nodes T times. At each time, the extension is realized by coupling the message bits in the same way as that in the mother chain. Specifically, the number of the extra check nodes and parity bit

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nodes is the same as that of the mother chain, and the connection way between the message bit nodes and the extra added check nodes is also the same as that in the mother chain. By doing so, the inherent advantage of RA codes can be preserved. Hence, the designed RC-SC-RA codes are simple in encoding. Rate-compatibility is achieved by configuring the extension parameters and the proportion of the randomly punctured variable nodes. Then the closed form expressions for density evolution are derived to calculate the thresholds of the proposed RC-SC-RA codes. Analysis results show that the iterative decoding thresholds of all the member codes in the proposed RC-SC-RA code family are very close to Shannon limits over the BEC. Finite length simulation results are consistent with the asymptotic results well. Moreover, the proposed RC-SC-RA codes perform better than SC-LDPC codes in decoding performance especially in lower-rate region.

1 Rate-compatible SC-RA Codes

1.1 (J,K,L) SC-RA codes

As described in Ref. [10], an SC-RA ensemble is constructed by coupling a chain of $2L+1(J, K)$ regular RA code, where J is odd and $gcd(J,K) = J$. Fig. 1 gives an example for $J = 3$ and $K = 3$. The left hand side of Fig. 1 is the protograph of a standard (3, 3) regular RA code. Apparently, there is one message bit node (shown in solid circle), a parity bit node (shown in circle) and a check node (shown in square). The right hand side of Fig. 1 is the corresponding (J,K,L) coupled chain, which consists of $2L+1$ copies of the protograph of RA code. A coupled chain is formed by connecting each edge of one message bit node at position $i \in \{-L,\cdots,L\}$ to exactly one of the check nodes at position $j \in \{i-\hat{l},\cdots,i+\hat{l}\}$, where $\hat{l} = (J-1)/2$. This coupled chain is terminated by adding $\hat{l}$ extra check nodes on each end. To avoid creating any degree-1 check nodes, $2\hat{l}$ parity bit nodes should be added.

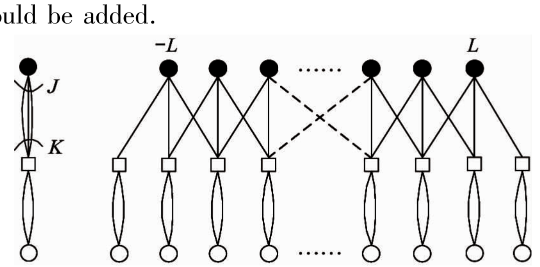

Fig. 1 The $(J=3, K=3, L)$ SC-RA coupled chain

A (J, K, L, M) SC-RA code can be obtained by taking an "*M-lifting*" of the (J, K, L) coupled chain[11]. The design rate of a (J,K,L,M) SC-RA code is

$$R_{RA} = \frac{(2L+1)M}{(2L+1)M + (2L+1+2\hat{l})(J/K)M} \tag{1}$$

1.2 (J, K, L, T) RC-SC-RA codes

As shown in Ref. [9], SC-RA codes have simple encoders and excellent BP thresholds. Therefore, SC-RA code is a good option to construct a rate-compatible code family. In this subsection, it is proposed that RC-SC-RA codes are obtained by extending a (J, K, L) SC-RA coupled chain (denoted by the mother chain) T times. Its protograph consists of a (J, K, L) SC-RA coupled chain and T extensions. The check nodes and parity bit nodes in each extension and the message bit nodes constitute a SC-RA coupled chain.

At tth extension, the message bits of the mother code are coupled with parameters J_t, K_t and L_t, where each message bit is connected to $(J_t-1)/2$ check nodes to the left and $(J_t-1)/2$ check nodes to the right. And when forming the tth extension, J_t-1 extra check nodes and J_t-1 parity bit nodes should be added. This code is denoted as $C(J,J_1,\cdots,J_T,K,K_1,\cdots,K_T,L,L_1,\cdots,L_T,T)$. For simplicity, all extensions are assumed to be coupled with the same parameters as the mother chain, i.e. $J_1 = \cdots = J_T = J$, $K_1 = \cdots = K_T = K$ and $L_1 = \cdots = L_T = L$. Such code is denoted by $C(J,K,L,T)$ briefly. In the rest of this paper, only the code ensemble $C(J,K,L,T)$ is discussed.

Fig. 2 gives the protograph of RC-SC-RA code $C(J=3,K=3,L,T)$. The solid circles are the message bit nodes. The squares are check nodes and the circles are parity bit nodes. The lower part of Fig. 2 is the $(J=3,K=3,L)$ SC-RA coupled chain and the upper part corresponds to T extensions. As illustrated in Fig. 2, the connection way between the check nodes

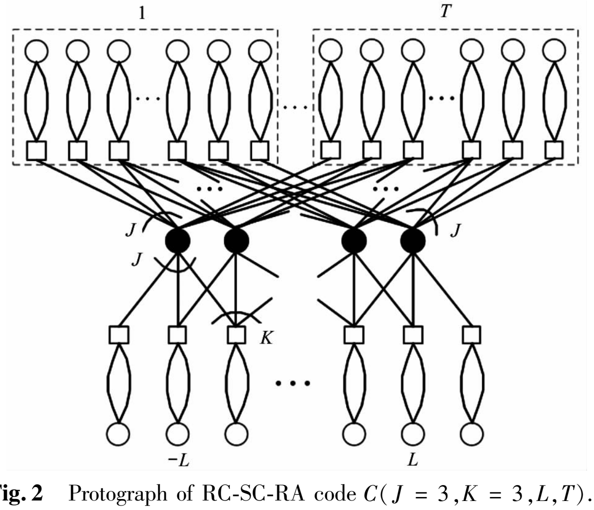

Fig. 2 Protograph of RC-SC-RA code $C(J=3,K=3,L,T)$.

and the message bit nodes in each extension is the same as that in the mother chain.

Denote the base matrix of the mother SC-RA code as $B_M = [B_{M1} \quad B_{M2}]_{m \times n}$, where $m = (J/p)(2L+p)$, $n = (K/p)(2L+1) + m$ and p is the greatest common divisor of J and K. Here B_{M1} corresponds to the connection between the message bit nodes and check nodes, and B_{M2} corresponds to the connection between the check nodes and parity bit nodes. The base matrix of the proposed code $C(J,K,L,T)$ is

$$B_T = \begin{bmatrix} B_{M1} & B_{M2} & \mathbf{0} & \cdots & \mathbf{0} \\ B_{M1} & & B_{M2} & \cdots & \mathbf{0} \\ \vdots & & & \ddots & \vdots \\ B_{M1} & & & & B_{M2} \end{bmatrix}_{m' \times n'} \quad (2)$$

where $m' = (T+1)m$ and $n' = n + Tm$. When constructing a code from the ensemble $C(J, K, L, T)$, "*M-lifting*" is performed for the left column of B_T, which represents the relation between the message bit nodes and all the check nodes. For each B_{M2}, the parity bit nodes are connected to the check nodes in a traditional accumulator pattern. The design rate of the rate-compatible SC-RA code is

$$R_T = \frac{(2L+1)M}{(2L+1)M + (T+1) \times (2L+J)\left(\frac{J}{K}\right)M} \quad (3)$$

As described above, given the mother code $C(J, K, L, T = 0)$ a priori, then code $C(J,K,L,T = 1)$ is obtained by encoding the message bits of the mother code in the same way as that in $C(J,K,L,T=0)$. Recursively, all the member codes $\{C(J,K,L,0), \cdots, C(J,K,L,T)\}$ in the RC-SC-RA code family are obtained. Given L and T, the rates of the code family are decided and discrete. To achieve rate-compatibility, puncture technique is required to apply to all the member codes. As pointed in Ref. [12], randomly punctured SC-LDPC code ensembles with large coupling length display near capacity thresholds for a wide range of rates and simulation results also confirm the excellent decoding performances. Therefore, in this paper, rate-compatibility can be obtained by directly puncturing the variable nodes at each coupled position by a certain percentage, denoted by α. The resulting rate will be equal to

$$R(\alpha) = \frac{R_t}{1-\alpha}, \ \alpha \in [0,1), \ t \in [1, T] \quad (4)$$

1.3 (J,K,L,ω,T) RC-SC-RA codes

Similar to the work in Ref. [6], the edge connections of coupled chains of RC-SC-RA codes can be randomized by adding "smoothing" parameter ω. Denote this code ensemble as $C(J,K,L,\omega,T)$, which is not used in practice but is useful for density evolution analysis.

Instead of requiring each edge of a message bit node at position $i \in \{-L, \cdots, L\}$ is connected to exactly one of the check nodes at position $j \in \{i - \hat{l}, \cdots, i + \hat{l}\}$, each edge of a message bit node at position i is assumed to be uniformly and independently connected to one of the check nodes at the position range of $[i, \cdots, i + \omega - 1]$. Correspondingly, each edge of a check node at position i is independently connected to one of the message bit nodes at the position range of $[i - \omega + 1, \cdots, i]$.

The design rate of the code ensemble $C(J,K,L,\omega,T)$ is

$$R_{T,\omega} = \frac{2L+1}{2L + 1 + \frac{J}{K}(T+1)\left[2L - \omega + 2\left(\omega + 1 - \sum_{i=0}^{\omega}\left(\frac{i}{\omega}\right)^K\right)\right]} \quad (5)$$

2 Density evolution analysis

In this section, the closed form expressions for density evolution of the code ensemble $C(J,K,L,\omega,T)$ over the BEC are derived. Let the channel erasure probability be ε. The illustration of the message passing within the protograph at the ith coupled position is shown in Fig. 3. The symbols are defined as follows.

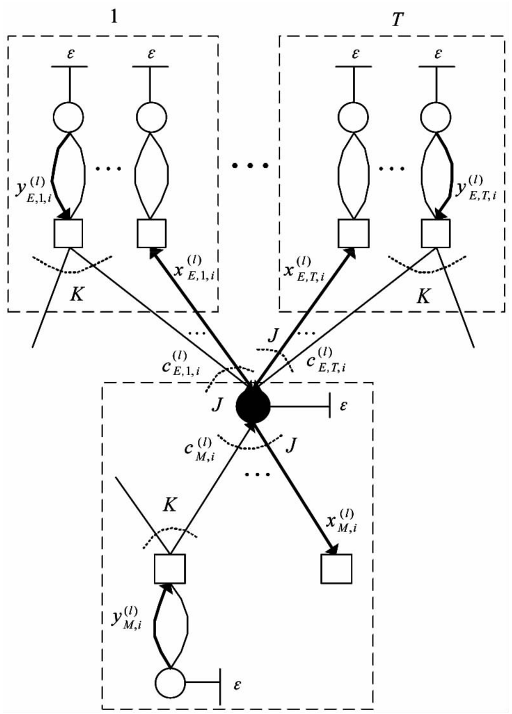

Fig. 3 Illustration of message passing between different types of nodes within protograph at the ith position

$x_{M,i}^{(l)}$ is the erasure probability of the message from the message bit node at position i to its every connected check node of the mother code at the lth iteration.

$x_{E,t,i}^{(l)}$ is the erasure probability of the message from the message bit node at position i to its every connected check node in the tth extension at the lth iteration, where $t \in [1, T]$.

$y_{M,i}^{(l)}$ is the erasure probability of the message from the parity bit node at position i to its one connected check node of the mother code at the lth iteration.

$y_{E,t,i}^{(l)}$ is the erasure probability of the message from the parity bit node at position i to its one connected check node in the tth extension at the lth iteration, where $t \in [1, T]$.

$c_{M,i}^{(l)}$ is the erasure probability of the message from the check node at position i to its every connected message bit node of the mother code at the lth iteration.

$c_{E,t,i}^{(l)}$ is the erasure probability of the message from the check node at position i in the tth extension to its every connected message bit node at the lth iteration, where $t \in [1, T]$.

Remark: Since each extension is identical to each other and is also the same as the mother code, it is easy to know that the erasure probabilities $x_{E,1,i}^{(l)} = \cdots = x_{E,T,i}^{(l)} = x_{M,i}^{(l)}$, and also $y_{E,1,i}^{(l)} = \cdots = y_{E,T,i}^{(l)} = y_{M,i}^{(l)}$ and $c_{E,1,i}^{(l)} = \cdots = c_{E,T,i}^{(l)} = c_{M,i}^{(l)}$ are also obtained. For simplicity, denote the symbols $x_{E,t,i}^{(l)}$ and $x_{M,i}^{(l)}$ as $x_i^{(l)}$, the symbols $y_{E,t,i}^{(l)}$ and $y_{M,i}^{(l)}$ as $y_i^{(l)}$, the symbols $c_{E,t,i}^{(l)}$ and $c_{M,i}^{(l)}$ as $c_i^{(l)}$, $t = 1, \cdots, T$.

In the following, the updated equations of the density evolution at the lth iteration ($l \geqslant 1$) are derived.

Firstly, compute the average erasure probability $c_i^{(l)}$, which is the message from the check node to each message bit node at the ith position. It can be obtained by

$$c_i^{(l)} = 1 - (1 - y_i^{(l-1)})^2 \times \left(1 - \frac{1}{\omega}\sum_{k=0}^{\omega-1} x_{i-k}^{(l-1)}\right)^{K-1} \tag{6}$$

Secondly, update for the message bit nodes. As shown in Fig. 3, the computation of $x_i^{(l)}$ consists of two parts: one is from the check node in the mother code and the other is from the T extensions. Hence, the average erasure probability $x_i^{(l)}$ can be calculated by

$$\begin{aligned} x_i^{(l)} &= \varepsilon \times \left(\frac{1}{\omega}\sum_{j=0}^{\omega-1} c_{i+j}^{(l)}\right)^{J-1} \times \left[\left(\frac{1}{\omega}\sum_{j=0}^{\omega-1} c_{i+j}^{(l)}\right)^{J}\right]^{T} \\ &= \varepsilon \times \left(\frac{1}{\omega}\sum_{j=0}^{\omega-1} c_{i+j}^{(l)}\right)^{(T+1)J-1} \end{aligned} \tag{7}$$

Finally, update for the parity bit nodes. As shown in Fig. 3, $y_i^{(l)}$ is equal to the erasure probability of the message from the check node at position i to the parity bit node. Hence, the computation of $y_i^{(l)}$ is given by

$$y_i^{(l)} = \varepsilon \times \left[1 - (1 - y_i^{(l-1)}) \times \left(1 - \frac{1}{\omega}\sum_{k=0}^{\omega-1} x_{i-k}^{(l-1)}\right)^{K}\right] \tag{8}$$

Eqs(6) – (8) are derived based on the randomization of the edge connections in the neighboring ω coupled positions. The initial values are

$$x_i^{(0)} = y_i^{(0)} = \begin{cases} \varepsilon & -L \leqslant i \leqslant L \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

Define the BP threshold of the RC-SC-RA code ensemble $C(J, K, L, \omega, T)$ as

$$\varepsilon^{BP}(J,K,L,\omega,T) \triangleq \sup\{\varepsilon \in [0,1]: \boldsymbol{x}^{(l-1)}(\varepsilon) \xrightarrow{l \to \infty} 0\} \tag{10}$$

where vector $\boldsymbol{x}^{(l)} = \{x_{-L}^{(l)}, \cdots, x_L^{(l)}\}$.

Using the update equations, one can determine the BP threshold of the RC-SC-RA codes.

3 Numerical results

3.1 Asymptotic results

In this subsection, the performance of the proposed RC-SC-RA codes is evaluated via density evolution. When calculating the BP thresholds, 10^4 is set as the maximum iteration number and 10^{-7} is set as the breakout condition of the erasure probability at each coupled position.

Table 1 illustrates the BP thresholds of $C(3,3,50,3,T)$ with T varying from 0 to 7. The gap in this table means the difference between Shannon limit ε^{SL} and BP threshold ε^{BP}. As shown in Table 1, the smallest gap is 0.0106 and the largest gap is 0.0406. Although the gaps to Shannon limits are becoming slightly larger as $T (T \neq 0)$ increases, the thresholds of the code ensemble $C(3, 3, 50, 3, T)$ for various T are still very close to Shannon limits.

Table 1 BP threshold of $C(3,3,50,3,T)$

T	R	ε^{SL}	ε^{BP}	Gap
0	0.4951	0.5049	0.4643	0.0406
1	0.3290	0.6710	0.6604	0.0106
2	0.2463	0.7537	0.7419	0.0118
3	0.1969	0.8031	0.7842	0.0189
4	0.1640	0.8360	0.8102	0.0258
5	0.1405	0.8595	0.8283	0.0312
6	0.1229	0.8771	0.8420	0.0351
7	0.1092	0.8908	0.8528	0.0380

Fig. 4 gives the thresholds of the codes $C(4,4,L,4,T)$ with different coupling length L over the BEC. When L is fixed, the gaps to Shannon limits will become slightly larger as the rates become lower. And at *fixed* T, the gaps to Shannon limits will become smaller when L becomes larger. However, the improvement can be negligible when L is larger than 15. That is to say, $L = 15$ is good enough.

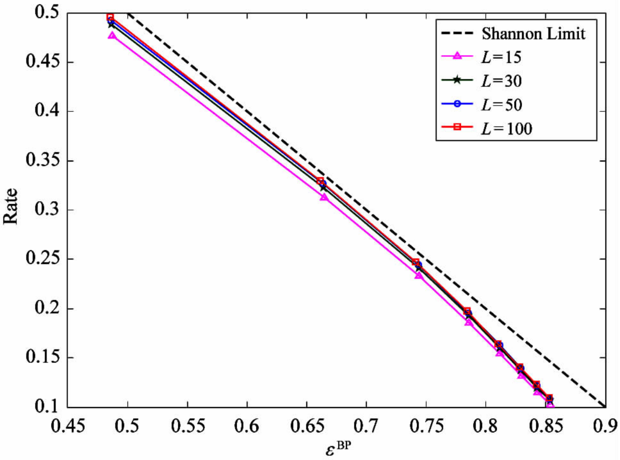

Fig. 4 Rate versus BP threshold of the codes $C(4,4,L,4,T)$ with different L

In the following, the influence of different check node degree K on our proposed codes is investigated. We consider the code ensembles $C(3,K,50,3,T)$ with $K = 3, 6, 9$ and 12. The performance of rate versus BP thresholds is shown in Fig. 5. Larger K means larger design rate of the underlying (J, K) regular RA code. From this figure, one can observe that when K is large, the range of the included rates becomes large. And as T becomes larger, the rates will become smaller and the gaps to Shannon limits become slightly larger.

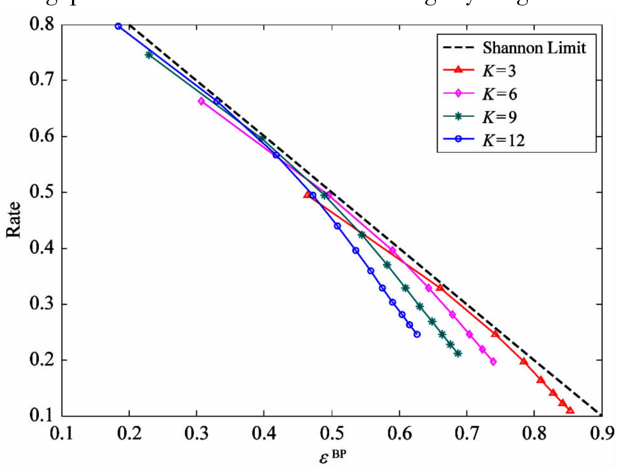

Fig. 5 Rate versus BP threshold of codes $C(3,K,50,3,T)$ with varying K

Finally, the decoding thresholds versus rates for the proposed RC-SC-RA codes and the rate-compatible SC-LDPC codes are compared in Ref. [9]. Since the rates of the codes in Ref. [9] vary from 0.7250 to 0.2880, for comparison, codes $C(3,9,10,3,T)$ with almost same rate range (from 0.7326 to 0.2812) and identical coupling length $L = 10$ are considered. The results are presented in Fig. 6, from which one can observe that the proposed RC-SC-RA codes perform slightly better in threshold performance than the rate-compatible SC-LDPC codes in Ref. [9].

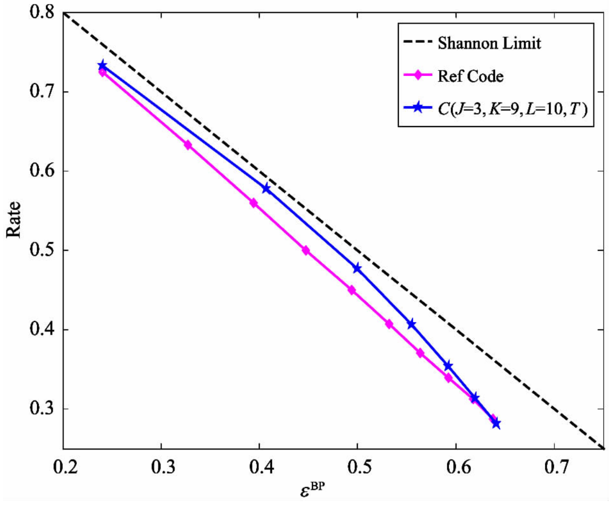

Fig. 6 Comparison of the rates versus BP thresholds between the codes $C(3,9,10,3,T)$ and the codes in Ref. [9]

3.2 Finite length simulations

In this subsection, the decoding performances of the RC-SC-RA codes $C(3,3,15,T)$ with $T = 0, 1, 2, 3$ transmitted over the BEC are shown in Fig. 7. These four codes are constructed randomly with lifting factor $M = 256$. A standard BP decoding algorithm is used with maximum iteration 100. The corresponding rates and thresholds of these codes are calculated and presented in Table 2.

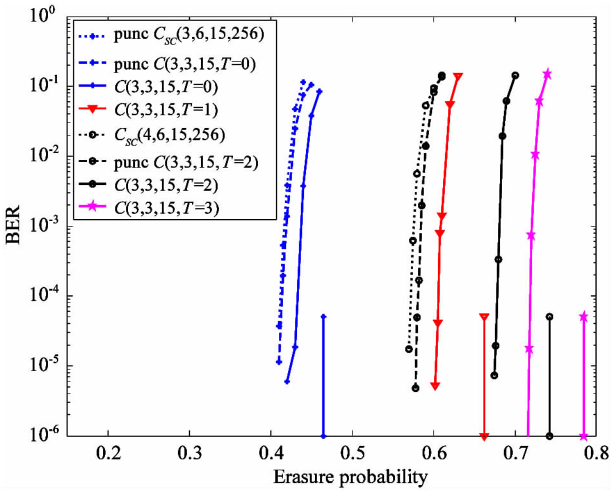

Fig. 7 BEC decoding performances and BP decoding thresholds of RC-SC-RA codes $C(3,3,15,T)$ for $T = 0, 1, 2, 3$ (from left to right) with lifting factor $M = 256$. Also shown for comparison between the proposed RC-SC-RA codes and SC-LDPC codes. The vertical lines are Shannon limits.

Table 2 BP thresholds of $C(3,3,15,3,T)$

T	R	ε^{SL}	ε^{BP}	Gap
0	0.4844	0.5156	0.4649	0.0507
1	0.3196	0.6804	0.6619	0.0185
2	0.2385	0.7615	0.7427	0.0188
3	0.1902	0.8098	0.7847	0.0251

As shown in Fig. 7, it is observed that each code has excellent decoding performance and displays a gap from its respective threshold of approximately 0.04 to 0.06 at a BER of 10^{-5}, for only a moderate lifting factor $M = 256$. This gap is expected to decrease as the lifting factor M increases.

In the following, the decoding performance of the proposed RC-SC-RA codes is compared with that of the SC-LDPC codes.

Denote a regular SC-LDPC ensemble by (l, r, L), where l and r are the variable and check node degree respectively[6]. An SC-LDPC code drawn from this ensemble with lifting factor M is denoted by $C_{SC}(l, r, L, M)$. To be fair, the lifting factor is set to 256 for all the codes in our simulations.

An SC-LDPC code $C_{SC}(3,6,15,256)$ is compared with the proposed code $C(3,3,15,T = 0)$. The rate and threshold of $C_{SC}(3,6,15,256)$ are 0.4677 and 0.4879, while for $C(3,3,15,T = 0)$ they are 0.4844 and 0.4649. The rates of these two codes can be increased to 0.5 by randomly puncturing some variable nodes with fraction α equal to 0.0646 and 0.0312 respectively. The comparison result is presented in the first two curves of Fig. 7, from which one can observe that the proposed code $C(3,3,15,T = 0)$ performs slightly better than the SC-LDPC code $C_{SC}(3,6,15,256)$ in BER performance. Besides, it is considered another SC-LDPC code $C_{SC}(4,6,15,256)$, whose rate and threshold are 0.3118 and 0.6609. Here, only code $C(3,3,15,T = 2)$ is punctured with the fraction α equal to 0.2351, yielding the same rate as $C_{SC}(4,6,15,256)$ and the corresponding threshold is 0.6796. The error performance is shown in Fig. 7. As stated above, the performance of our proposed RC-SC-RA code is better than that of the SC-LDPC code in decoding performance. Comparing these two case, it can be concluded that the proposed RC-SCRA codes perform better especially in lower-rate region.

4 Conclusions

In this paper, a family of rate-compatible SC-RA codes is proposed, whose protograph can be obtained by graph extension. The proposed rate-compatible SC-RA codes are simple in encoding and have excellent decoding thresholds. Density evolution analysis shows that the iterative decoding thresholds of all the member codes in the proposed rate-compatible SC-RA code family are very close to Shannon limits over the BEC. Finite length simulation results are consistent with the calculated thresholds well and it is shown that the proposed RC-SC-RA codes perform better than SC-LDPC codes in decoding performance especially in lower-rate region. However, it is found that when T is larger, the gap between the threshold and Shannon limit is slightly larger. It is conjectured that it is caused by choosing the same parameters for each extension as the mother code and making no optimization for them. How to scale the parameters for each extension to improve the thresholds when T is large is the future work.

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