doi:10.3772/j. issn. 1006-6748. 2018. 04. 014

Determinate mathematical model of couple bearing force of magnetic-liquid suspension guide-way with complicate constraint①

Weng Gangmin(翁钢民)②* **, Tian Xuefei** ***, Zhao Jianhua****, Zhang Bin****, Wang Qiang****, Chen Tao****, Liu Jingcheng****, Gao Dianrong****

(* School of Economics and Management, Yanshan University, Qinhuangdao 066004, P. R. China)

(** College of Humanities and Law, Yanshan University, Qinhuangdao 066004, P. R. China)

(*** Qinhuangdao Beidaihe Middle School, Qinhuangdao 066004, P. R. China)

(**** Hebei Provincial Key Laboratory of Heavy Machinery Fluid Power Transmission and Control, Yanshan University, Qinhuangdao 066004, P. R. China)

Abstract

Magnetic-liquid suspension guide-way system (MLSGS) is coupled supported by permanent magnetic suspension and hydrostatic bearing. The structure and bearing mechanism of MLSGS of heavy computer numerical control (CNC) machine tools are introduced and the mathematical expression of bearing force of bearing unit is derived and it can be broken apart into six parts which sustains directly the coordinate components of broad external load, where the air gap of permanent magnet and hydrostatic oil film can be simplified as elastic supports, the compatibility equations of deformations for oil films and air gap are presented, and then the bearing capacity calculation of the bearing unit is transformed into a determinate problem. Considering the guide-way as a rigid body, the mathematics expression between the bearing unit's bearing force and the oil film variation is linearized, and the oil pocket's bearing capacity which bears different components of the external load is calculated separately. The six components of the bearing unit are added up, and the final general mathematics expression is derived. The proposed research offers a general simple method for calculating the bearing capacity of MLSGS with complicate constraint, which can be mastered simply by engineering designer and can improve design efficiency and accuracy.

Key words: heavy computer numerical control (CNC) machine tool, magnetic-liquid suspension guide-way system(MLSGS), bearing unit, compatibility equations of deformation, linear treatment

0 Introduction

Hydrostatic guide-way is a supporting and guiding component of heavy computer numerical control (CNC) machine tools. The bearing capacity and static stiffness are the most important indexes which directly affect machining precision[1]. At present, oil film stiffness is generally in the range of 10 – 30 kN/μm, the thickness of the oil film basically varies within a few microns, and then machining precision of the heavy CNC machine tools varies within the range of 10 μm[2,3].

At present, machining precision of heavy CNC machine tools is generally 1 – 0.1 μm. In order to improve the machining precision from 10 μm to 1 μm, the pressure of the oil pocket should be increased by 10 times[4]. Take GTMXXX as an example, if the oil pocket pressure varies from 2.5 MPa to 25 MPa, then the flow and power of supply system should increase by 10 times and 100 times[5] respectively. At this point, heating of hydraulic oil and thermal deformation of the guide-way will be inconceivable, and then the hydrostatic system will not operate properly[2]. Therefore, it is infeasible to increase oil pocket pressure to improve the bearing capacity and stiffness of the guide-way system. Under heavy load condition, the bearing capacity and stiffness of hydrostatic guide-way have become the bottleneck of the development of heavy CNC machine tools[6].

According to the lack of bearing capacity and stiff-

① Supported by the National Natural Science Foundation of China (No. 51705445), the Natural Science Foundation of Hebei Province (No. E2016203324) and the Provincial and Ministerial Level Scientific Research Institutions Open Fund of Civil Aviation University of China.

② To whom correspondence should be addressed. E-mail: wenggangmin@126.com

Received on Oct. 12, 2017

ness of hydrostatic guide-way[7], Hyprostatik Company developed progressive mengen (PM) flow controller[8], Shao et al.[9] proposed ACFCVM. In order to improve flow, bearing capacity and stiffness of hydrostatic guide-way, Zhang et al.[10] proposed an adaptive load supply system. The range of the improvement of the bearing capacity, stiffness and the precision machining is limited by the conventional way (Adjust the oil pressure and fuel supply way to improve performance).

Based on the phenomenon that same magnetic poles repel each other, the permanent magnetic suspension can support two objects with no contact, no friction suspension. It can produce the same repulsion support effect with hydrostatic bearing, and then it is suitable to be the auxiliary support of hydrostatic support system. Therefore, the maglev theory is introduced into the hydrostatic guide-way system to construct magnetic-liquid suspension guide-way system (MLSGS).

MLSGS consists of hydrostatic and permanent magnetic suspension systems. The positions of bearing unit are related to structure of guide-way, form and position of drive unit, and arrangement of bearing unit is uncertain and asymmetrical. At the same time, there is the coupling relationship between permanent magnetic system and hydrostatic supporting system. Consequently, the bearing capacity of the bearing unit becomes very complex, and there is a lack of general calculation formula of bearing capacity of MLSGS. The bearing capacity calculation of the support unit is the foundation of design and analysis of MLSGS.

Therefore, the paper introduces the structure characteristics and support mechanism of MLSGS, and then derives a general determinate mathematical model of MLSGS with complex constraint condition, which lays a foundation for practical application of MLSGS.

1 Brief introduction of MLSGS

1.1 Structure characteristics

MLSGS adopts hydraulic oil as lubricating medium. A closed opposing skate board is arranged around the guide-way. Four rectangular inlet pocket and a large liquid return groove of '田' shape are arranged in upper skate board and lower skate board. A permanent magnet is arranged in the center of the skate board (Fig. 1). The rectangular inlet pocket and liquid return groove of '日' shape are arranged in side the skate board. The position of permanent magnet in the guide-way is corresponding to permanent magnet in the upper and lower skate board, and the magnetic pole is the same, produces the repulsion force. Each of hydrostatic supporting pocket and permanent magnets form a bearing unit, as shown in Fig. 2 and Fig. 3.

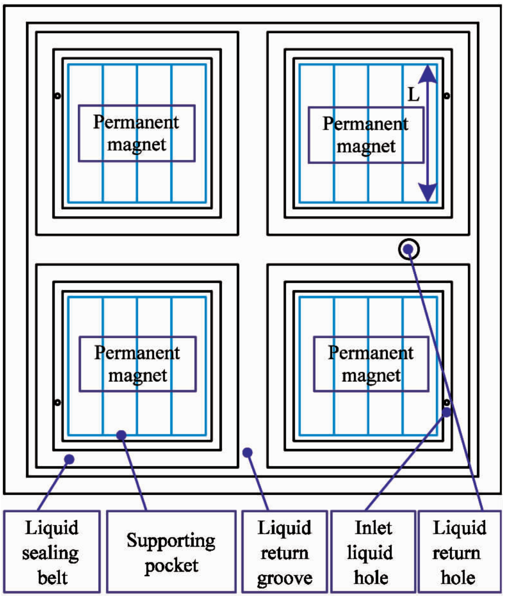

Fig. 1 Diagram of skate board

MLSGS can realize coupling support of hydrostatic force and permanent magnetic force, and then can increase bearing capacity and stiffness of guide-way system.

1.2 Bearing capacity of bearing unit

Assume that there are m oil pockets on the skate board, which are supplied with a constant flow q_0. The oil pocket in the skate board is rectangular, as shown in Fig. 4.

The position $\boldsymbol{L}_i$ and unit vector $\boldsymbol{l}_i$ of bearing force of supporting unit can be expressed as

$$\begin{cases} \boldsymbol{L}_i = [x \quad y \quad z]^{\mathrm{T}} \\ \boldsymbol{l}_i = [u \quad v \quad w]^{\mathrm{T}} \end{cases} \tag{1}$$

where $u^2 + v^2 + w^2 = 1$; $i = 1, 2, 3 \cdots m$.

According to Ref. [11], hydrostatic bearing force f_y of hydrostatic oil pocket can be expressed as

$$f_y = \frac{\mu q A_e}{\bar{B} h^3} \tag{2}$$

where, μ is the Kinetic viscosity of lubricants (Pa · s); h is the thickness of oil film (m); $\bar{B}$ is the bearing flow coefficient, $\bar{B} = \frac{A - a_1}{6b_1} + \frac{B - b_1}{6a_1}$, dimensionless; A and B are the length, width of bearing pocket (m); a_1 and b_1 are the width of sealing belt (m); p_r is the pressure of bearing pocket (MPa); p_s is the pressure of system (MPa); A_e is the effective bearing area of bearing pocket, $A_e = (A - a_1)(B - b_1)$ (m^2).

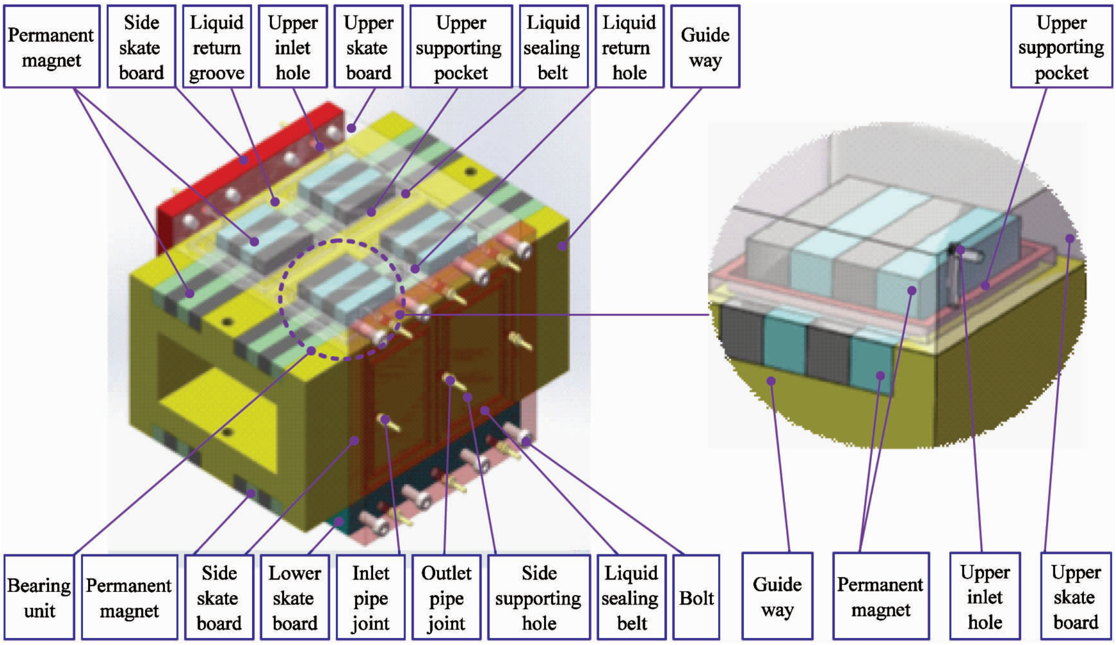

Fig. 2 Diagram of MLSGS

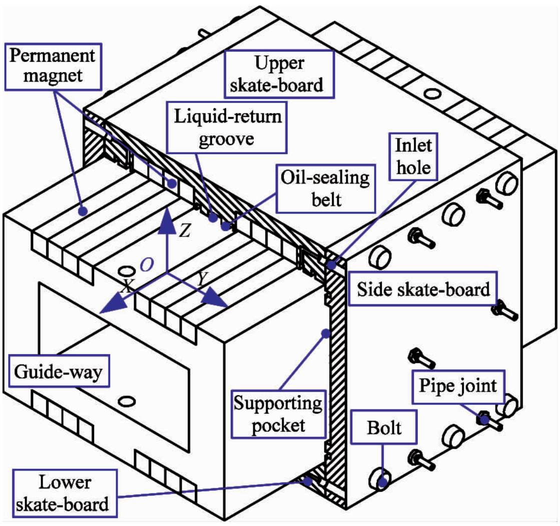

Fig. 3 Local section view of MLSGS

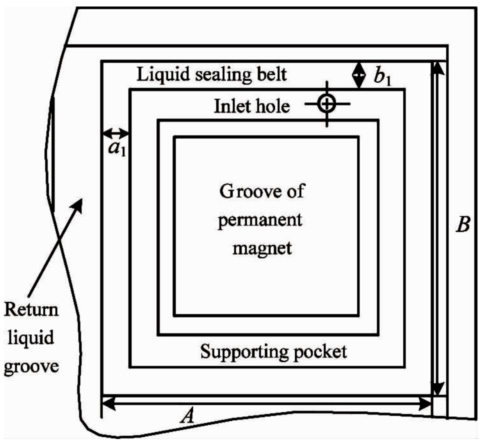

Fig. 4 Structure of oil pocket

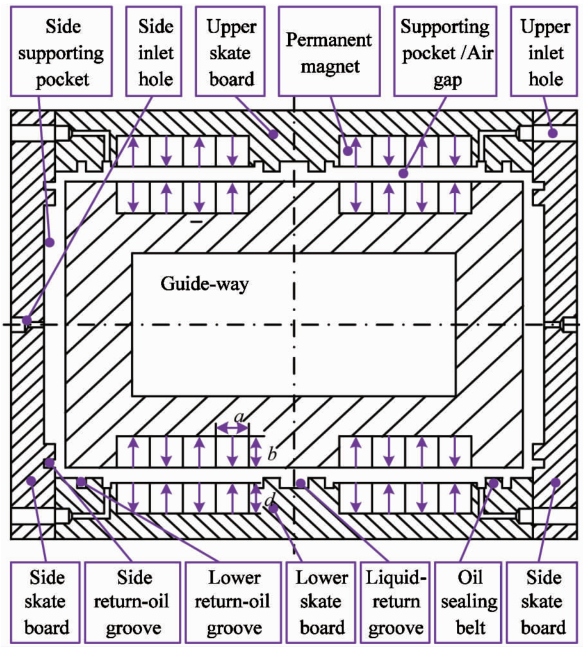

Fig. 5 Full profile intention of MLSGS

According to Ref. [12], bearing force f_d of upper and lower permanent magnet can be expressed as

$$f_d = \frac{B_{r1} B_{r2} L \times 10^{-6}}{4\pi\mu_0}\varphi(h) \qquad (3)$$

where, B_{r1}, B_{r2} are the residual magnetic flux density of permanent magnet (T); L is the length of permanent magnet (m); μ_0 is the magnetic permeability of air gap (H/m); (Shown as Fig. 1). The expression of $\varphi(h)$ in Eq. (3) is

$$\varphi(h)=\left\{\begin{array}{l}2\psi(c,d,h)-2\psi(c,b+d,h)\\-\psi(c+a,d,h)+\psi(c+a,b+d,h)\\-\psi(c-a,d,h)+\psi(c-a,b+d,h)\\-2\psi(c,0,h)+\psi(c-a,0,h)\\+2\psi(c,b,h)-\psi(c-a,b,h)\\+\psi(c+a,0,h)-\psi(c+a,b,h)\end{array}\right\} \quad (4)$$

where, a is the width of the permanent magnet (mm); b, d are the thickness of the permanent magnet (mm); c is 0 (mm); (Shown as Fig. 5). The expression of ψ in Eq. (4) is as follows:

$$\psi(\xi,\zeta,h)=\left\{\begin{array}{l}(\zeta+l_0+h)\ln[\xi^2+(\zeta+l_0+h)^2]\\+2\xi arctan\left(\dfrac{\zeta+l_0+h}{\xi}\right)\end{array}\right\} \quad (5)$$

where, ζ, ξ are the undetermined coefficient, l_0 is the thickness of zinc coating, h is the thickness of oil film.

According to Eq. (2) and Eq. (3), resultant force f_h of bearing unit can be expressed as

$$f_h=f_y+f_d=\frac{\mu q A_e}{\bar{B}h^3}+\frac{B_{r1}B_{r2}L\times 10^{-6}}{4\pi\mu_0}\varphi(h) \quad (6)$$

2 Bearing force of bearing unit

2.1 Solution of bearing force

External load $\boldsymbol{F}$ of guide-way can be expressed as vector form as

$$\boldsymbol{F}=[F_x \quad F_y \quad F_z \quad M_x^F \quad M_y^F \quad M_z^F]^{\mathrm{T}} \quad (7)$$

Bearing force f of supporting unit can keep balance with external load $\boldsymbol{F}$ of guide-way, so the force balance equation of guide-way can be expressed as

$$\left\{\begin{array}{l}\sum_{i=1}^{m} f_i\boldsymbol{l}_i=[F_x \quad F_y \quad F_z]^{\mathrm{T}}\\ \sum_{i=1}^{m} f_i\boldsymbol{L}_i\times\boldsymbol{l}_i=[M_x^F \quad M_y^F \quad M_z^F]^{\mathrm{T}}\end{array}\right. \quad (8)$$

The external load of guide-way can be broken apart into six parts, F_x, F_y, F_z, M_x, M_y, M_z. The bearing force of supporting unit can be expressed as the form of sum of six parts, f^{Fx}, f^{Fy}, f^{Fz}, f^{Mx}, f^{My}, f^{Mz}:

$$f=f^{Fx}+f^{Fy}+f^{Fz}+f^{Mx}+f^{My}+f^{Mz} \quad (9)$$

So Eq. (8) can be expressed as follows:

$$\sum_{i=1}^{m} f_i^{Fx}u_i=F_x \quad (10a)$$

$$\sum_{i=1}^{m} f_i^{Fy}v_i=F_y \quad (10b)$$

$$\sum_{i=1}^{m} f_i^{Fz}w_i=F_z \quad (10c)$$

$$\sum_{i=1}^{m} f_i^{Mx}(y_iw_i-z_iv_i)=M_x^F \quad (10d)$$

$$\sum_{i=1}^{m} f_i^{My}(z_iu_i-x_iw_i)=M_y^F \quad (10e)$$

$$\sum_{i=1}^{m} f_i^{Mz}(x_iv_i-y_iu_i)=M_z^F \quad (10f)$$

Solve the component f^{Fx}, f^{Fy}, f^{Fz}, f^{Mx}, f^{My}, f^{Mz} of supporting unit in turn, then bearing force f of supporting unit can be solved under external load.

2.2 Solution of bearing force f^{Fx}

Due to the material of guide-way and skate boarding being steel alloys (HT200-300), large stiffness and small deformation, the guide-way and skate boarding can be assumed to be rigid body.

The skate boarding translates as a whole without deformations and rotation when it bears external force. Similarly, the skate boarding rotates without deformation and translation when it bears external torque.

Fig. 6 is the force diagram of the skate boarding when it bears external force.

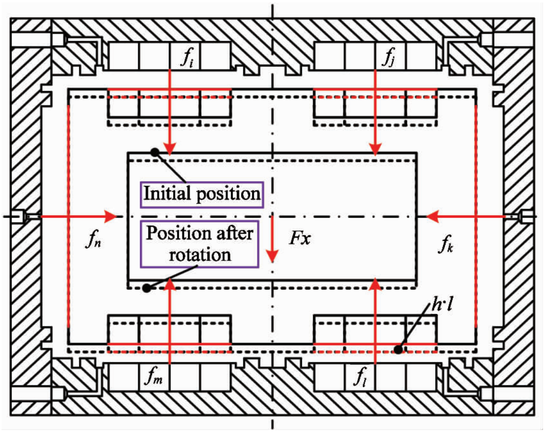

Fig. 6 Force diagram of guide-way under external force

The bearing force of supporting unit is expressed as f^{Fx} when external force F_x acts on the skate boarding. The unit direction vector of supporting unit is l, and the bearing force f^{Fx} can keep balance with external force F_x in the direction of x axis. So the force balance equation of bearing force f^{Fx} and external force F_x can be expressed as Eq. (10a). The number of unknown solution f^{Fx} is m, and then Eq. (10a) belongs to indeterminate problem[13], so the extra equation must be sought to solve Eq. (10a).

2.2.1 Extra equation——deformation compatibility equation of air gap and oil film

The skate boarding translates Δx in the direction of x axis as a whole when the external force F_x acts on the guide-way, and then the thickness variation of oil film of supporting unit is $u\Delta x$.

Using the solution method of the determinate problem in Ref. [13], the expression of bearing force of supporting unit and thickness variation of oil film can be derived.

According to Eq. (6), the variation of bearing force of supporting unit can be expressed as

$$f^{Fx} = \frac{\mu q A_e}{\bar{B}}\left[\frac{1}{(h_0 - u\Delta x)^3} - \frac{1}{h_0^3}\right] + \frac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0}[\varphi(h_0 - u\Delta x) - \varphi(h_0)] \tag{11}$$

2.2.2 Linearization and calculation of bearing force

Nonlinear equation Eq. (11) is very complicated, and it is necessary to translate it to the form of Taylor series and ignore the higher order term, so the linearization equation can be expressed as follows:

$$f^{Fx} = \left[\frac{3\mu q A_e}{\bar{B}h_0^4} - \frac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0}\varphi'(h_0)\right]u\Delta x \tag{12}$$

where, expression of $\varphi'(h_0)$ is

$$\varphi'(h_0) = \left\{\begin{array}{l} 2\psi'(c,d,h_0) - 2\psi'(c,b+d,h_0) \\ -\psi'(c+a,d,h_0) + \psi'(c+a,b+d,h_0) \\ -\psi'(c-a,d,h_0) + \psi'(c-a,b+d,h_0) \\ -2\psi'(c,0,h_0) + \psi'(c-a,0,h_0) \\ +2\psi'(c,b,h_0) - \psi'(c-a,b,h_0) \\ +\psi'(c+a,0,h_0) - \psi'(c+a,b,h_0) \end{array}\right\} \tag{13}$$

where, expression of ψ' is

$$\psi'(\xi,\zeta,h_0) = \ln[\xi^2 + (\zeta + l_0 + h_0)^2] + 2 \tag{14}$$

Substitute Eq. (12) into Eq. (10a), and Δx can be expressed as

$$\Delta x = \frac{F_x}{\sum_{i=1}^{m}\left[\frac{3\mu q A_e}{\bar{B}h_0^4} - \frac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0}\varphi'(h_0)\right]u_i^2} \tag{15}$$

Substitute Eq. (15) into Eq. (12), and f^{Fx} can be expressed as

$$f_i^{Fx} = \frac{\Phi_i u_i}{\sum_{i=1}^{m}\Phi_i u_i^2}F_x \tag{16}$$

where, $\Phi_i = \dfrac{3\mu q_i A_{e,i}}{\bar{B}h_0^4} - \dfrac{B_{r1}B_{r2}L_i \times 10^{-6}}{4\pi\mu_0}\varphi'(h_0)$.

2.3 Solution of bearing force f^{Fy}, f^{Fz}

Similarly, bearing force f^{Fy}, f^{Fz} can be expressed as follows when external force F_y, F_z acts on skate boarding:

$$\begin{cases} f^{Fy} = \left\{\begin{array}{l}\frac{\mu q A_e}{\bar{B}}\left[\frac{1}{(h_0 - v\Delta y)^3} - \frac{1}{h_0^3}\right] \\ + \frac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0}[\varphi(h_0 - v\Delta y) - \varphi(h_0)]\end{array}\right\} \\ f^{Fz} = \left\{\begin{array}{l}\frac{\mu q A_e}{\bar{B}}\left[\frac{1}{(h_0 - w\Delta z)^3} - \frac{1}{h_0^3}\right] \\ + \frac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0}[\varphi(h_0 - w\Delta z) - \varphi(h_0)]\end{array}\right\} \end{cases} \tag{17}$$

Eq. (17) is translated into the form of Taylor series and higher order term is ignored, and then the linearization equation can be expressed as follows.

$$\begin{cases} f^{Fy} = \left[\frac{3\mu q A_e}{\bar{B}h_0^4} - \frac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0}\varphi'(h_0)\right]v\Delta y \\ f^{Fz} = \left[\frac{3\mu q A_e}{\bar{B}h_0^4} - \frac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0}\varphi'(h_0)\right]w\Delta z \end{cases} \tag{18}$$

Similarly, substitute Eq. (18) into Eq. (10b), Eq. (10c), and f^{Fy}, f^{Fz} can be expressed as

$$\begin{cases} f_i^{Fy} = \frac{\Phi_i v_i}{\sum_{i=1}^{m}\Phi_i v_i^2}F_y \\ f_i^{Fz} = \frac{\Phi_i w_i}{\sum_{i=1}^{m}\Phi_i w_i^2}F_z \end{cases} \tag{19}$$

where, $\Phi_i = \dfrac{3\mu q_i A_{e,i}}{\bar{B}h_0^4} - \dfrac{B_{r1}B_{r2}L_i \times 10^{-6}}{4\pi\mu_0}\varphi'(h_0)$.

2.4 Solution of bearing force f^{Mx}

The skate boarding only rotates in a small angle as a whole when it bears external torque.

Fig. 7 is the force diagram of the skate boarding when it bears the external torque.

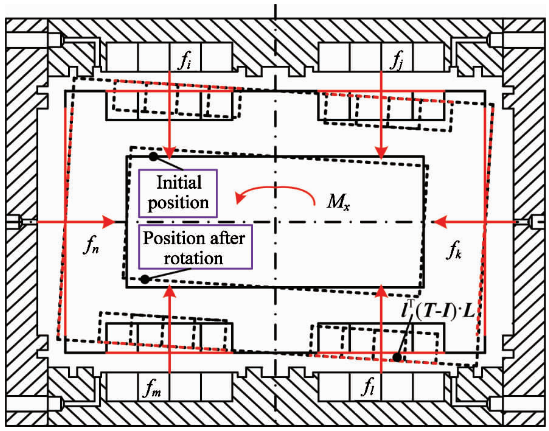

Fig. 7 Force diagram of guide-way under external torque

The bearing force of supporting unit is expressed as f^{Fx} when external torque M_x acts on the skate boarding. The unit direction vector and position of supporting unit are $\boldsymbol{l}$ and $\boldsymbol{L}$, and bearing force f^{Fx} can keep balance with external torque M_x. So the force balance equation of bearing force f^{Mx} and external torque M_x can be expressed as Eq. (10d), Eq. (10e), Eq. (10f). The number of unknown solution f^{Mx} is m, and then Eq. (10d), Eq. (10e), Eq. (10f) belong to indeterminate problem[13], so the extra equation must be sought for solving force balance equation.

2.4.1 Extra equation——deformation compatibility equation of air gap and oil film

The skate boarding rotates in a small angle γ in the direction of x axis as a whole when external torque M_x acts on the guide-way. Due to small angle γ, the direction of the bearing force of supporting unit remains the same.

According to positions and orientations position[14], the following is got:

$$ {}^{Q}\boldsymbol{L} = \boldsymbol{T}\boldsymbol{L} + \boldsymbol{R} \tag{20} $$

where, $\boldsymbol{T}$ is 3×3 rotation matrix, $\boldsymbol{R}$ is 3×1 translation matrix, $\boldsymbol{R} = (0\ 0\ 0)^{\mathrm{T}}$. ${}^{Q}\boldsymbol{L}, \boldsymbol{L}$ are the position of the skate boarding before and after rotation. The expression of matrix $\boldsymbol{T}$ is shown as follows:

$$ \boldsymbol{T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma & -\sin\gamma \\ 0 & \sin\gamma & \cos\gamma \end{bmatrix} \tag{21} $$

The position variation of skate boarding is shown as

$$ \Delta \boldsymbol{L} = {}^{Q}\boldsymbol{L} - \boldsymbol{L} = (\boldsymbol{T} - \boldsymbol{I})\boldsymbol{L} \tag{22} $$

According to position $\Delta\boldsymbol{L}$ and direction vector $\boldsymbol{l}$ of supporting unit, the position variation of supporting unit can be shown as follows:

$$ \boldsymbol{l} \cdot \Delta\boldsymbol{L} = (yv + zw)(\cos\gamma - 1) + (yw - zv)\sin\gamma \tag{23} $$

Due to small angle γ, so :

$$ \begin{cases} \lim\limits_{\gamma \to 0} \sin\gamma \to \gamma \\ \lim\limits_{\gamma \to 0} \cos\gamma - 1 \to \sqrt{1 - \gamma^2} - 1 \end{cases} \tag{24} $$

where, $\sin\gamma$, $\cos\gamma - 1$ are separately first order and second order small quantity[12], so Eq. (23) can be simplified into the following form when angle γ approaches to zero:

$$ (yw - zv)\sin\gamma \tag{25} $$

Similarly, according to Eq. (5) and Eq. (11), the bearing force variation of supporting unit can be expressed as

$$ f^{Mx} = \frac{\mu q A_e}{\bar{B}} \left\{ \frac{1}{[h_0 - (yw - zv)\sin\gamma]^3} - \frac{1}{h_0^3} \right\} + \frac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0} \{ \varphi[h_0 - (yw - zv)\sin\gamma] - \varphi(h_0) \} \tag{26} $$

2.4.2 Linearization and calculation of bearing force

Eq. (26) is very complicated, and it is necessary to be translated to the form of Taylor series and ignore the higher order term, so the linearization equation can be expressed as

$$ f^{Mx} = \left[\frac{3\mu q A_e}{\bar{B} h_0^4} - \frac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0} \varphi'(h_0) \right] (yw - zv)\Delta\gamma \tag{27} $$

Similarly, substitute Eq. (27) into Eq. (10d), and f^{Mx} can be expressed as

$$ f_i^{Mx} = \frac{\Phi_i (y_i w_i - z_i v_i)}{\sum\limits_{i=1}^{m} \Phi_i (y_i w_i - z_i v_i)^2} M_x \tag{28} $$

where, $\Phi_i = \dfrac{3\mu q_i A_{e,i}}{\bar{B} h_0^4} - \dfrac{B_{r1}B_{r2}L_i \times 10^{-6}}{4\pi\mu_0} \varphi'(h_0)$.

2.5 Solution of bearing force f^{My}, f^{Mz}

Similarly, bearing force f^{My}, f^{Mz} when external torque M_y, M_z acts on skate boarding can be expressed as follows:

$$ \begin{cases} f^{My} = \dfrac{\mu q A_e}{\bar{B}} \left\{ \dfrac{1}{[h_0 - (xw - zu)\sin\beta]^3} - \dfrac{1}{h_0^3} \right\} \\ \quad + \dfrac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0} \{ \varphi[h_0 - (xw - zu)\sin\beta] - \varphi(h_0) \} \\ f^{Mz} = \dfrac{\mu q A_e}{\bar{B}} \left\{ \dfrac{1}{[h_0 - (xv - yu)\sin\alpha]^3} - \dfrac{1}{h_0^3} \right\} \\ \quad + \dfrac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0} \{ \varphi[h_0 - (xv - yu)\sin\alpha] - \varphi(h_0) \} \end{cases} \tag{29} $$

Eq. (29) is translated into the form of Taylor series and higher order term is ignored, then the linearization equation can be expressed as follows:

$$ \begin{cases} f^{My} = \left[\dfrac{3\mu q A_e}{\bar{B} h_0^4} - \dfrac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0} \varphi'(h_0) \right] (xw - zu)\Delta\beta \\ f^{Mz} = \left[\dfrac{3\mu q A_e}{\bar{B} h_0^4} - \dfrac{B_{r1}B_{r2}L \times 10^{-6}}{4\pi\mu_0} \varphi'(h_0) \right] (xv - yu)\Delta\alpha \end{cases} \tag{30} $$

Similarly, substitute Eq. (30) into Eq. (10e), Eq. (10f), and f^{My}, f^{Mz} can be expressed as follows:

$$\begin{cases} f_i^{My} = \dfrac{\Phi_i(x_i w_i - z_i u_i)}{\sum_{i=1}^{m} \Phi_i (x_i w_i - z_i u_i)^2} M_y \\ f_i^{Mz} = \dfrac{\Phi_i(x_i v_i - y_i u_i)}{\sum_{i=1}^{m} \Phi_i (x_i v_i - y_i u_i)^2} M_z \end{cases} \quad (31)$$

where, $\Phi_i = \dfrac{3\mu q_i A_{e,i}}{\bar{B} h_0^4} - \dfrac{B_{r1} B_{r2} L_i \times 10^{-6}}{4\pi\mu_0} \varphi'(h_0)$.

2.6 Summary of bearing force of supporting unit

The bearing force of supporting unit can be expressed as follows when external load $\boldsymbol{F}$ acts on the skate boarding.

$$f_i = \Gamma_1 F_x + \Gamma_2 F_y + \Gamma_3 F_z + \Gamma_4 M_x + \Gamma_5 M_y + \Gamma_6 M_z \quad (32)$$

where, $\Gamma_1 = \dfrac{\Phi_i u_i}{\sum_{i=1}^{m} \Phi_i u_i^2}$, $\Gamma_4 = \dfrac{\Phi_i(y_i w_i - z_i v_i)}{\sum_{i=1}^{m} \Phi_i (y_i w_i - z_i v_i)^2}$;

$\Gamma_2 = \dfrac{\Phi_i v_i}{\sum_{i=1}^{m} \Phi_i v_i^2}$, $\Gamma_5 = \dfrac{\Phi_i(x_i w_i - z_i u_i)}{\sum_{i=1}^{m} \Phi_i (x_i w_i - z_i u_i)^2}$;

$\Gamma_3 = \dfrac{\Phi_i w_i}{\sum_{i=1}^{m} \Phi_i w_i^2}$, $\Gamma_6 = \dfrac{\Phi_i(x_i v_i - y_i u_i)}{\sum_{i=1}^{m} \Phi_i (x_i v_i - y_i u_i)^2}$;

$$\Phi_i = \frac{3\mu q_i A_{e,i}}{\bar{B} h_0^4} - \frac{B_{r1} B_{r2} L_i \times 10^{-6}}{4\pi\mu_0} \varphi'(h_0).$$

3 Calculation Example

As shown in Fig. 3, O is the center of the track geometry, and there are 12 oil supporting pocket on the Table 1. The position $\boldsymbol{L}_i$ and the bearing capacity of each oil pocket $\boldsymbol{l}_i$ are:

$$\boldsymbol{L}_i = \begin{bmatrix} -0.1000 & -0.1000 & -0.1500 \\ -0.1000 & 0.1000 & -0.1500 \\ 0.1000 & -0.1000 & -0.1500 \\ 0.1000 & 0.1000 & -0.1500 \\ -0.1000 & -0.1000 & 0.1500 \\ -0.1000 & 0.1000 & 0.1500 \\ 0.1000 & -0.1000 & 0.1500 \\ 0.1000 & 0.1000 & 0.1500 \\ -0.0950 & -0.2170 & 0 \\ 0.0950 & -0.2170 & 0 \\ -0.0950 & 0.2170 & 0 \\ 0.0950 & 0.2170 & 0 \end{bmatrix}^{\mathrm{T}}$$

$$\boldsymbol{l}_i = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & -1 \\ 1 & 1 & 1 & 1 & -1 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \quad (33)$$

Table 1 Parameters of MLSGS

Number of oil pockets	B(m)	b_1(m)	A(m)	a_1(m)
12	0.176	0.015	0.176	0.015
Bearing area A_e(m^2)	Oil film thickness δ_0(μm)	Thickness of galvanized layer l_0(mm)	Residual magnetic induction B_1/B_2(T)	Dynamic viscosity μ(Pa · s)
0.0259	30	0.5	1.4	0.04136
a(mm)	b, d(mm)	c(mm)	L (mm)	Initial flow $q_{0,0}$(L/min)
30	30	0	120	0.1446

In operation, the cutting force is (20 000, 20 000, 20 000) N, the cutting torque is (10 000, 10 000, 10 000) N · m, and the oil bearing capacity of the rotary table is obtained according to Eq. (32), as shown in Table 2.

4 Discussion

From Table 2 in the 1 – 12 groups, it can be seen that the greater the change in oil film thickness is, the greater the total bearing force changes . Take the maximum change in film thickness of the 10,12 group data as an example: in the 10th group, the change in film thickness is 11.71 μm. The Bearing force is 31.31 kN, and the magnetic force is 21.36 kN, the hydrostatic force is 9.95 kN; In the 11th group, the change in film thickness is –11.71 μm. The Bearing force is –31.31 kN, and the magnetic force is –149.31 kN, the hydrostatic force is –31.31 kN; It can be seen that the change of the permanent magnetic force plays a major role in the change of the total bearing force. According to the data of 1 – 12 groups, when the force and torque are applied, the oil chamber (9 – 12 groups) on both sides has only a small rotation

Table 2 Calculation result of oil pocket

Number	1	2	3	4	5	6
Bearing force f_i (kN)	-22.5	2.5	2.5	27.5	22.5	-2.5
Hydrostatic force f_{iy} (kN)	71.78	24.39	24.39	11.05	12.74	29.41
Magnetic force f_{id} (kN)	-94.28	-21.89	-21.89	16.45	9.76	-31.91
Film thickness $\triangle h$ (μm)	-8.41	0.93	0.93	10.28	8.41	-0.93
Rotation angle γ (°)	-0.4795	0.0533	-0.0533	-0.5861	-0.4795	0.0533
Rotation angle β (°)	0.4795	0.0533	-0.0533	0.5861	0.4795	0.0533
Rotation angle α (°)	0	0	0	0	0	0
Number	7	8	9	10	11	12
Bearing force f_i (kN)	-2.5	-27.5	-21.31	31.31	21.31	-31.31
Hydrostatic force f_{iy} (kN)	29.41	94.19	67.54	9.95	13.19	118
Magnetic force f_{id} (kN)	-31.91	-121.69	-88.86	21.36	8.12	-149.31
Film thickness $\triangle h$ (μm)	-0.93	-10.28	-7.97	11.71	7.97	-11.71
Rotation angle γ (°)	-0.0533	-0.5861	0	0	0	0
Rotation angle β (°)	-0.0533	0.5861	0	0	0	0
Rotation angle α (°)	0	0	0.4543	-0.6674	0	-0.6674

in the z-axis direction. The rotation angle of the upper and lower oil chambers (1 - 8 groups) around the z axis does not change, the rotation angle exists around the x, y axis. For all of the group data, although the bearing capacity of the corresponding oil chamber is different, the change trend of the bearing capacity of the oil chamber is the same, which proves the correctness of the calculation method.

5 Conclusion

With the external load, the oil pocket parameters and the parameters of the permanent magnet are determined, the bearing unit coupling load of MLSGS in the complex confinement form is deduced when the guideway with permanent magnet suspension and hydrostatic pressure is adopted. According to this mathematical model, the relationship between the change of bearing force caused by the change of oil film thickness is obtained, that is, the larger the change of oil film thickness is, the greater the change of total bearing force is, and the permanent magnetism changes play a leading role. The study provides a static calculation method for the coupling capacity of MLSGS under complex constraint condition, which is convenient for the engineering designers to master and improve the design efficiency and precision.

References

[1] Kai Y, Fukuta M, Matsubara A, et al. Dynamic characteristics of ultra-precision machine tools (evaluation for stiffness of guide ways by impact test)[J]. *Transactions of the Japan Society of Mechanical Engineers*, Part C. 2013, 79(808): 4624-4633

[2] Zhao J H. Theoretical analysis and experimental research of liquid hydrostatic slide's performance of gantry moving milling center [D]. Qinhuangdao: Yanshan University, School of Mechanical Engineering, 2013, 1-14. (In Chinese)

[3] Shenyang Machine Tool Co., Ltd. GTM series vertical turning and milling complex machining center heavy moving column dynamic beam vertical turning center. [EB/OL]. http://www.smtcl.com/Products/Turning/GTM/: Smtel, 2017

[4] Zhao J H, Liang Y N, Gao D R. Oil pocket's bearing capacity analysis of liquild hydrostatic worktable in gentry moving milling center[J]. *Chinese Journal of Mechanical Engineering*, 2014, 27(5): 1008-1017 (In Chinese)

[5] Zhao J H, Zhou S L, Lu X H, et al. Numerical simulation and experimental study of heat-fluid-solid coupling of double flapper-nozzle servo valve[J]. *Chinese Journal of Mechanical Engineering*, 2015, 28(5): 1030-1038 (In Chinese)

[6] Wang Z W, Zhao W H, Lu B H. Influencing Factors on Dynamic Response of Hydrostatic Guide ways [C]. In: Proceedings of the 2nd International Conference on Advances in Materials and Manufacturing, Guilin, China, 2011. 2095-2101

[7] Heyne M, Kowallik T, Erbe T, et al. Measurement of the percentage contact area of high precision planar ball guide

ways [C]. In: Proceedings of the 10th International Conference of the European Society for Precision Engineering and Nanotechnology, Delft, Netherlands, 2010: 445-448

[8] Gao D R, Zhao J H, Zhang Z C, et al. Study on the influence of PM flow controller parameters on the performance of liquid hydrostatic guide way[J]. *Chinese Journal of Mechanical Engineering*, 2011, 47(18): 186-194 (In Chinese)

[9] Shao J P, Zhang X T. Study on control scheme of film thickness of liquid hydrostatic guide[J]. *Energy Conservation Technology*, 2006, (24): 558-561

[10] Zhang L, Dai H L, Liu S R. Research on adaptive oil supply system of static pressure guide based on AMESim [J]. *Manufacturing Technology & Machine Tool*, 2010 (12): 25-28

[11] Ding Z Q. Design of Hydrostatic Bearing[M]. Shanghai: Shanghai Scientific & Technical Publishers, 1989. 95-100(In Chinese)

[12] Tian L L, Zhang K S, Wang D Y et al. Research on the levitation and guidance magnetic force of permanent magnetic guide way[J]. *Proceedings of the CSEE*, 2008, 28 (21): 135-139

[13] Bai X Z. Mechanics of Materials [M]. Beijing: China Building Material Industry Publishing House, 2005. 31-40(In Chinese)

[14] Xiong Y L. Fundamentals of Robotics [M]. Wuhan: Huazhong University of Science and Technology Press, 2007. 75-80(In Chinese)

Weng Gangmin, born in 1963, Wuhan, P. R. China. He graduated from School of Management, Tianjin University. He is a Grammar School of Yanshan University. He is a professor of Yanshan University, P. R. China. His main research directions include the tourism economy, tourism business management, industrial economy.