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Robust adaptive beamforming for constant modulus signal of interest[①]

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Abstract

It is required in the diagonally loaded robust adaptive beamforming the automatic determination of the loading level which is practically a challenging problem. A constant modulus restoral method is herein presented to choose the diagonal loading level adaptively for the extraction of a desired signal with constant modulus (a common feature of the phase modulation signals). By introducing the temporal smoothing technique, the proposed constant modulus restoral diagonally loaded robust adaptive beamformer provides increased capability compared with some existing robust adaptive beamformers in rejecting interferences and noise while protecting the signal-of-interest. Simulation results are included to illustrate the performance of the proposed beamformer.

Key words: array signal processing, robust adaptive beamforming, diagonal loading, constant modulus

0 Introduction

Adaptive beamforming is widely used in many practical applications such as radar, sonar, seismology, wireless communications, space science, and medical imaging[1]. However, unlike the traditional sum-and-delay beamforming, adaptive beamforming is known to be much sensitive to even a small model error, for instance, mismatch in steering vector of the signal-of-interest (SOI) caused by imperfect array calibration look direction error and/or the presence of finite data samples[1]. For this reason, developing robust adaptive beamformers is of great interest.

Early typical robust schemes include gain or derivative linear constraint, signal plus interference subspace projection, and norm constraint[1]. Recently established robust techniques include uncertainty-set constraint[2-4], covariance matrix enhancing or fitting[5-7], power matching[8], steering vector estimation[9], interference and noise covariance matrix reconstruction[10]. Other robust methods can be found in Ref. [11] and the references therein. The above mentioned approaches can be used conditionally to prevent cancellation of SOI while rejecting interferences and noise.

Some of the above methods can also be interpreted as a special diagonal loading technique which is itself a popular robust approach. However, the adaptive determination of diagonal loading factor is very difficult[12]. Recently, automatic determination of diagonal loading level has been fulfilled from a SOI property restoral point of view. In Refs[13,14], the robustness of diagonally loaded beamformers is achieved by taking into account the noncircularity of SOI. In Refs[15,16], two robust beamformers termed the minimum constant modulus errors and the constant modulus diagonal loading (COMDIAL) are proposed, respectively, wherein the diagonal loading level is determined by exploiting the constant modulus feature of SOI. The purpose of this contribution is to enhance the performance of the COMDIAL beamformer via temporal smoothing without any user parameters or training procedures.

The rest of the paper is outlined as follows. Section 1 introduces the constant modulus feature of a signal, the array signal model and the Capon beamformer. Section 2 describes the diagonally loaded beamformers and proposes a method exploiting the constant modulus feature of the SOI to determine the diagonal loading level automatically. Section 3 verifies the effectiveness of the proposed beamformer by simulations. Conclusions are drawn in Section 4.

1 Problem formulation

Consider a zero-mean complex-valued SOI, say $s_0(t)$, whose modulus is constant and thus

$$|s_0(t)s_0(t+\tau)| = E\{|s_0(t)|^2\} = \sigma_0^2 \quad (1)$$

where "E" denotes statistical expectation, "$|\cdot|$" and

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"$|\cdot|^2$" denote the modulus and the square of the modulus, respectively, τ is the lag parameter, and σ_0^2 is the SOI power.

Consider further an N-sensor beamforming array receiving the above mentioned SOI which is corrupted by M co-channel interferences, say $\{s_m(t)\}_{m=1}^{M}$. The array output vector $\boldsymbol{x}(t)$ can be expressed as

$$\boldsymbol{x}(t) = \underbrace{\boldsymbol{a}_0 s_0(t)}_{=s(t)} + \underbrace{\sum_{m=1}^{M} \boldsymbol{a}_m s_m(t)}_{=i(t)} + \boldsymbol{n}(t) \tag{2}$$

where $\boldsymbol{a}_0$ is the steering vector of the SOI, $\boldsymbol{a}_m$ is the steering vector of the m-th interference $s_m(t)$, and $\boldsymbol{n}(t)$ is the noise vector. Throughout the paper, SOI interferences and noise are assumed to be independent zero-mean stationary random processes. The noise process is further assumed to be spatially white and circular.

The output of the beamformer is given by

$$y_w(t) = \boldsymbol{w}^{\mathrm{H}} \boldsymbol{x}(t) \tag{3}$$

where $\boldsymbol{w}$ is the beamformer weight vector, and superscript "H" denotes Hermitian transpose. In the popular minimum variance distortion less response (MVDR) beamformer, weight vector $\boldsymbol{w}$ is designed as

$$\min_{w} \boldsymbol{w}^{\mathrm{H}} \boldsymbol{R}_{xx} \boldsymbol{w} \quad \text{s. t.} \quad \boldsymbol{w}^{\mathrm{H}} \boldsymbol{a}_0 = 1 \tag{4}$$

where $\boldsymbol{R}_{xx} = E\{\boldsymbol{x}(t)\boldsymbol{x}^{\mathrm{H}}(t)\}$ is the array output covariance matrix. The solution to Eq. (4) can be determined by using the Lagrange multiplier technique, as

$$\boldsymbol{w}_{\mathrm{MVDR}} = \frac{\boldsymbol{R}_{xx}^{-1} \boldsymbol{a}_0}{\boldsymbol{a}_0^{\mathrm{H}} \boldsymbol{R}_{xx}^{-1} \boldsymbol{a}_0} \tag{5}$$

The practical counterpart of the above MVDR beamformer is the sample matrix inversion (SMI) beamformer wherein the presumed value of $\boldsymbol{a}_0$ and the sample estimate of $\boldsymbol{R}_{xx}$ are used instead.

In the presence of SOI steering vector mismatch and finite data samples, SOI may be misinterpreted by the SMI beamformer as an interfering signal to be suppressed, which causes signal cancellation problem. The purpose of this paper is to tackle this problem on the basis of diagonal loading and SOI temporal structure restoral.

2 Proposed beamformer

The weight vector of the diagonally loaded robust beamformer is determined as

$$\min_{w} \boldsymbol{w}^{\mathrm{H}} [(1-\alpha)\hat{\boldsymbol{R}}_{xx} + \alpha \boldsymbol{I}] \boldsymbol{w} \quad \text{s. t.} \quad \boldsymbol{w}^{\mathrm{H}} \hat{\boldsymbol{a}}_0 = 1 \tag{6}$$

where $\hat{\boldsymbol{R}}_{xx}$ is the sample estimate of the array output covariance matrix:

$$\hat{\boldsymbol{R}}_{xx} = \frac{1}{K} \sum_{k=0}^{K-1} \boldsymbol{x}(t_k) \boldsymbol{x}^{\mathrm{H}}(t_k) \tag{7}$$

with K being the snapshot number, and $\boldsymbol{I}$ denotes the identity matrix, $\alpha \in [0,1]$ is the regularization parameter to be determined, and $\hat{\boldsymbol{a}}_0$ is the presumed SOI steering vector.

The solution to Eq. (6) can be also determined by using the Lagrange multiplier technique, as

$$\boldsymbol{w}_{\alpha} = \frac{[(1-\alpha)\hat{\boldsymbol{R}}_{xx} + \alpha \boldsymbol{I}]^{-1} \hat{\boldsymbol{a}}_0}{\hat{\boldsymbol{a}}_0^{\mathrm{H}} [(1-\alpha)\hat{\boldsymbol{R}}_{xx} + \alpha \boldsymbol{I}]^{-1} \hat{\boldsymbol{a}}_0} \tag{8}$$

Note that SOI is assumed to have a constant modulus, to suppress significantly the interferences and noise while preserving SOI, the diagonal loading level should be selected such that the output of the beamformer would be as close as possible to $s_0(t)$ and, thus

$$\begin{aligned} r(t_k, t_{k+n}) &= | y_{w_\alpha}(t_k) y_{w_\alpha}(t_{k+n}) | \to | s_0(t) s_0(t+\tau) | \\ &= \sigma_0^2 \end{aligned} \tag{9}$$

where $y_{w_\alpha}(t) = \boldsymbol{w}_\alpha^{\mathrm{H}} \boldsymbol{x}(t)$, and n is an integer.

More precisely, regularization parameter α can be determined as follows:

$$\hat{\alpha}_{\mathrm{CCOMDIAL}} = \arg \min_{\alpha \in [0,1]} \sum_{n=0}^{J-1} \sum_{k=0}^{K-n-1} \left| 1 - \frac{| y_{w_\alpha}(t_k) y_{w_\alpha}(t_{k+n}) |}{\hat{\sigma}_0^2(\alpha)} \right| \tag{10}$$

where $1 \leqslant J \leqslant K$, and $\hat{\sigma}_0^2(\alpha) = \boldsymbol{w}_\alpha^{\mathrm{H}} \hat{\boldsymbol{R}}_{xx} \boldsymbol{w}_\alpha$.

If $J = 1$, then scheme in Eq. (10) reduces to the COMDIAL method in Ref. [16]:

$$\alpha_{\mathrm{COMDIAL}} = \arg \min_{\alpha \in [0,1]} \sum_{k=0}^{K-1} \left| 1 - \frac{| y_{w_\alpha}(t_k) |^2}{\hat{\sigma}_0^2(\alpha)} \right| \tag{11}$$

It can be obtained from Eq. (9):

$$\frac{1}{L} \sum_{n=0}^{J-1} \sum_{k=0}^{K-n-1} | s_0(t_k) s_0(t_{k+n}) | = \sigma_0^2 \tag{12}$$

where $L = KJ - (J-1)J/2$. Therefore, regularization parameter α can be alternatively determined as

$$\alpha_{\mathrm{CRDL}} = \arg \min_{\alpha \in [0,1]} \left| 1 - \frac{1}{L} \sum_{n=0}^{J-1} \sum_{k=0}^{K-n-1} \left[\frac{| y_{w_\alpha}(t_k) y_{w_\alpha}(t_{k+n}) |}{\hat{\sigma}_0^2(\alpha)} \right] \right| \tag{13}$$

The scheme for the determination of α in Eq. (13) is preferable since the constant modulus restore is exploited after the effect of the residual noise contained in the beamformer output is reduced by the temporal smoothing process, whereas α determined in either Eq. (10) or Eq. (11) is more sensitive to the noise residue in the beamformer output.

In addition, note that

$$\begin{aligned} \frac{1}{K} \sum_{k=0}^{K-1} | y_{w_\alpha}(t_k) |^2 &= \boldsymbol{w}_\alpha^{\mathrm{H}} \underbrace{\left[\frac{1}{K} \sum_{k=0}^{K-1} \boldsymbol{x}(t_k) \boldsymbol{x}^{\mathrm{H}}(t_k) \right]}_{\hat{\boldsymbol{R}}_{xx}} \boldsymbol{w}_\alpha \\ &= \hat{\sigma}_0^2(\alpha) \end{aligned} \tag{14}$$

The scheme in Eq. (13) thus can be simplified as

$$\alpha_{\mathrm{CRDL}} = \arg \min_{\alpha \in [0,1]} \left| 1 - \frac{1}{L'} \sum_{n=1}^{J-1} \sum_{k=0}^{K-n-1} \left[\frac{| y_{w_\alpha}(t_k) y_{w_\alpha}(t_{k+n}) |}{\hat{\sigma}_0^2(\alpha)} \right] \right| \tag{15}$$

where

$$L' = K(J-1) - \frac{(J-1)J}{2} \tag{16}$$

The above method is called the constant modulus restoral diagonal loading (CRDL) beamformer. The computational complexity of the proposed beamformer is composed of the complexity in the iterative optimization of regularization parameter α and the complex multiplications. A complex modulus calculation requires two times of multiplications while a complex multiplication requires four real multiplications. The whole iterative optimization process for determining regularization parameter α thus needs $6KJ + 3J(1-J)$ multiplications. Note also that the main computational complexity of the proposed CRDL rule is consisted in the determination of the diagonal loading level and it is about $O(9J^2)$.

3 Simulation results

In this section, several numerical examples are presented to illustrate the performance of the proposed CRDL beamformer. A uniform linear array with eight sensors spaced half wavelength apart is used to extract a constant modulus BPSK SOI from additive white Gaussian noise and two interferences, one is a BPSK signal, and the other is a Gaussian random process. The presumed SOI DOA is 0°. The interference DOAs are 30° and −40°, respectively. The signal-to-interference ratio (SIR) is −10dB. All the curves shown are the averaged results of 500 Monte-Carlo simulation trials.

In the simulations, the output signal-to-interference-plus-noise ratio (OSINR) of the proposed CRDL beamformer is examined compared with the existing COMDIAL beamformer[16], robust Capon beamformer (RCB)[3], generalized linear combination (GLC) beamformer[7], midway (MW) beamformer[8], and also the maximally attainable OSINR:

$$\text{MAX-SINR} = \sigma_0^2 \boldsymbol{a}_0^{\mathrm{H}} \boldsymbol{R}_{i+n}^{-1} \boldsymbol{a}_0 \tag{17}$$

where $\boldsymbol{R}_{i+n}$ is the interference plus noise covariance matrix defined as

$$\boldsymbol{R}_{i+n} = E\{[\boldsymbol{i}(t) + \boldsymbol{n}(t)][\boldsymbol{i}(t) + \boldsymbol{n}(t)]^{\mathrm{H}}\} \tag{18}$$

Example 1: The curves of the beamformers' OSINR versus the input signal-to-noise ratio (ISNR), snapshot number and look direction error.

The results shown in Fig. 1 and Fig. 2 are the OSINR curves of beamformers against ISNR, where the look direction errors are 1° and 3°, respectively. The signal-to-noise ratio (SNR) is varied from 0dB to 20dB. In all the simulations, the ideal user parameter for RCB is chosen to ensure its best performance. The OSINR curves shown in Fig. 3 and Fig. 4 are for the same simulation conditions except that the snapshot number is 300. It can be seen from Fig. 1 and Fig. 2 that CRDL outperforms other beamformers in the presence of short data samples, especially when the ISNR is equal to 20dB. Nearly 3dB is gained in OSINR of CRDL than COMDIAL. From Fig. 3 and Fig. 4, it is that CRDL has a superior OSINR over the other beamformers regardless of the ISNR values.

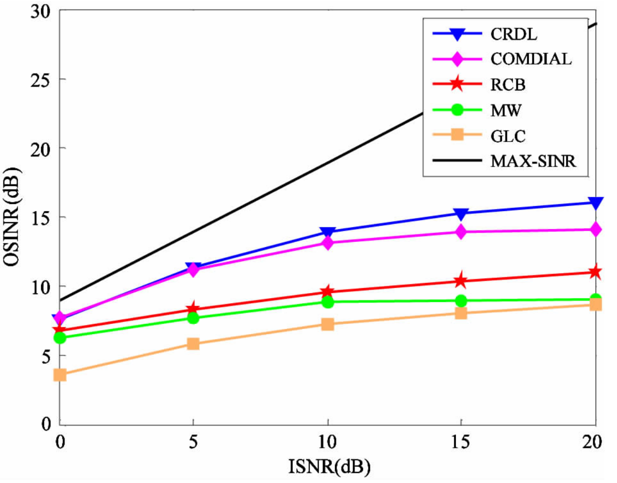

Fig. 1 OSINR versus ISNR: the snapshot number is 50, look direction error is 1°

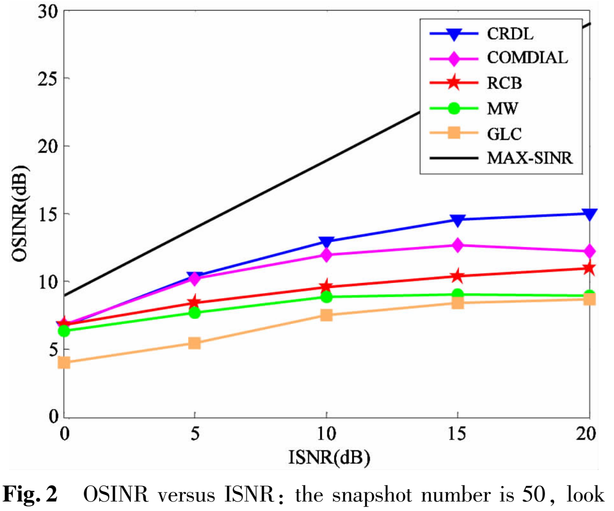

Fig. 2 OSINR versus ISNR: the snapshot number is 50, look direction error is 3°

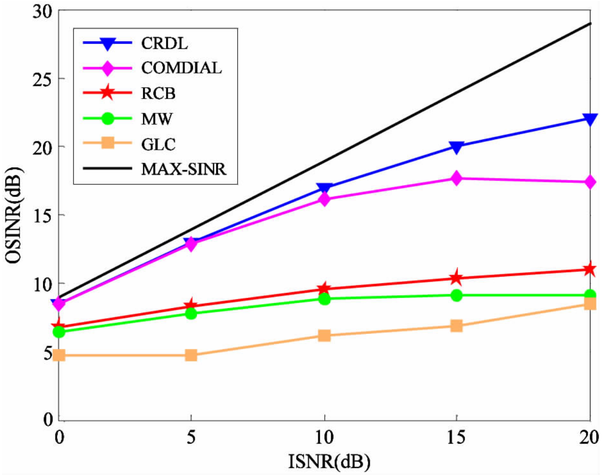

Fig. 3 OSINR versus ISNR: the snapshot number is 300, look direction error is 1°

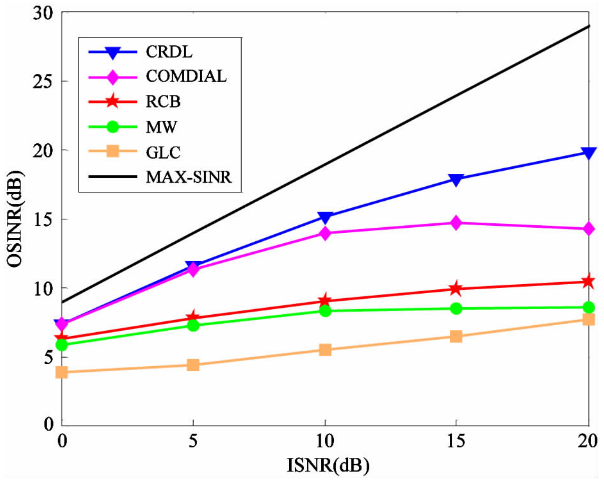

Fig. 4 OSINR versus ISNR: the snapshot number is 300, look direction error is 3°

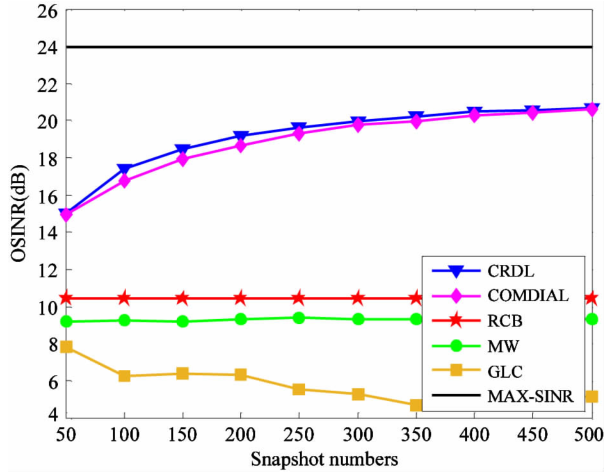

Fig. 5 OSINR versus snapshot number: the ISNR is 15dB, look direction error is 1°

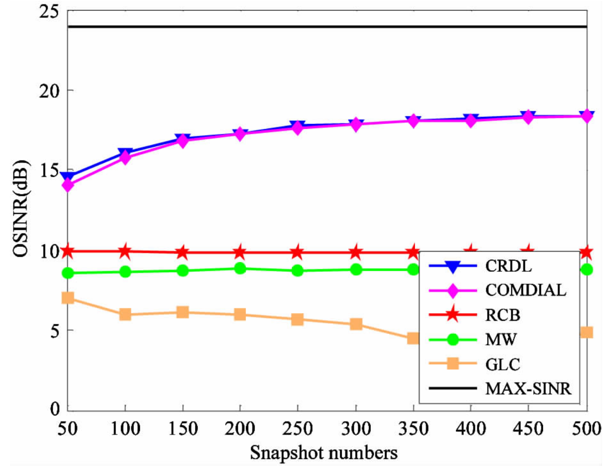

Fig. 6 OSINR versus snapshot number: the ISNR is 15dB, look direction error is 3°

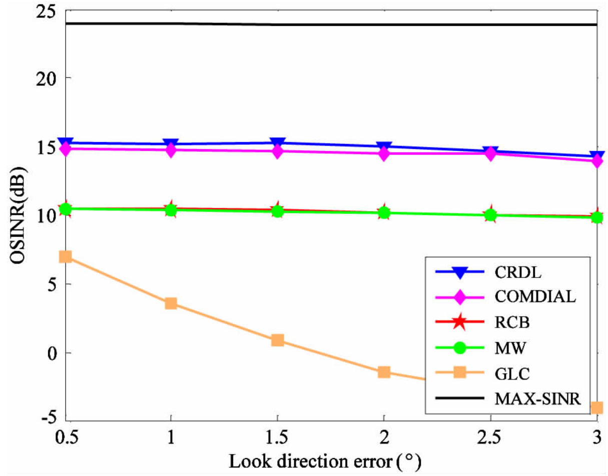

Fig. 7 OSINR versus look direction error: the ISNR is 15dB, snapshot number is 50

In addition, fixing SNR as 15dB, it is considered further the effect of the number of snapshot and the look direction error on the beamformers' OSINR. The results shown in Fig. 5 and Fig. 6 are the beamformers' OSINR versus the snapshot number, where the look direction errors are 1° and 3°, respectively. The results shown in Fig. 7 are the OSINR curves against look direction errors, where the snapshot number is 50.

It is observed that CRDL has a better performance than the other tested beamformers. From Fig. 5 and Fig. 6, it is seen that CRDL's OSINR is 5dB higher than other beamformers when the snapshot number is beyond 70. Fig. 7 shows that CRDL still outperformers other beamformers. As the look direction error becomes larger, all beamformers' performances degrade.

Also, by using the temporal smoothing technique, CRDL outperforms COMDIAL especially for the case of large look direction error. In the presence of short data samples, CRDL still outperforms COMDIAL.

Example 2: The curves of the beamformers' single experimental running time (SRT) versus the snapshot number K and the sensor number N.

Under the condition of the same hardware and software (Intel i3 dual-core processor, 3.30GHz of faster, 4GB of memory; the Matlab simulation software), the result shown in Fig. 8 is the beamformers' SRT against the sensor number, where the snapshot number is 100, the look direction error is 3° and the ISNR is 20dB. Fig. 9 shows the beamformers' SRT against the snapshot number, where the sensor number is 8 and the other simulation conditions are the same as above. The CRDL beamformer, COMDIAL beamformer and the MW beamformer are compared because all of the three beamformers need iterative computations for the determination of a regularization parameter.

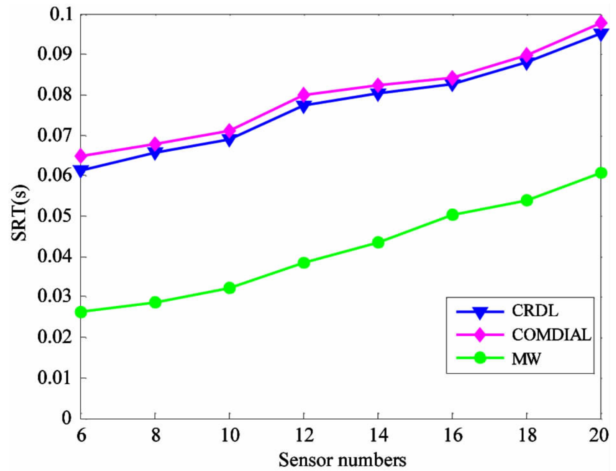

Fig. 8 SRT versus sensor number: the ISNR is 20dB, snapshot number is 50, look direction error is 3°

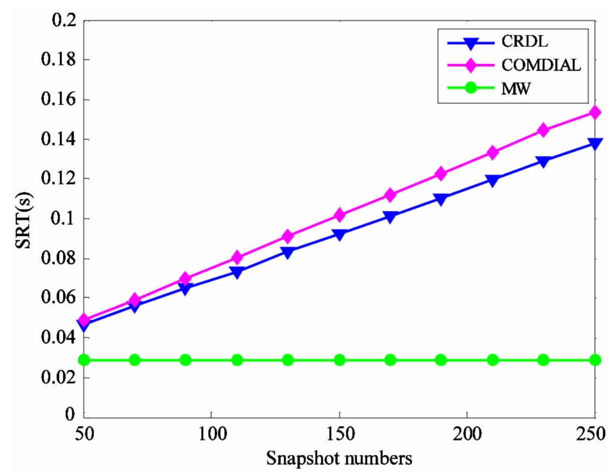

Fig. 9 SRT versus snapshot number: the ISNR is 20dB, sensor number is 8, look direction error is 3°

It can be seen from Fig. 8 and Fig. 9 that the SRTs of all the tested beamformers become longer as the sensor numbers or snapshot numbers increase. Fig. 8 also shows that CRDL takes about 0.08s to run a single experiment while MW takes about 0.05s to run. The SRT of COMDIAL is slightly higher than CRDL. From Fig. 9, an average single running time of CRDL is about 0.09s while the SRT of COMDIAL is about 0.11s, and MW' SRT is almost not changed as the snapshot number increases.

4 Conclusion

This paper has proposed a diagonally loaded robust beamformer (CRDL) based on constant modulus restoral to extract a constant modulus SOI. By using the constant modulus feature of SOI, the proposed method could determine an appropriate diagonal loading level without any user-parameter and training procedure. Under the tested scenarios, CRDL has been observed to outperform the existing COMDIAL, RCB, MW and GLC against the look direction error. Moreover, by using the temporal smoothing technique, CRDL also has a higher OSINR than other beamformers under scenarios of short data samples. The OSINR of CRDL is about 5dB higher than the others in the case of large look direction error and small snapshot number.

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