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Application of non-probabilistic reliability on aerostat capsule①

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Abstract

Aerostat capsule is small sample data, so designing reliability is very difficult to be obtained accurately by conventional probabilistic reliability method. Based on the interval non-probabilistic reliability theory, an instability mathematics model of envelope structure is studied, and the calculation formula of interval reliability index is put forward. Through the mechanical experiments of three capsule structures, the experimental results of the interval reliability are obtained. By comparing the theoretical and measured values, it is found that the theoretical reliability index is more conservative. Non-probabilistic reliability method can reflect the reliability degree of the capsule body under different loading conditions, which can provide some guidance for engineering application.

Key words: non-probabilistic, aerostat, reliability index, capsule

0 Introduction

Aerostat[1,2] refers to the aircraft lighter than air density, which is referred to as the tethered balloon and airship. With the emergence of new technology and the development of material science, the aerostat ushered in the rapid development stage. So far, there are at least a dozen of countries developing aerostat. The number and type of aerostat applied in economic life and military struggle are increasing, such as the rapidly elevated aerostat platform (REAP) deployed in Iraq by United States, the rapid aerostat initial development (RAID) deployed in Afghanistan and the stratospheric airship program under development.

Many domestic scholars have done extensive researches on aerostat. Cheng et al. [3] and Wang[4] discussed the shape and structure of the aerostat and verified the inflatable structure strength by experiments.

Aerostat belongs to complex and expensive aviation products. It is not mass produced usually, so extreme lack of statistical data related to reliability exists, which belongs to the small sample and poor information. Traditional methods of reliability evaluation usually adopt probabilistic methods. But when the real statistical data of the product is difficult to obtain or the model is not accurate enough, it is difficult to obtain high confidence in the probabilistic reliability assessment method. In the middle of 1990s, Benhaim[5] proposed the concept of non-probabilistic reliability, and Qiu and Elishakoff[6] made further research and improvement. The advantage of non-probabilistic reliability is low requirement of the original data, which only needs to know uncertain parameter bounds, not requiring distribution form, not involving the concept of probability, nor requiring to calculate the probability density function and membership function. After the introduction of non-probabilistic reliability, a lot of scholars have paid attention to it. They have put forward the methods of interval reliability, robust reliability, and so on, and have carried on many aspects of application research.

According to the small sample characteristics of aerostat, the interval reliability theory is introduced to determine aerostat reliability, by which the interval reliability index of the software structure of aerostat is obtained. Through experiments, the non-probabilistic reliability index of actual structure under steady state is compared with the theoretical interval reliability index.

1 Representations of interval variables and interval non-probabilistic reliability

If uncertain parameter V changes within interval $[V^L, V^U]$, then V is an interval variable and there is $V \in V^I = [V^L, V^U]$, in which, V^L is the interval lower bound of V, V^U is the interval upper bound of V, and V^I is the real valued interval of V.

Let:

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$$V^C = \frac{V^U + V^L}{2}, \quad V^R = \frac{V^U - V^L}{2} \tag{1}$$

there is

$$V^L = V^C - V^R, \quad V^U = V^C + V^R \tag{2}$$

and, V^I and V are represented respectively by

$$\begin{aligned} V^I &= [V^L, V^U] = V^C + V^R\Delta^I \\ V &= V^C + V^R\Delta \end{aligned} \tag{3}$$

in which, $\Delta^I = [-1,1]$ is a standardized interval, $\Delta \in \Delta^I$ is a standardized interval variable.

Obviously, V^I is uniquely determined by V^C and V^R, which are mean value and deviation of interval $[V^L, V^U]$ respectively.

The measure of interval non-probabilistic reliability is described as follows. Set two mutually dependent interval variables, and the corresponding function M^I is expressed as

$$M^I = R^I - S^I \tag{4}$$

where, R^I and S^I are strength and stress interval variables respectively. By Eq. (3), the following is got:

$$\begin{aligned} R^I &= R^c + R^r\Delta^I \\ S^I &= S^c + S^r\Delta^I \end{aligned} \tag{5}$$

where, R^c and S^c are mean values of R^I and S^I respectively, while R^r and S^r are deviations of R^I and S^I respectively.

Putting Eq. (5) into Eq. (4), then

$$M^I = (R^c - S^c) + (R^r - S^r)\Delta^I \tag{6}$$

Defining the non-probabilistic reliability index η as

$$\eta = \begin{cases} (R^c - S^c)/(R^r - S^r) & R^c > S^c \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

When the structural uncertain parameters are interval variables, it can be assumed that the structure is in just two deterministic states: reliable or unreliable. Dimensionless quantity $\eta = 1$ is the reliable critical value. When $\eta > 1$, the system is reliable. The greater the value of η is, the higher the degree of safety of the structure is. Thus, η can measure both structural safety and reliability.

2 Critical bending load of inflatable model[4,7]

When the closed membrane structure is in the inflated state, there is prestress in the film. When the film is bent under a bending load, one side of the cross section of the pipe wall is under the additional axial tensile stress, and the other side is under the additional axial compressive stress. When the bending load reaches a certain value, the compressive stress is greater than the axial tensile stress produced by the internal pressure of the pipe, and then the pipe wall at the compression side starts to fold, as shown in Fig. 1. For further bending, the fold expands from the compression side along the circumference of the cross section of the tube wall until it is covered with the entire cross section[7].

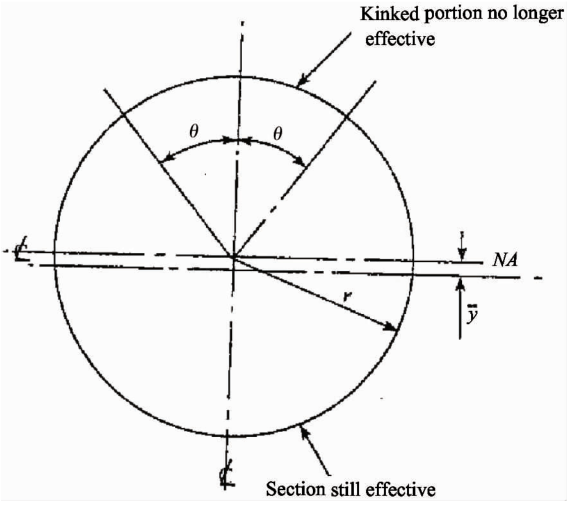

Fig. 1 Extent of kink around the cross section

In Fig. 1, θ is the angle of the fold region, NA is the position of neutral axis of cross section, $\bar{y}$ is the distance between the neutral axis and the initial neutral axis, and r is the section radius.

When an elastic film inflatable tube is inflated, if regardless of film thickness, the axial pretension in the film is expressed as

$$T = \frac{P\pi r^2}{2\pi r} = \frac{Pr}{2} \tag{8}$$

in which the unit of T is N/m, P is the internal and external pressure difference of air pipe, and r is the radius of cross section of the air pipe.

When deflection occurs under lateral force, the axial tension of the moment on the cross section is expressed as follows[3] (Fig. 2):

$$T' = \frac{My}{I_z} = \frac{FLy}{I_z} \tag{9}$$

where, M is the bending moment of the cross section of the inflated tube, y is the distance between the section of the inflated pipe and the center of inertia moment, I_z is the moment of inertia of the inflated tube, F is the bending load, and L is the distance between the loading point and the fixed end.

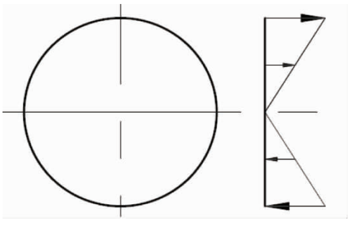

Fig. 2 Axial stress produced by bending moment

I_z is expressed as

$$I_z = 4\int_0^{\frac{\pi}{2}} y^2 r\mathrm{d}\theta = \pi r^3 \tag{10}$$

When $y = \pm r$, the axial tension on the cross section has the maximum value. At the bottom of the fixed end, the axial compressive stress produced by bending moment is the largest, where wrinkles appear first. The axial tension at this point is

$$T'' = T - T' = \frac{Pr}{2} - \frac{FL}{\pi r^3} \tag{11}$$

When T'' is zero, structure is unstable, and the resulting load is defined as a critical load, which is noted as F_{cr}.

$$F_{cr} = \frac{P\pi r^3}{2L} \tag{12}$$

When the load exceeds the critical load, the folded region of the capsule is no longer valid, and the moment of inertia and the central axis also change, no longer linearly.

3 Non-probabilistic reliability model of inflatable model

After the aerostat capsule is produced, it could be considered as deterministic structure. Because the body of the capsule has a certain permeability, the deformation of the capsule will also cause changes in internal pressure. After the balloon inflation, internal pressure P has a certain amount of change. Due to the lack of large sample statistics, it is impossible to determine the pressure changes consistent with a probabilistic statistical model. So internal pressure P is defined as an interval variable, which changes in the interval $[P^L, P^U]$. According to Eq. (12), there is

$$F_{cr} = \frac{\pi r^3}{2L}[P^L, P^U] \tag{13}$$

From Eq. (13), it can be seen that F_{cr} is an interval variable and $F_{cr} \in F_{cr}^I = [F_{cr}^L, F_{cr}^U]$. F_{cr}^L is the interval lower bound and $F_{cr}^L = \frac{\pi r^3}{2L}P^L$. F_{cr}^U is the interval upper bound and $F_{cr}^U = \frac{\pi r^3}{2L}P^U$.

The inflation can make the closed membrane structure obtain the flexural strength and rigidity that the original film material does not have. One can increase the inflation pressure to improve the carrying capacity. In engineering, the structure of inflatable capsule not only requires sufficient strength and stiffness, but also requires good stability. The capsule body structure bending failure is a gradual process. When the load is less than the critical load value, the central axis and the moment of inertia of the membrane structure can be approximated by a linear change. When the load exceeds the critical load, the capsule wrinkle region is no longer valid, and moment of inertia and the central axis are changed, being no longer a linear process change. When the capsule folds occur, deformation of the inflatable capsule will be mutated, and then it can be considered that the structure of the capsule is unreliable and can not meet the structural stability requirements. Based on the first passage failure theory, an interval non-probabilistic interference model is established:

$$M^I = F_{cr}^I - F^I \tag{14}$$

in which, F^I is an interval bending load and $F^I = [F^L, F^U]$. F^L is the interval lower bound and F^U is the interval upper bound.

Then the interval non-probabilistic reliability index of capsule structure can be expressed as

$$\eta = \begin{cases} (F_{cr}^c - F^c)/(F_{cr}^r - F^r) & F_{cr}^c > F^c \\ 0 & \text{otherwise} \end{cases} \tag{15}$$

where, F_{cr}^c and F_{cr}^r are mean values and deviations of F_{cr}^I respectively, while F^c and F^r are mean values and deviations of F^I respectively.

4 Inflatable model test

4.1 Test device

The test device is shown in Fig. 3. The model length is 100 cm, the diameter is 16 cm, the loading point is 80 cm from the fixed end, and the measuring point is 60 cm from the fixed end. The two are not in one position, mainly considering the local deformation of the loading point, which will affect the measurement accuracy[4].

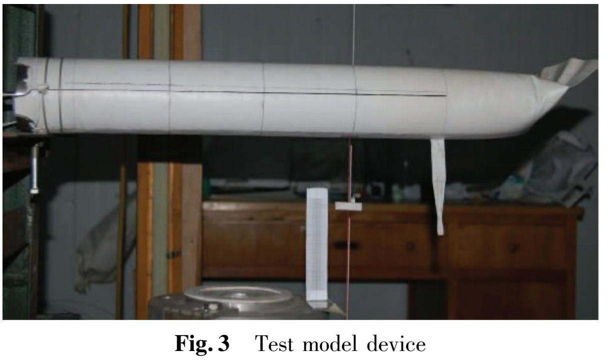

Fig. 3 Test model device

According to the concentrated load deflection formula of cantilever beam, there is

$$\delta = \frac{FL^3}{3EI} = \frac{FL^3}{3\pi Er^3} \tag{16}$$

where, δ is deflection, F is bending load, L is the distance from the loading point to the fixed end, E is the modulus of elasticity, and I is the cross sectional moment of inertia.

Seen from Eq. (16), the deflection of the cantilever beam is linearly related to the external force, thus the stress failure model can be mapped to a deflection failure model. That is to say, when the deflection of the specified point of the capsule structure exceeds the critical deflection, the instability of the capsule structure occurs. Therefore, the external interference model can be equivalently mapped into a deflection interference model, noted as

$$\eta = \begin{cases} (\delta_{cr}^c - \delta^c)/(\delta_{cr}^r - \delta^r) & \delta_{cr}^c > \delta^c \\ 0 & \text{otherwise} \end{cases} \tag{17}$$

in which, δ_{cr}^c and δ_{cr}^r are mean values and deviations of interval critical deflection δ_{cr}^I respectively, while δ^c and δ^r are mean values and deviations of interval deflection δ^I respectively.

4.2 Test procedure

The internal pressure of the test is 10 kPa, the model is filled to the specified pressure, and the end load is applied in the form of weights, starting from zero load until the model loses load capacity, and the test is repeated three times.

The experiment provides three identical structural test pieces, repeat the same experimental steps, draw three sets of experimental data, and finally analyze the experimental data in line with the concept of small sample size and small data.

4.3 Test results

The unstable state of the inflatable model is shown in Fig. 4. A large number of folds are produced at the root of the model, and the deformation of the model increases rapidly and eventually loses its carrying capacity.

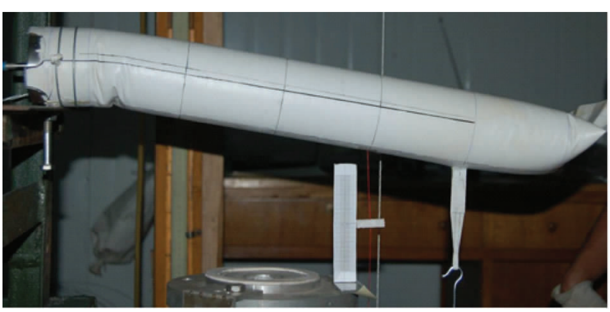

Fig. 4 Model instability state

When internal pressure is 10 kPa, the test load is plotted and the deformation of the measurement is as a curve, as shown in Fig. 5. The first half of the test data curve is linear. After the wrinkling of the model, displacement of the model accelerates too. In the final stage, the displacement increases dramatically and the model loses its load-carrying capacity, and the ultimate buckling load of the model is 19.4 N.

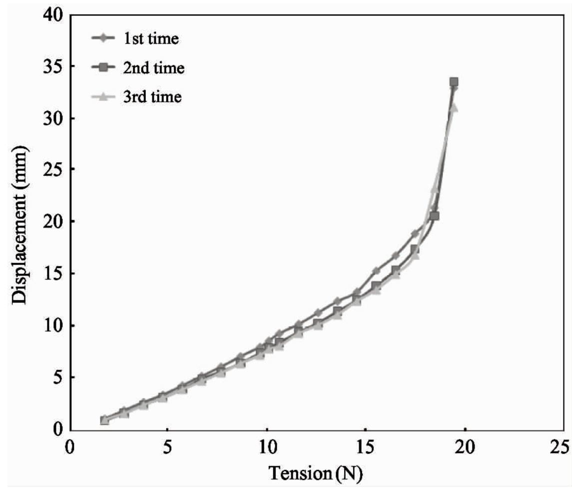

Fig. 5 Load and displacement curves of models

The experimental data is translated into the interval number, the interval change curve is drawn, as shown in Fig. 6. Respectively, taking the theoretical critical load 10.05 N and the experimental instability load 19.4 N as the threshold, the non-probabilistic reliability index curve is plotted, as shown in Fig. 7.

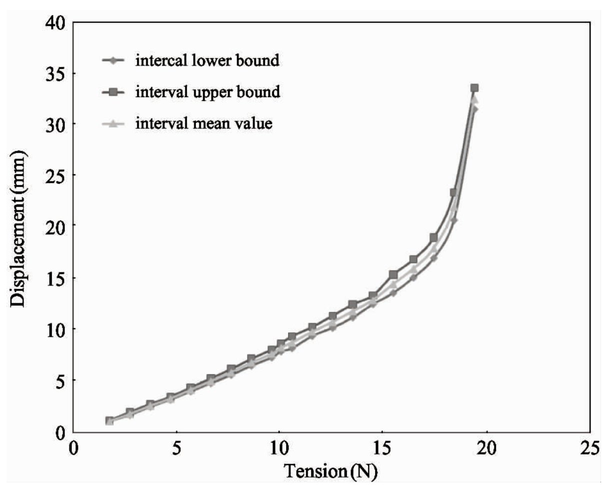

Fig. 6 Load and displacement interval curve of models

5 Results analysis

When the inflation pressure is 10 kPa, the theoretical calculation value of critical buckling load of the test model is 10.05 N, while the test value is 19.4 N. The test value is 1.93 times of the theoretical calculation value, which shows that the model still has a strong carrying capacity after folding.

After translating the test data into interval number and creating the interval model, the interval non-probability is calculated. Seen from Fig. 6, at the initial stage of the experiment and before the instability, the interval changes are little, and then the interval changes gradually. It shows that the structure changes little at low load and the experimental data is relatively concentrated; while the closer the structure is to the unstable state, the greater the change in the test results is, and the more divergent the experimental data distribution is.

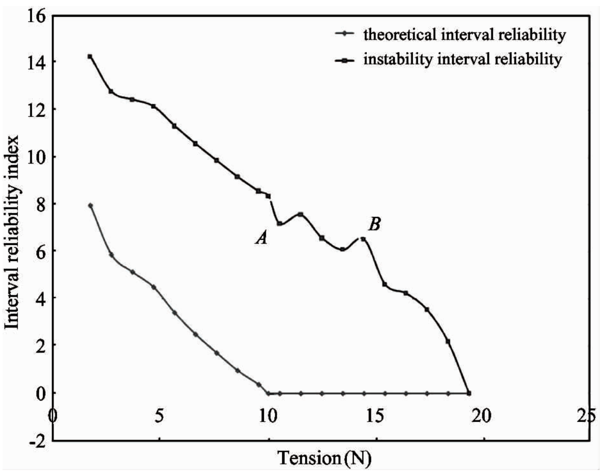

Fig. 7 Interval reliability curve of models

In Fig. 7, the downward trend of the interval reliability index before the theoretical critical load is steady and approximately linear. When the external load is more than the theoretical critical load value, the theoretical interval reliability index is zero, while the unstable interval reliability index changes sharply. In the graph, the slope of the curve increases gradually and the failure accelerates. In Fig. 7, there are two points of mutation, which is because only three experiments have been done, the experimental samples are relatively few and the coverage is small. Point A interval variance increases relative to two adjacent points, while point B interval variance becomes smaller relative to two adjacent points, which causes two probability interval reliability index mutations. The overall trend is declining, consistent with the actual situation of the project.

6 Conclusion

In this paper, critical load and experimental analysis of the inflatable model are carried out, the experimental data are processed by interval parameters, and the structural interval non-probabilistic reliability data curve is obtained. Before the theoretical critical buckling load, the relationship between the deformation of the model and the load is approximately linear. After the theoretical critical load, the model produces a fold. As the load increases, the fold region expands and eventually loses its carrying capacity. The interval non-probabilistic reliability model is a good reflection of this trend. According to the different strength indexes, the corresponding non-probabilistic reliability curve is plotted to reflect the reliability of the structure under different load conditions, which can provide some guidance for the engineering application.

References

[1] Hembree B, Slegers N. Tethered aerostat modeling using an efficient recursive rigid-body dynamics approach[J]. *Journal of Aircraft*, 2015, 48(2): 623-632

[2] Zhang P, Wang P A. Simulation and experiment of mechanical properties of load patch structure of an aerostat [J]. *Electro-Mechanical Engineering*, 2016, 32(5): 30-33

[3] Cheng Q, Yuan J H, Tang X, et al. A preliminary research on shape optimization of tethered balloon envelope [J]. *Journal of Xi'an Aerotechnical University*, 2012, 30(3):42-45, 70

[4] Wang P A. Research on destabilization load of inflatable fabric beam[J]. *Journal of Xi'an Aeronautical University*, 2014, 32(3): 7-10

[5] Benhaim Y. Convex models of uncertainty in radial pulse buckling of shells [J]. *Journal of Applied Mechanics*, 1993, 60(3): 683-688

[6] Qiu Z P, Elishakoff I. Anti-optimization of structures with large uncertain but non-random parameters via interval analysis[J]. *Computer Methods in Applied Mechanics and Engineering*, 1998, 152(3): 361-372

[7] Khoury G A. Airship Technology[M]. 2nd Edition. England: Cambridge University Press, 2012. 165-166

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