doi:10.3772/j.issn.1006-6748.2018.04.004

Marginalized cubature Kalman filtering algorithm based on linear/nonlinear mixed-Gaussian model①

Hu Yumei(胡玉梅)*, Hu Zhentao②**, Jin Yong**
(* School of Automation, Northwestern Polytechnical University, Xi'an 710129, P. R. China)
(** College of Computer and Information Engineering, Henan University, Kaifeng 475004, P. R. China)

Abstract

Aiming at improving the estimation accuracy and real-time of nonlinear system with linear Gaussian sub-structure, a novel marginalized cubature Kalman filter is proposed in Bayesian estimation framework. Firstly, the marginalized technique is adopted to model the target system dynamics with nonlinear state and linear state separately, and the two parts are estimated by cubature Kalman filter and standard Kalman filter respectively. Therefore, the linear part avoids the generation and propagation process of cubature points. Accordingly, the computational complexity is reduced. Meanwhile, the accuracy of state estimation is improved by taking the difference of nonlinear state estimation as the measurement of linear state. Furthermore, the computational complexity of marginalized cubature Kalman filter is discussed by calculating the number of floating-point operation. Finally, simulation experiments and analysis show that the proposed algorithm can improve the performance of filtering precision and real-time effectively in target tracking system.

Key words: state estimation, marginalized modeling, mixed-Gaussian model, cubature Kalman filter

0 Introduction

State estimation is widely used in military and defense fields such as aerial reconnaissance and early warning, ballistic missile defense and battlefield surveillance, civilian fields such as air traffic control, intelligent vehicles and robot vision. However, with the rapidly development of technology of sensors, communication and computing ability, the higher requirements of accuracy and real-time in practical target system application are needed. State estimation methods are often accompanied with nonlinear problems. Unfortunately, endless parameters are needed to describe posterior probability density function for nonlinear system estimation, which can be hardly applied in practice. Only some specific problems can obtain the optimal solution[1,2]. For this reason, many domestic and foreign scholars have carried on the approximations of the posterior, and proposed massive suboptimal methods. Among them, the random sampling filter represented by particle filter (PF)[3,4] and the determination sampling filter represented by unscented Kalman filter (UKF)[5-7], cubature Kalman filter (CKF)[8-10] are widely used for nonlinear estimation.

Particle filter uses a set of random weighted particles in the state space to approximate the probability density function. It is not constrained by the model assumption of linear and Gaussian and suitable for most nonlinear non-Gaussian dynamic system. However, the random sampling nonlinear filter is computationally intensive and requires a large number of particles to ensure the accuracy and convergence. For such problems, the theoretical derivation of marginalized particle filter (MPF) for three types of models is given in Ref. [11] according to the system modeling from special to general. It provided an effective filtering method for nonlinear system with linear Gaussian sub-structure. Subsequently, Schön et al[4]. applied this method to the felids of submarine topographic location and target tracking to estimate the submarine's own position. It was pointed out that it is difficult to obtain more accurate linear measurement of map database dur-

① Supported by the National Natural Science Foundation of China (No. 61771006), the Open Foundation of Key Laboratory of Spectral Imaging Technology of the Chinese Academy of Sciences (No. LSIT201711D), the Outstanding Young Cultivation Foundation of Henan University (No. 0000A40366) and the Excellent Chinese and Foreign Youth Exchange Programme of China Science and Technology Association (2017CASTQNJL046).

② To whom correspondence should be addressed. E-mail: huzhentao2011@126.com

Received on Nov. 29, 2017

ing terrain-assisted location. But the use of multi-sensor measurement in MPF can effectively avoid the error caused by linearization[4].

Karlsson et al.[12,13] analyzes the computational complexity of MPF in theory and simulation by calculating the number of floating-point operation and the equivalent floating-point operations, proving that MPF has better real-time performance. Hu and Liu[14] optimizes the consistency of particle weights by constructing consistency distance and the consistency matrix to improve particle filter accuracy in nonlinear system. However, due to the inherent defect of random sampling filter, the computation of marginalized particle filter is still large. Compared with the random sampling filter, UKF generates a set of points through deterministic sampling and nonlinearly transform strategy to characterize the distribution of system state variable. It is proved that UKF performs an effectively weighted approximation to nonlinear posterior distribution, and is strongly universal for nonlinear Gaussian system[15,16]. To improve the filtering accuracy, an alternative framework for iterated unscented Kalman filter is presented in Ref. [7]. However, whether the parameter selection of UKF is reasonable directly influences the positive define of filter covariance and the state estimation accuracy. Similaritily, CKF uses the third-order cubature rule to approximate weighted Gaussian integral, and the nonlinear state posterior probability density function is approximated by a set of weighted deterministic sampling points. In the process of sampling and filtering the weight of CKF is always positive, ensuring the filtering covariance positive definite[17]. In addition, because the number of samples of deterministic sampling method is smaller than random sampling, CKF has better real-time than PF[18].

Considering the nonlinear system with linear Gaussian sub-structure and the superiority of CKF, the nonlinear state and linear state of target tracking system are modeled separately by marginalized theory. As a result, a novel marginalized cubature Kalman filter (MCKF) algorithm is proposed to improve tracking accuracy and real-time. The MCKF has two advantages.

1) In the standard CKF, both non-linear state and linear state need to calculate and propagate cubature points, while this process can be avoided in the linear state estimation in MCKF, and its real-time performance can be improved.

2) Since the linear state and nonlinear state are filtered by CKF and KF respectively, the optimal estimation condition is satisfied in the linear estimate process, which improves the accuracy of global state estimation.

1 Marginalized modeling

Considering a class of nonlinear systems with linear Gaussian sub-structure, the system is modeled as follows according to marginalized principle:

$$\boldsymbol{x}_{N,k} = \boldsymbol{f}^1_{N,k}(\boldsymbol{x}_{N,k-1}) + \boldsymbol{f}^2_{N,k}(\boldsymbol{x}_{L,k-1}) + \boldsymbol{\omega}_{N,k-1} \quad (1)$$

$$\boldsymbol{x}_{L,k} = \boldsymbol{F}^3_{L,k}\boldsymbol{x}_{N,k-1} + \boldsymbol{F}^4_{L,k}\boldsymbol{x}_{L,k-1} + \boldsymbol{\omega}_{L,k-1} \quad (2)$$

where, $\boldsymbol{x}_{N,k}$ and $\boldsymbol{x}_{L,k}$ denote the nonlinear and linear states respectively, $\boldsymbol{f}^i_{N,k}(\cdot)$ and $\boldsymbol{F}^i_{L,k}$ denote the transfer function matrices of nonlinear state and linear state respectively. Nonlinear system noise $\boldsymbol{\omega}_{N,k}$ and linear system noise $\boldsymbol{\omega}_{L,k}$ are independent, and with respect to $\boldsymbol{\omega}_{N,k} \sim \mathcal{N}(0, \boldsymbol{\sigma}^2_{\boldsymbol{\omega}_{N,k}})$ and $\boldsymbol{\omega}_{L,k} \sim \mathcal{N}(0, \boldsymbol{\sigma}^2_{\boldsymbol{\omega}_{L,k}})$, respectively. The system nonlinear state measurement and linear state measurement are given as follows.

$$z_{N,k} = \boldsymbol{h}_k(\boldsymbol{x}_{N,k}) + \boldsymbol{v}_{N,k} \quad (3)$$

$$z_{L,k} = \boldsymbol{C}_k\boldsymbol{x}_{L,k} + \boldsymbol{v}_{L,k} \quad (4)$$

Among them, $\boldsymbol{h}(\cdot)$ is measurement matrix, denoting the mapping relationship from $\boldsymbol{x}_{N,k}$ to $\boldsymbol{z}_{N,k}$ function, $\boldsymbol{C}_k$ is linear measurement matrix. The measurement noise $\boldsymbol{v}_{N,k}$ and $\boldsymbol{v}_{L,k}$ are independently Gaussian white noise, and $\boldsymbol{v}_{N,k} \sim \mathcal{N}(0, \boldsymbol{\sigma}^2_{\boldsymbol{v}_{N,k}})$, $\boldsymbol{v}_{L,k} \sim \mathcal{N}(0, \boldsymbol{\sigma}^2_{\boldsymbol{v}_{L,k}})$. In particular, when the linear state only contains velocity:

$$z_{L,k} = (\boldsymbol{x}_{N,k} - \boldsymbol{x}_{N,k-1})/\tau + \boldsymbol{v}_{L,k} \quad (5)$$

where, τ denotes the sampling interval.

2 Marginalized cubature Kalman Filter

According to Bayesian method, the system state estimation can be realized by calculating posterior probability density function (PDF). However, the analytical solution PDF can't be obtained in nonlinear systems. MCKF filtering for nonlinear state and linear state separately, that is, the posterior distribution of the nonlinear state is calculated by cubature point approximation, and the analytical solution of PDF is represented by mean and variance in linear part. To achieve the overall system estimates, the steps are given as follows.

1) Nonlinear state prediction

- Calculating cubature points

$$\boldsymbol{P}_{N,k-1|k-1} = \boldsymbol{S}_{k-1|k-1}(\boldsymbol{S}_{k-1|k-1})^{\mathrm{T}} \quad (6)$$

$$\boldsymbol{X}^i_{N,k-1|k-1} = \boldsymbol{S}_{k-1|k-1}\boldsymbol{\xi}_i + \hat{\boldsymbol{x}}_{N,k-1|k-1} \quad (7)$$

where, n denotes the number of nonlinear state dimension, $L = 2n$ and $\boldsymbol{\xi}_i = \sqrt{L/2}[\boldsymbol{\delta}]_i, i = 1,2,\cdots,L$, $[\boldsymbol{\delta}]_i \in \mathcal{R}^{n\times 1}$ denotes the ith column element in $[\boldsymbol{I}^{n\times n}, -\boldsymbol{I}^{n\times n}] \in \mathcal{R}^{n\times L}$.

- Calculating propagation cubature points $\boldsymbol{X}^{*,i}_{N,k|k-1}$, state one-step prediction $\hat{\boldsymbol{x}}_{N,k|k-1}$ and state prediction error covariance $\boldsymbol{P}_{N,k|k-1}$

$$\boldsymbol{X}^{*,i}_{N,k|k-1} = \boldsymbol{f}^1_k(\boldsymbol{X}^i_{N,k-1|k-1}) + \boldsymbol{f}^2_k(\boldsymbol{x}_{L,k-1|k-1}) \quad (8)$$

$$\hat{\boldsymbol{x}}_{N,k|k-1} = \sum\nolimits_{i=1}^{L} \boldsymbol{X}^{*,i}_{N,k|k-1}/L \quad (9)$$

$$\boldsymbol{P}_{N,k|k-1} = \sum\nolimits_{i=1}^{L} \boldsymbol{X}^{*,i}_{N,k|k-1}(\boldsymbol{X}^{*,i}_{N,k|k-1})^{\mathrm{T}}/L - \hat{\boldsymbol{x}}_{N,k|k-1}(\hat{\boldsymbol{x}}_{N,k|k-1})^{\mathrm{T}} + \boldsymbol{\sigma}^2_{\omega_{N,k-1}} \quad (10)$$

- Calculating measurement cubature points

$$\boldsymbol{P}_{N,k|k-1} = \boldsymbol{S}_{k|k-1} \times (\boldsymbol{S}_{k|k-1})^{\mathrm{T}} \quad (11)$$

$$\boldsymbol{X}^i_{N,k|k-1} = \boldsymbol{S}_{k|k-1}\boldsymbol{\xi}_i + \hat{\boldsymbol{x}}_{N,k|k-1} \quad (12)$$

- Calculating propagation cubature points $\boldsymbol{Z}^i_{N,k|k-1}$ and measurement prediction $\hat{\boldsymbol{z}}_{N,k|k-1}$

$$\boldsymbol{Z}^i_{N,k|k-1} = \boldsymbol{h}(\boldsymbol{X}^i_{N,k|k-1}) \quad (13)$$

$$\hat{\boldsymbol{z}}_{N,k|k-1} = \sum\nolimits_{i=1}^{L} \boldsymbol{Z}^i_{N,k|k-1}/L \quad (14)$$

- Calculating covariance $\boldsymbol{P}^{zz}_{N,k|k-1}$ and the mutual interaction covariance between estimation and measurement $\boldsymbol{P}^{xz}_{N,k|k-1}$

$$\boldsymbol{P}^{zz}_{N,k|k-1} = \sum\nolimits_{i=1}^{L} \boldsymbol{Z}^i_{N,k|k-1}(\boldsymbol{Z}^i_{N,k|k-1})^{\mathrm{T}}/L - \hat{\boldsymbol{z}}_{N,k|k-1}(\hat{\boldsymbol{z}}_{N,k|k-1})^{\mathrm{T}} + \boldsymbol{\sigma}^2_{v_{N,k}} \quad (15)$$

$$\boldsymbol{P}^{xz}_{N,k|k-1} = \sum\nolimits_{i=1}^{L} \boldsymbol{X}^i_{N,k|k-1}(\boldsymbol{Z}^i_{N,k|k-1})^{\mathrm{T}}/L - \hat{\boldsymbol{x}}_{N,k|k-1}(\hat{\boldsymbol{z}}_{N,k|k-1})^{\mathrm{T}} \quad (16)$$

- Calculating filtering gain $\boldsymbol{K}_{N,k}$

$$\boldsymbol{K}_{N,k} = \boldsymbol{P}^{xz}_{N,k|k-1}(\boldsymbol{P}^{zz}_{N,k|k-1})^{-1} \quad (17)$$

2) Nonlinear state update

$$\hat{\boldsymbol{x}}_{N,k|k} = \hat{\boldsymbol{x}}_{N,k|k-1} + \boldsymbol{K}_{N,k}(\boldsymbol{z}_{N,k} - \hat{\boldsymbol{z}}_{N,k|k-1}) \quad (18)$$

$$\boldsymbol{P}_{N,k|k} = \boldsymbol{P}_{N,k|k-1} - \boldsymbol{K}_{N,k}\boldsymbol{P}^{zz}_{N,k|k-1}(\boldsymbol{K}_{N,k})^{\mathrm{T}} \quad (19)$$

3) Linear state generalized prediction

- Calculating state prediction $\boldsymbol{x}_{L,k|k-1}$ and its covariance $\boldsymbol{P}_{L,k|k-1}$

$$\boldsymbol{x}_{L,k|k-1} = \boldsymbol{F}^3_k\boldsymbol{x}_{N,k-1|k-1} + \boldsymbol{F}^4_k\boldsymbol{x}_{L,k-1|k-1} \quad (20)$$

$$\boldsymbol{z}_{L,k|k-1} = \boldsymbol{H}_L\boldsymbol{x}_{L,k|k-1} \quad (21)$$

$$\boldsymbol{P}_{L,k|k-1} = \boldsymbol{F}^4_k\boldsymbol{P}_{L,k-1|k-1}(\boldsymbol{F}^4_k)^{\mathrm{T}} + \boldsymbol{\sigma}^2_{\omega_{l,k-1}} \quad (22)$$

- Calculating filtering gain $\boldsymbol{K}_{L,k}$

$$\boldsymbol{K}_{L,k} = \boldsymbol{P}_{L,k|k-1}\boldsymbol{H}^{\mathrm{T}}_{L,k}(\boldsymbol{H}_{L,k}\boldsymbol{P}_{L,k|k-1}\boldsymbol{H}^{\mathrm{T}}_{L,k} + \boldsymbol{\sigma}^2_{v_{L,k}})^{-1} \quad (23)$$

4) Linear state update

Considering the difference between current estimation and the previous one in nonlinear state estimation as the measurement of linear state

$$\boldsymbol{z}_{L,k} = (\hat{\boldsymbol{x}}_{N,k|k} - \hat{\boldsymbol{x}}_{N,k-1|k-1})/\tau \quad (24)$$

$$\hat{\boldsymbol{x}}_{L,k|k} = \hat{\boldsymbol{x}}_{L,k|k-1} + \boldsymbol{K}_{L,k}(\boldsymbol{z}_{L,k} - \boldsymbol{H}_{L,k}\hat{\boldsymbol{x}}_{L,k|k-1}) \quad (25)$$

$$\boldsymbol{P}_{L,k|k} = \boldsymbol{P}_{L,k|k-1} + \boldsymbol{K}_{L,k}\boldsymbol{H}_{L,k}\boldsymbol{P}_{L,k|k-1} \quad (26)$$

5) Merging state estimation and covariance

$$\hat{\boldsymbol{x}}_{k|k} = [\hat{\boldsymbol{x}}_{L,k|k} \quad \hat{\boldsymbol{x}}_{N,k|k}]^{\mathrm{T}} \quad (27)$$

$$\boldsymbol{P}_{k|k} = \begin{bmatrix} \boldsymbol{P}_{N,k|k} & 0_{n\times l} \\ 0_{l\times n} & \boldsymbol{P}_{L,k|k} \end{bmatrix} \quad (28)$$

Considering the marginalized theory and the superiority of CKF, the nonlinear state and linear state of target tracking system are modeled separately. To illustrate the realization mechanism of MCKF, the framework of the proposed algorithm is given in Fig. 1. On this basis, we propose a novel marginalized cubature Kalman filter algorithm to improve filtering performance, the implementation is summarized in Algorithm 1.

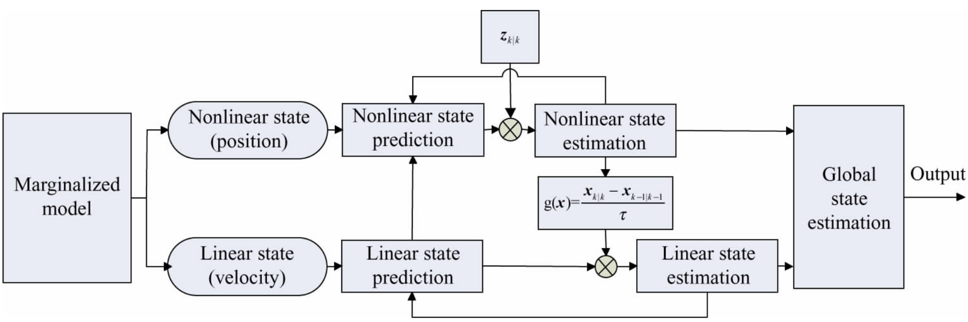


Fig. 1 The framework of MCKF

Algorithm 1 The Realization of MCKF

a) Initialize the linear and nonlinear states separately.
$\boldsymbol{x}_{N,1} = [\boldsymbol{x}_1(1,1) \quad \boldsymbol{x}_1(2,1)]^{\mathrm{T}}$
$\boldsymbol{x}_{L,1} = [\boldsymbol{x}_1(3,1) \quad \boldsymbol{x}_1(4,1)]^{\mathrm{T}}$

b) For $k = 1, 2, \cdots, n$

c) According to equations from Eq. (6) to Eq. (17) for nonlinear state prediction to obtain $\hat{\boldsymbol{x}}_{N,k|k-1}$, $\boldsymbol{P}_{N,k|k-1}$, and $\boldsymbol{K}_{N,k}$.

d) According to Eq. (18) and Eq. (19) for nonlinear state update to obtain $\hat{\boldsymbol{x}}_{N,k|k}$ and $\boldsymbol{P}_{N,k|k}$.

e) According to equations from Eq. (20) to Eq. (23) for linear state generalized prediction to obtain $\boldsymbol{x}_{L,k|k-1}$, $\boldsymbol{P}_{L,k|k-1}$ and $\boldsymbol{K}_{L,k}$.

f) According to equations from Eq. (24) to Eq. (26) for linear state update to obtain $\hat{\boldsymbol{x}}_{L,k|k}$ and $\boldsymbol{P}_{L,k|k}$.

g) According to Eq. (27) and Eq. (28) for linear state generalized prediction to amalgamate, and obtain $\hat{\boldsymbol{x}}_{k|k}$ and $\boldsymbol{P}_{k|k}$.

h) $k = k + 1$, return to the second step.

3 Computational complexity analysis

We derive computational complexity to further analyze the proposed algorithm. For time k, it is assumed that system state and measurement satisfy $\boldsymbol{x}_k \in \mathscr{R}^{n\times 1}$ and $\boldsymbol{z}_k \in \mathscr{R}^{m\times 1}$, respectively. In MCKF implementation process, nonlinear state and linear state are $\boldsymbol{x}_{N,k} \in \mathscr{R}^{n_1\times 1}$ and $\boldsymbol{x}_{L,k} \in \mathscr{R}^{n_2\times 1}$. The equivalent computational complexity of matrix Cholesky factorization with $n_1 \times n_1$ and $n \times n$ dimension are $c_1 = n_1^3/3 + 2n_1^2$ and $c'_1 = n^3/3 + 2n^2$, respectively. The equivalent computational complexity of $m \times m$ matrix inversion is $c_2 = m^3$. The computations for standard CKF[7] and MCKF are given in Table 1 and Table 2.

Table 1 MCKF computational complexity

	Reality multiplication	Reality plus	Other
Step c)	$4n_1^3 + 5n_1^2 + (m^2 + 3m + 1)n_1 + 3m^2 + m$	$6n_1^3 + (3 + 3m)n_1^2 + (2m^2 + m - 1)n_1 + m^2 - m$	$2c_1 + c_2$
Step d)	$mn_1^2 + (m^2 + m)n_1$	$mn_1^2 + (m^2 + 1)n_1$	0
Step e)	$4n_2^3 + (1 + 2m)n_2^2 + 2m^2 n_2$	$4n_2^3 + (2m - 2)n_2^2 + (2m^2 - 3m)n_2 - m^2$	0
Step f)	$n_2^3 + (m + 1)n_2^2 + mn_2$	$n_2^3 + mn_2^2 + mn_2$	c_2
In total	Reality multiplication: $M_{MCKF} = 4n_1^3 + (m + 5)n_1^2 + (2m^2 + 4m + 1)n_1 + 3m^2 + m + 5n_2^3 + (3m + 2)n_2^2 + (2m^2 + m)n_2$ Reality plus: $A_{MCKF} = 6n_1^3 + (4m + 3)n_1^2 + (3m^2 + m)n_1 - m + 5n_2^3 + (3m - 2)n_2^2 + (2m^2 - 2m)n_2$ Other: $E_{MCKF} = 2(c_1 + c_2) = 2n_1^3/3 + 4n_1^2 + 2m^3$		

Table 2 Standard CKF computational complexity

Reality multiplication: $M_{CKF} = 6n^3 + (m + 3)n^2 + (2m^2 + 4m + 1)n + 3m^2 + m$
Reality plus: $A_{CKF} = 9n^3 + (2m + 1)n^2 + (2m^2 + 3m - 1)n + 2m^2 - m$
Other: $E_{CKF} = 2c'_1 + c_2 = 2n^3/3 + 4n^2 + m^3$

From the known conditions, $n_1 = n - n_2$, n is an integer, then the following formulae are workable.

The difference between M_{CKF} and M_{MCKF} is given:

$$\begin{aligned} M_{CKF} - M_{MCKF} &= 6(n_1 + n_2)^3 + (m + 3)(n_1 + n_2)^2 \\ &\quad + (2m^2 + 4m + 1)(n_1 + n_2) + 3m^2 \\ &\quad + m - (4n_1^3 + (m + 5)n_1^2 \\ &\quad + (2m^2 + 4m + 1)n_1 + 3m^2) \\ &\quad - (m + 5n_2^3 + (3m + 2)n_2^2 \\ &\quad + (2m^2 + m)n_2) \\ &= 2n_1^3 + 18n_1n_2^2 + 18n_1^2n_2 + 2n_1^2 \\ &\quad + (1 - 2m)n_2^2 \\ &\quad + 2(m + 3)n_1n_2 + (3m + 1)n_2 \\ &> 2n_1^3 + 18n_1n_2^2 + 18n_1^2n_2 + 2n_1^2 \\ &\quad + (3m + 1)n_2 > 0 \end{aligned} \tag{29}$$

The difference between A_{CKF} and A_{MCKF} is given:

$$\begin{aligned} A_{CKF} - A_{MCKF} &= (3n_1^3 - 2(m + 1)n_1^2) + (27n_1n_2^2 \\ &\quad + (3 - m)n_2^2) \\ &\quad + (27n_1^2n_2 - (m - 1)^2 n_1) \\ &\quad + 4n_2^3 + 2(2m + 1)n_1n_2 \\ &\quad + (5m - 1)n_2 \\ &= (3n_1 - 2(m + 1))n_1^2 \\ &\quad + (27n_1 + 3 - m)n_2^2 \\ &\quad + (27n_1n_2 - (m - 1)^2)n_1 + 4n_2^3 \\ &\quad + 2(2m + 1)n_1n_2 + (5m - 1)n_2 \end{aligned} \tag{30}$$

Eq. (30) is reduced to:

$$\begin{aligned} A_{CKF} - A_{MCKF} &= 3n_1^3 + (27n_1 - m + 3)n_2^2 \\ &\quad + (27n_1n_2 - (m - 1)^2)n_1 \\ &\quad + (4n_2^3 - 2(m + 1)n_1^2) \\ &\quad + 2(2m + 1)n_1n_2 + (5m - 1)n_2 \end{aligned} \tag{31}$$

The difference between E_{CKF} and E_{MCKF} is given:

$$\begin{aligned} E_{CKF} - E_{MCKF} &= 2(n_1 + n_2)^3/3 + 4(n_1 + n_2)^2 \\ &\quad + m^3 - 2n_1^3/3 - 4n_1^2 - 2m^3 \\ &= 2n_1^2n_2 + 2n_1n_2^2 + 2n_2^3/3 + 8n_1n_2 \\ &\quad + 4n_2^2 - m^3 \end{aligned} \tag{32}$$

According to physical measurement characteristics, the number of sensor measurement dimension is smaller than state's. The difference between A_{CKF} and A_{MCKF} is discussed for two cases.

Case 1 Assuming that the number of nonlinear state dimension n_1 is larger than linear state dimension n_2, $n > n_1 > n_2 \geqslant m \geqslant 1$, the following formula is obtained according to Eq. (30).

$$\begin{cases} 3n_1 - 2(m+1) = 2(n_1 - m) + n_1 - 2 \geqslant 0 \\ 27n_1 + 3 - m > 0 \\ 27n_1 n_2 - (m-1)^2 > 27n_1^2 - (m-1)^2 > 0 \\ 5m - 1 > 0 \end{cases} \tag{33}$$

Accordingly, $A_{CKF} > A_{MCKF}$.

Case 2 Assuming that the number of nonlinear state dimension n_1 is less than the linear state dimension n_2, $n > n_2 > n_1 \geqslant m \geqslant 1$, the following formula is obtained according to Eq. (31).

$$\begin{cases} 27n_1 - m + 3 > 0 \\ 27n_1 n_2 - (m-1)^2 > 0 \\ 4n_2^3 - 2(m+1)n_1^2 > 2(2n_2 - m - 1)n_1^2 \geqslant 0 \\ 5m - 1 > 0 \end{cases} \tag{34}$$

Accordingly, $A_{CKF} > A_{MCKF}$.

On the known conditions $n_1 \geqslant m$, $n_1 \geqslant m$ and Eq. (32), $2n_1 n_2^2 - m^3 > 0$ can be got. Accordingly, $E_{CKF} > E_{MCKF}$.

According to analysis above, the sum computational complexity of reality multiplication, reality plus, Cholesky and matrix inversion in MCKF are lower than that of standard CKF.

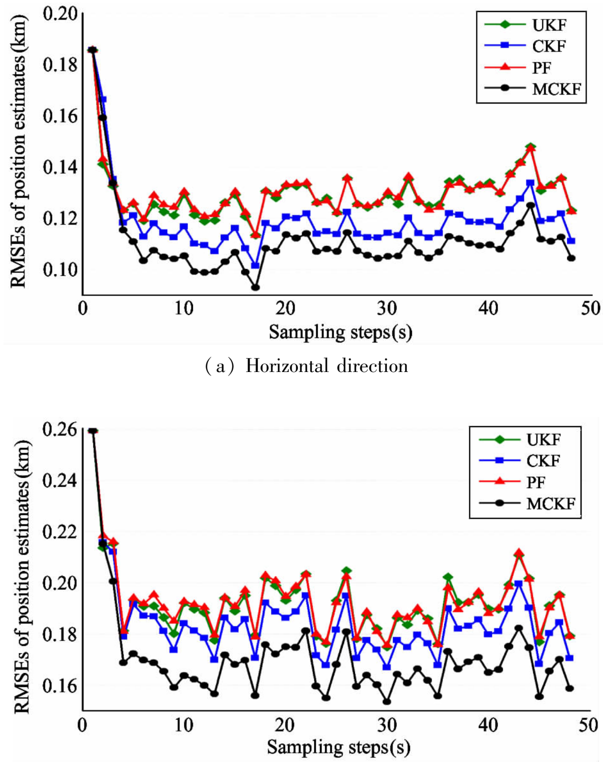
4 Simulations and analysis

To verify the feasibility and effectiveness of proposed algorithm, Monte Carlo simulations are performed, and the number of Monte Carlo is 200. Taking root-mean-square (RMSE) as the performance evaluation.

$$RMSE = \sqrt{\sum_{m=1}^{M} (x_k - x_{k|k})^2 / M} \tag{35}$$

In Cartesian coordinate system, assuming that state transition matrix in Eq. (1) are $\boldsymbol{F}_k^1 = \boldsymbol{I}_{2\times 2}$ and $\boldsymbol{F}_k^2 = \mathrm{diag}([\tau, \tau])$. State transition matrix in Eq. (2) are $\boldsymbol{F}_k^3 = 0_{2\times 2}$ and $\boldsymbol{F}_k^4 = I_{2\times 2}$. Nonlinear measurement matrix and linear measurement matrix in Eq. (3) are given as $\boldsymbol{h}_k(\boldsymbol{x}_{N,k}) = (\sqrt{x^2 + y^2}\ \mathrm{atan}(y/x))^{\mathrm{T}}$ and $\boldsymbol{C}_k = \boldsymbol{I}_{2\times 2}$, respectively. System noise variance is set as $\boldsymbol{Q}_k = \mathrm{diag}([0.3^2 \quad 0.3^2 \quad 0.05^2 \quad 0.05^2])$, and measurement noise variance is $\boldsymbol{\sigma}_{v_k}^2 = \mathrm{diag}([0.3^2 \ (0.1\pi/180)^2])$. The initial system state is given as $x_0 = [15\text{ km} \quad 25\text{ km} \quad 0.3\text{ km/s} \quad 0.5\text{ km/s}]$.

Fig. 2 shows RMSE comparison of MCKF and other three algorithms. In MCKF, velocity is modeled separately to construct measurement to provide the innovation in update process. At the same time, MCKF has higher precision compared with CKF in linear estimation process. It can be verified by the real-time RMSE in Fig. 1 and RMSE means values in Table 3, MCKF is superior to the other three algorithms in estimation accuracy.



(a) Horizontal direction

(b) Vertical direction

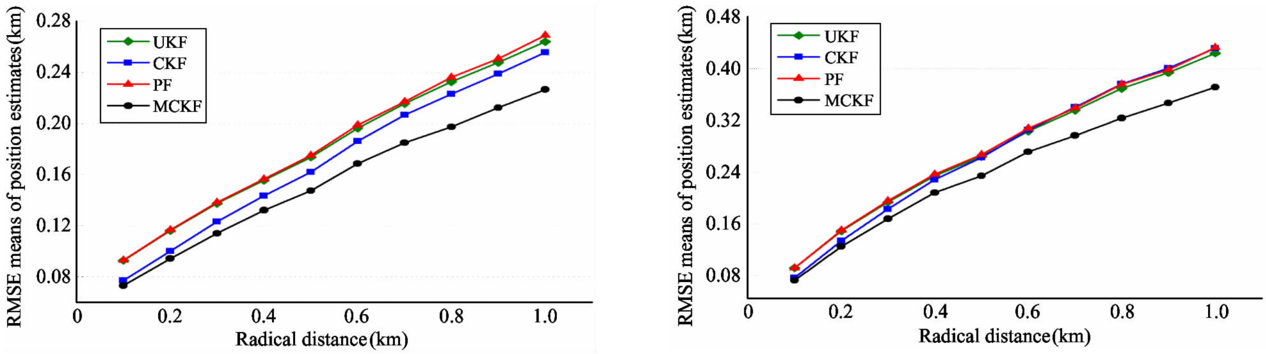
Fig. 2 Comparison of position estimation RMSE

Table 3 shows the comparison of RMSE means and time consumption for one operation of MCKF and CKF. MCKF avoids the process of cubature points calculation in the linear state. Meanwhile, the amount of inversion calculation is reduced by decomposing larger dimension matrix into two smaller parts according to marginalized modeling mechanism. The values in Table 4 illustrate that MCKF has lower time consumption, which verifies the correctness of calculation complexity derivation.

Table 3 Time and RMSE means of MCKF and CKF

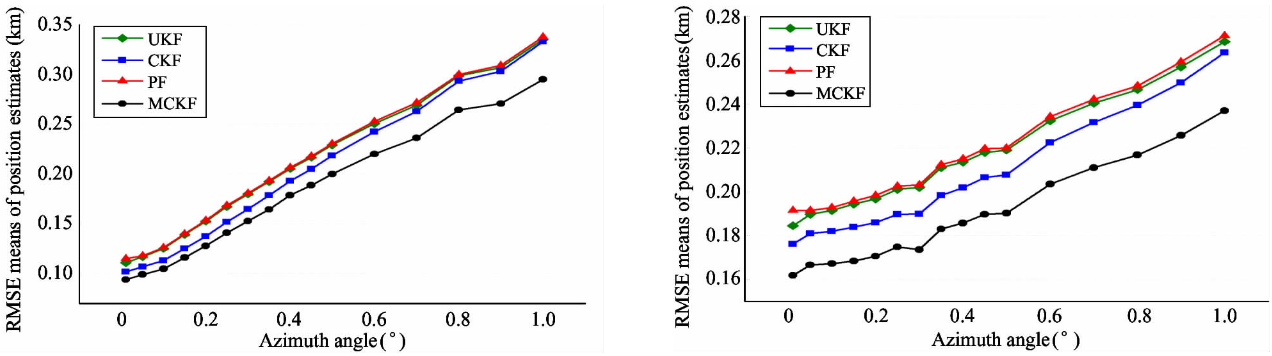
Algorithm	CKF	MCKF
Consumption for one operation (s)	0.0048	0.0032
Horizontal direction RMSE (km)	0.1164	0.1079
Vertical direction RMSE (km)	0.1809	0.1163

In order to verify the superiority of MCKF under the condition of noises, the RMSE means comparison of the four algorithms under different radial distance error levels are given in Fig. 3. As shown in the figure, RMSE means increase along with the increase of radial distances error from 0.1 km to 1.0 km, however the accuracy of MCKF is higher than that of UKF, CKF and PF in overall. The phenomenon is more obvious when radial distance noises increase.



(a) Horizontal direction (b) Vertical direction

Fig. 3 Comparison of RMSE means for different measurement noises (different radial distances)



(a) Horizontal direction (b) Vertical direction

Fig. 4 Comparison of RMSE means for different measurement noises (different azimuth angles)

Fig. 4 presents the comparison of RMSE means of the four algorithms under different azimuth error levels. Similar to Fig. 3, the RMSE means increase along with azimuth measurement noise from 0.01° to 1.0°, but the increase of MCKF is minimum. Both Fig. 3 and Fig. 4 show that MCKF has better ability to suppress measurement noise and improve the accuracy of state estimation. The reason is that velocity is directly involved in state update process. At the same time, the condition of optimal estimation is satisfied in linear estimation process.

5 Conclusion

Based on the characteristics of nonlinear system with linear Gaussian sub-structure, the system model with the nonlinear part and linear part separately by marginalized method is established, and nonlinear estimation and linear estimation are carried out by combining with cubature Kalman filter and Kalman filter respectively. Note that in the process of linear state update, the measurement is obtained by solving the function of state estimation and step interval. Then, in order to show the proposed algorithm has the improved performance of estimation accuracy and real-time, it is deduced theoretically that the MCKF has lower computational complexity than standard CKF. Finally, simulation experiments and analysis show that the proposed algorithm outperforms the compared algorithms in performance of accuracy and real-time.

References

[1] Arulampalam M S, Maskell S, Gordon N A. Tutorial on particle filters for online nonlinear/non-Gaussian Bayesian tracking[J]. *IEEE Transactions on Signal Processing*, 2002, 50(2): 174-188

[2] Zhang X Y. Simultaneous optimization for robust correlation estimation in partially observed social network[J]. *Neurocomputing*, 2016,205(C): 455-62

[3] Gordon N J, Salmond D J, Smith A M. Novel approach to non-linear/non-Gaussian Bayesian state estimation[J]. *Radar &Signal Processing IEEE Proceedings-F*, 1993, 140(2): 107-113

[4] Schön T B, Karlsson R, Gustafsson F. The marginalized particle filter in practice[J]. *Aerospace Conference*, 2006, 55: 409-419

[5] Julier S, Uhlmann J, Durrant-Whyte H F. A new method for the nonlinear transformation of means and covariances

in filters and estimators[J]. *IEEE Transactions on Automatic Control*, 2000, 45(3): 477-482

[6] Kandepu R, Foss B, Imsland L. Applying the unscented Kalman filter for nonlinear state estimation[J]. *Journal of Process Control*, 2008, 18(7): 753-768

[7] Chang G, Xu T, Wang Q. Alternative framework for the iterated unscented Kalman filter[J]. *IET Signal Processing*, 2017, 11(3): 258-264

[8] Arasaratnam I, Haykin S. Cubature Kalman filters[J]. *IEEE Transactions on Automatic Control*, 54(6): 1254-1269, 2009

[9] Singh A, Bhaumik S. Transformed cubature quadrature Kalman filter[J]. *IET Signal Processing*, 2017, 11(9): 1095- 1103

[10] Jia B, Xin M. Adaptive cubature Kalman filter with directional uncertainties[J]. *IEEE Transactions on Aerospace & Electronics Systems*, 2016, 52(3): 1477-1486

[11] Zhang Q, Chen Z. Yin F, et al. Distributed marginalized auxiliary particle filter for speaker tracking in distributed microphone networks[J]. *IEEE Transactions on Audio Speech & Language Processing*, 2016, 24(11): 1921-1934

[12] Karlsson R, Schön T B. Complexity analysis of the marginalized particle filter[J]. *IEEE Transactions on Signal Processing*, 2005, 53(11): 4408-4411

[13] Karlsson R. Particle filter for positioning and tracking applications[D]. Linköping: Linköping University, 2005

[14] Hu Z T, Liu X X. Real-time marginalized particle filter based on weights consistency optimization[J]. *Acta Electronica Sinica*, 2014, 42(10): 1970-1976

[15] Zhuang Z X, Zhang J Q, Yin J J. Rao-blackwellized particle probability hypothesis density filter[J]. *Acta Aeronautica Astronautica Sinica*, 2009, 30(4): 698-705

[16] Li W, Wei G., Han F. Weighted average consensus-based unscented Kalman filtering[J]. *IEEE Transactions on Cybernetics*, 2017, 46(2): 558-567

[17] Wang X X, Pan Q, Huang H, Gao A. Overview of deterministic sampling filtering algorithms for nonlinear system [J]. *Control and Decision*, 2012, 27(6): 801-821

[18] Arasaratnam I, Haykin S, Hurd T R. Cubature Kalman filtering for continuous-discrete systems: theory and simulations[J]. *IEEE Transactions on Signal Processing*, 2010, 58(10): 4977-4993

Hu Yumei, received her M. E. degree in the School of Computer and Information Engineering at Henan University in 2016. She is now a Ph. D candidate in the School of Automation, Northwestern Polytechnical University, Xi'an, P. R. China. Her main research interests include information fusion, nonlinear filtering, target tracking, and variational Bayesian theory.