

Two-stage DOA estimation method for passive radar based on sparse representation^①

Liu Nan (刘楠), Song Wenlong^②, Dong Guanghui

(College of Mechanical and Electronic Engineering, Northeast Forestry University, Harbin 150040, P. R. China)

Abstract

In a GPS illuminator based passive radar system, estimation of direction of arriving (DOA) of multiple targets is a difficult problem due to strong interference. A two-stage method combining extensive cancellation algorithm (ECA) and sparse representation is proposed. In the first stage, ECA algorithm is used to eliminate the direct-path and multi-path interference. In the second stage, sparse representation of improved weight constraints based on L1 norm is adopted to estimate DOA and suppress the interference. Simulation results show that the proposed method can effectively estimate DOA in low computation complexity without estimating the disturbance parameter.

Key words: passive radar, GPS illuminator, direction of arriving (DOA), extensive cancellation algorithm, sparse representation

0 Introduction

Passive radar is a radar system which exploits external non cooperative transmitter as the illuminator, receives signals reflected from the target, and extracts the source bearing, speed and other parameters^[1]. GPS has the characteristics of all-weather, anti-interference and real-time, and the idea of using reflected signals from GPS for the passive radar, becomes a hotspot recently^[2]. However, weak echo signal through an echo channel is inevitable interfered by direct wave, multipath signal and clutter sidelobe. How to make the direction of arriving (DOA) estimation of multiple targets in the interference has become an important content which must be solved.

To detect the multiple targets when applying interference suppression, many approaches have been presented. In Ref. [3], the method for interference suppression is accurate to dispose direct wave and multipath because of estimating the parameter of each disturbance, but it can not deal with echo sidelobe interference. The extensive cancellation algorithm (ECA)^[4] progressively detects the strongest delayed and frequency shifted replicas of the direct signal and removes its effects on the received signal without parameter estimation, although the technique can not estimate direction. MUSIC-like^[5] method uses a subspace

method of direction estimation which is more than the number of array element, applies fourth order cumulating matrix with high computational complexity. In Ref. [6], Sparse represents a signal processing theory as a rising thought system, can use high order accumulated to estimate DOA.

A two-stage DOA estimation method is proposed in this work. In the first stage, ECA is applied to remove the direct wave and multipath interference. In the second stage, a sparse DOA estimation model is constructed with improved L1 norm as sparse constraint conditions. And the equilibrium constraint is applied to restrain the residual echo sidelobe jamming and decrease the computation complexity. The paper is organized as follows. Section 1 describes a system model and reference scenario. Section 2 derives an improved re-weighted L1 norm sparse model and problem formulation. Section 3 describes the proposed algorithm as a processing procedure. Theoretical results are supported by numerical simulations, see Section 4. Finally the conclusions are drawn in Section 5.

1 System model

The system model is shown in Fig. 1. In the figure, L is a base line, R_T is the distance between the target and the radiation source, R_R is the distance between the target and the receiver. α is the angle be-

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② To whom correspondence should be addressed. E-mail: wlsong139@126.com

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tween the GPS satellite and ground, γ is the angle between the target and the receiver, θ is the angle between L and R_R , v is the projection from target to ground with angle bisector φ .

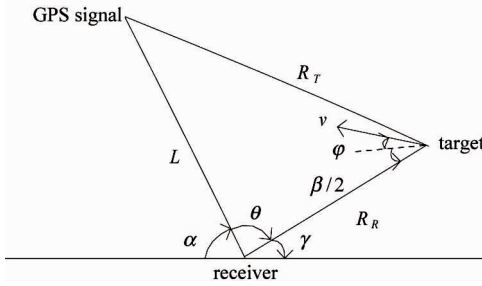


Fig. 1 Space system model

The ambiguity function is

$$\chi(R_R, R_{R0}, v, v_0, \theta) = \left| \int_{-\infty}^{\infty} C(t - \tau_0) C^*(t - \tau) \exp[-j2\pi(f_d - f_{d0})] dt \right|^2 \quad (1)$$

where, R_{R0} , v_0 , τ_0 and f_{d0} are the distance, speed, time delay and Doppler frequency shift of the initial target respectively. The receiving station is composed of reference channel and echo channel^[7]. The reference channel uses standard GPS receivers to collect the direct signal. In the echo channel, the weak target echoes are embedded in the background of strong direct signal, multipath targets and multi target sidelobe jamming. In the echo channel, down conversion to an intermediate frequency is processed after amplification and filter. The two-stage DOA estimation method is proposed. In the first stage, ECA algorithm is used to eliminate the direct interference and multipath interference using time domain interference cancellation and zero Doppler subsidence. In the second stage, multi objective DOA estimation is completed in low SNR using L1 Norm as a constraint condition.

2 Two-stage DOA estimation

2.1 ECA algorithm

Different from the conventional interference suppression method, the signal from base station is projected onto orthogonal subspace with base station direct wave and time delay expansion space by ECA algorithm, instead of estimating interference parameter. In the ECA algorithm, segmented cancellation can eliminate not only direct wave and multipath interference of zero frequency, but also near zero frequency clutter. Consequently, in the first stage ECA algorithm is adopted to suppress direct wave and multipath interference.

In echo channel, the target echoes are submerged in the background including target echo, direct wave interference, multipath interference and other noises. The signal in the echo channel is

$$s_{sur}(t) = s_{ref} + \sum_{i=1}^{N_m} A_{mi}s(t - \tau_{mi}) + \sum_{i=1}^{N_i} A_i s(t - \tau_i) e^{j\pi p \sin \theta_i} + n_{sur}(t) \quad (2)$$

where, the first part is direct wave interference, the second part is multipath interference, the third part is target echo signal, and the last part is other noise in the channel. Suppose that in the two-dimensional correlation unit of R distance, the strongest reflected signal is k distance, then the delay reference array is

$$s_{ref-k}(t) = s(t) \cdot \mathbf{D} \cdot \mathbf{F} \quad (3)$$

where, $\mathbf{D} = \{d_{ij}\}$ is Delay matrix, \mathbf{F} is Doppler frequency shift matrix with P frequency points. The projection operator \mathbf{M} is calculated by associative array $s_{ref-k}(t)$. The target echo is projected to the interference space orthogonal subspaces by projection operator. And then echo is

$$s_{sur-E}(t) = s_{sur}(t) \cdot [\mathbf{I} - \mathbf{M}(\mathbf{M}^H \mathbf{M})^{-1} \mathbf{M}^H] \quad (4)$$

where, $\mathbf{M} = \mathbf{B} \cdot s_{ref-k}$ is data matrix, $\mathbf{B} = \{b_{ij}\} = \begin{cases} 1 & i = j - R + 1 \\ 0 & \text{other} \end{cases}$, where \mathbf{M}^H is Hermitian matrix of \mathbf{M} , and \mathbf{I} is unit matrix. If Echo antenna receives N targets in the same range-Doppler unit, the target is to minimize error sum squares between echo signal and desired signal. The output after the first stage of ECA is

$$s_{sur-E}(\tau, f_d) = \sum_{i=1}^N G_i(\tau, f_d) e^{j\pi p \sin \theta_i} + Q_p(\tau, f_d) \quad (5)$$

where, $G_i(\tau, f_d) = \sum_{i=1}^N A_i s(t - \tau_i) s_{ref}(t - \tau_i - \tau) e^{j2\pi f_d(t - \tau_i)}$ is the complex envelope of main lobe, Q_p is the sum of residual direct wave, multipath wave and other targets sidelobe. Echo signal is projected onto an orthogonal subspace by ECA algorithm. Accordingly, direct wave and multipath interference are suppressed effectively. And then DOA estimation can be completed in the second stage.

2.2 DOA estimation sparse representation

MUSIC-like method is proposed in Ref. [8] for multiple targets in a range-Doppler unit, which uses all four order cumulation and subspace for direction finding. But this method requires information redundancy and high computation to coherent processing. Sparse representation for DOA estimation does not need signal coherence, therefore sparse representation is chosen for

high accuracy and few snapshots. When the relationship between number of antenna array elements P and target number satisfied $P \geq uN_0 \ln(N/N_0)$, it is defined that the target space is a sparse structure^[9]. Using sparse reconstruction, DOA of multiple target is accurately estimated with low computation complexity. The vectorization procedure of Eq. (5) is represented as

$$y = \text{vec}(s_{\text{sur-E}}(\tau, f_d)) = \mathbf{A}\boldsymbol{\beta} + n \quad (6)$$

where $\text{vec}(\cdot)$ is vector processing, y is the processed signal, $\boldsymbol{\beta}$ is a scalar matrix, \mathbf{A} is a coefficient matrix whose components are $[e^{j\pi \sin \theta_1} \dots e^{j\pi \sin \theta_i}]^T$. It is known that under appropriate conditions, it is possible to solve Eq. (6) with a sparsity constraint as follows:

$$\begin{aligned} \text{minimize } \|\hat{\boldsymbol{\beta}}\|_1, \text{ s. t. } \|\mathbf{y} - \mathbf{A}\hat{\boldsymbol{\beta}}\| &< \varepsilon \\ \|\text{Re}(\hat{\boldsymbol{\beta}}(i)), \text{Im}(\hat{\boldsymbol{\beta}}(i))\|_2 &< b_i \end{aligned} \quad (7)$$

where $\varepsilon, b_i (i = 1 \dots N)$ are parameters of convex optimization. However, this optimization problem is generally NP-hard. Therefore one seeks computationally efficient second-order cone programming algorithms that can approximately solve Eq. (7) with the RIP condition. After L1 regularization method there still remains error under low SNR because of amplitude of noise in $\hat{\boldsymbol{\beta}}$. Candes proposed a weighted norm method which is defined as

$$\begin{aligned} \text{minimize } \|\omega \hat{\boldsymbol{\beta}}\|_1, \text{ s. t. } \|\mathbf{y} - \mathbf{A}\hat{\boldsymbol{\beta}}\| &< \varepsilon \\ \|\text{Re}(\hat{\boldsymbol{\beta}}(i)), \text{Im}(\hat{\boldsymbol{\beta}}(i))\|_2 &< b_i \end{aligned} \quad (8)$$

where, diagonal matrix ω whose component is

$$\omega_i = \begin{cases} 1/(\|\hat{\boldsymbol{\beta}}(i)\| + \eta) & \|\hat{\boldsymbol{\beta}}(i)\| > \delta \\ \alpha\eta/(\|\hat{\boldsymbol{\beta}}(i)\| + \eta) & \|\hat{\boldsymbol{\beta}}(i)\| \leq \delta \end{cases} \quad (9)$$

where, δ is threshold value, large coefficient will be more than δ , α controls the degree of the adjustment, η is impact factor of weight. By adjusting the above parameters, Eq. (9) can be applied to balance the constraint value with the goal of recovering sparse signal in low SNR.

3 Algorithm descriptions

ECA algorithm is first used to eliminate the direct interference and multipath interference in zero Doppler subsidence. Correlation functions of target echo signals and reference signals can be handled as a linear combination of target and noise. So when sparse is the number of target k , $\boldsymbol{\beta}$ satisfies RIP condition^[10] $\delta_{2k}(\boldsymbol{\beta}) < \sqrt{2} - 1$, Eq. (7) can be adopted to sparse reconstruction to achieve the target DOA estimates. For the presence of error due to low SNR of k snapshots in practical range-Doppler unit, diagonal matrix which improves sparse model of weighted L1 norm using sparse regular-

ization is proposed. And the measurement can get DOA estimation with low computation complexity and high accuracy. The particular procedure is:

Step 1: construct matrix \mathbf{A} using ECA method according to Eq. (8)

Step 2: define cost function $J(\hat{\boldsymbol{\beta}}) = \|\mathbf{y} - \mathbf{A}\hat{\boldsymbol{\beta}}\|_2^2 + \|\omega \hat{\boldsymbol{\beta}}\|_1$ $\hat{\boldsymbol{\beta}} = \text{argmin}(J(\hat{\boldsymbol{\beta}}))$

Step 3: initial $\omega^0, \hat{\boldsymbol{\beta}}^0, \alpha, \eta, \delta$, end condition is $\rho = 0.01$

Step 4: update Eq. (9), for Eq. (8) to solve $\hat{\boldsymbol{\beta}}^{k+1}$

Step 5: update α, η ;

Step 6: $k = k + 1$;

Step 7: if $k > K$ or $|J(\hat{\boldsymbol{\beta}}^{k+1}) - J(\hat{\boldsymbol{\beta}}^k)| < \rho$, then iterated over, else go to Step 4

In the iterative process, the duality gap, gradient vector and Hesse matrix of the cost function $O(L)$ is applied to determine the searching direction, dual gap is applied to determine the termination condition, and finally liner echo search algorithm is adopted to determine the optimal step size.

4 Computer simulation and complexity analysis

In order to analyze the performance, GPS signal is constructed as a radiation source in a range-Doppler unit, assuming SNR is -20dB, and the DOA of two goals is -1° and 5° respectively. Fig. 2 shows the direction estimation result. Fig. 3 shows the reconstruction probability of 5° goal while 1° goals remains the same. As shown in the figure, the correct rate is more than 98% when two goals are 5.1° apart. Fig. 4 shows the probability estimation of two goals when SNR changes. It is obvious that this method can process target detection with SNR greater than -30dB. Fig. 5 shows the correct rate when gradually adjusting the array element number and remaining SNR the same. As shown in the figure, DOA estimation accuracy is higher when

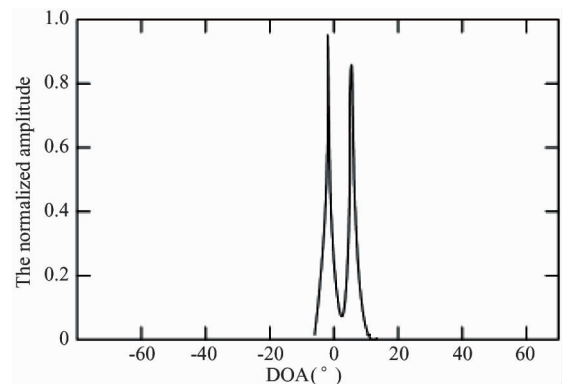


Fig. 2 DOA result

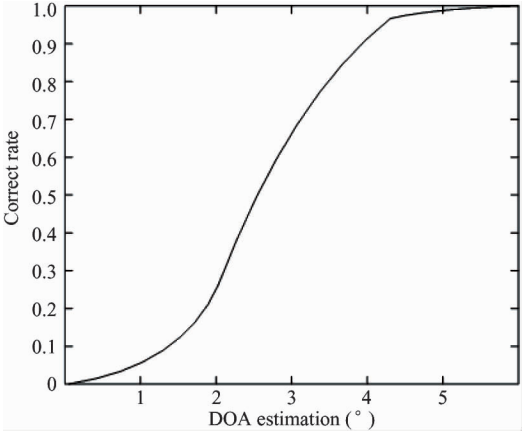


Fig. 3 Relationship between DOA estimation accuracy and correct rate

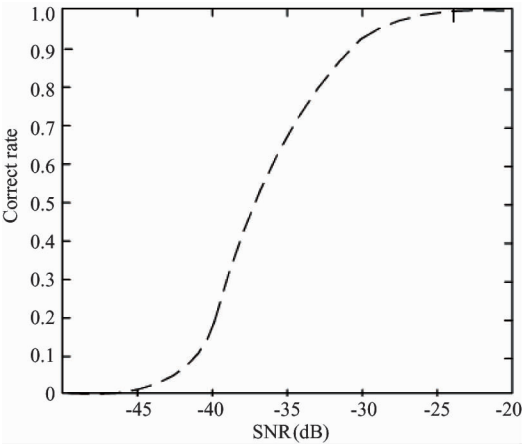


Fig. 4 Relationship between SNR and correct rate

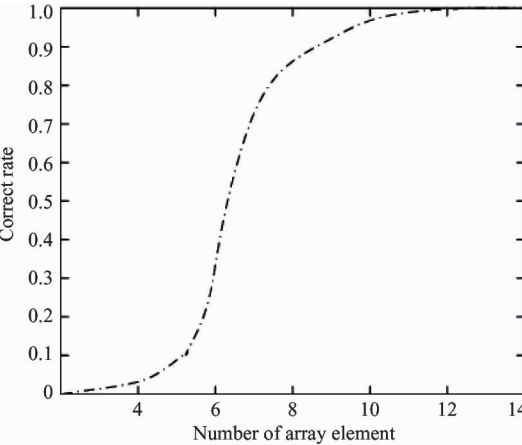


Fig. 5 Relationship between the number of array element and correct rate

the number of array element is larger than 10 because of spatial sparsity.

In simulation, 8 multipath interferences appear in the region of 0 ~ 30km, with SINR of 50dB. The distance, Doppler shift, DOA and SNR of 4 targets are:

(20km, 1MHz, 20°, - 8.04dB), (40km, 3MHz, 50°, - 10.07dB), (70km, 2MHz, 80°, - 17.28dB), (80km, - 2MHz, 100°, - 23.92dB). Fig.6 shows the range-Doppler correlation before processing interference suppression. As shown in Fig.6, target echo can not be detected because of channel interference. Fig.7 is zero Doppler section after ECA algorithm. ECA can eliminate interference within 10km to 60km. Fig.8 shows the contour of 2D correlation after processing ECA method for suppressing the direct wave and multipath in the region of 0 ~ 100km. Fig.9 shows the better effect detection after sparse representation. As shown in Fig.10, MSE (mean square error) of the method in this paper and Candes method is lower than that of MUSIC-like method. Fig.11 shows that the probability of resolution in this paper is higher than Candes method and MUSIC-like method. As shown in Fig.12, the RMSE (root mean square error) is stable when the threshold is less. When the threshold is greater than 0.3, RMSE increases for over constraint of much weight value.

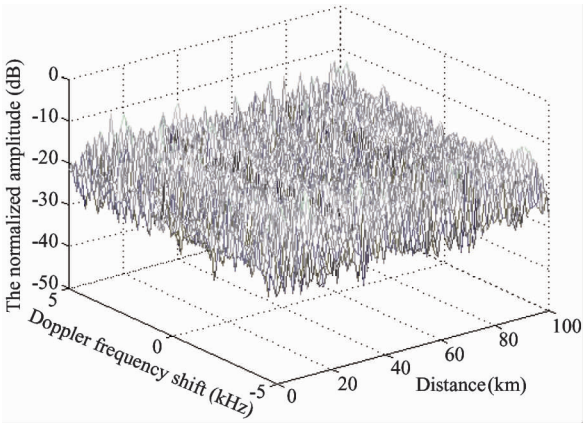


Fig. 6 Correlation before cancellation

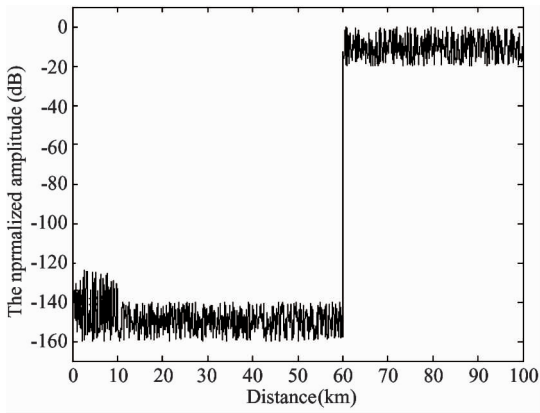


Fig. 7 Zero Doppler section after ECA

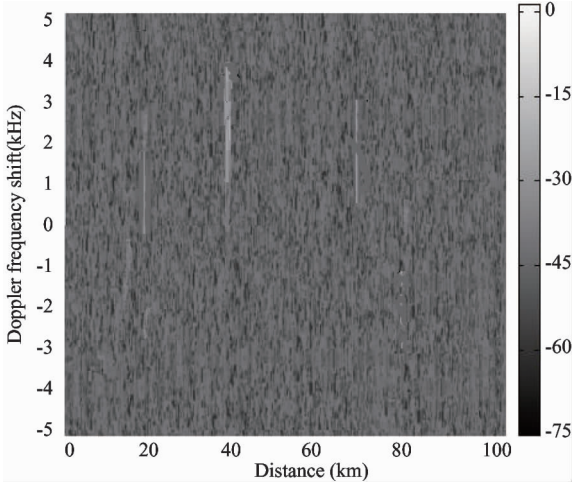


Fig. 8 2D correlation after ECA

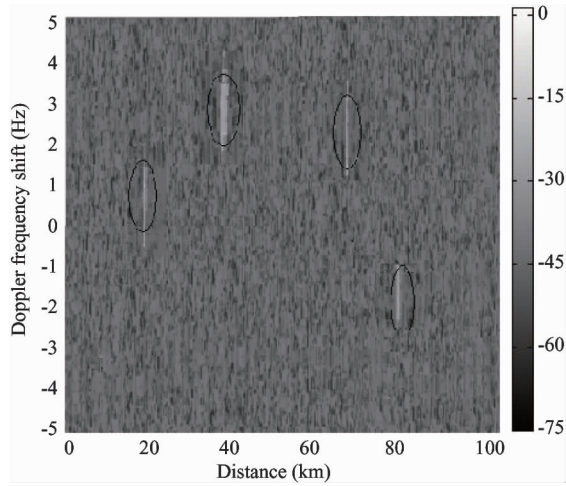


Fig. 9 2D correlation after sparse representation

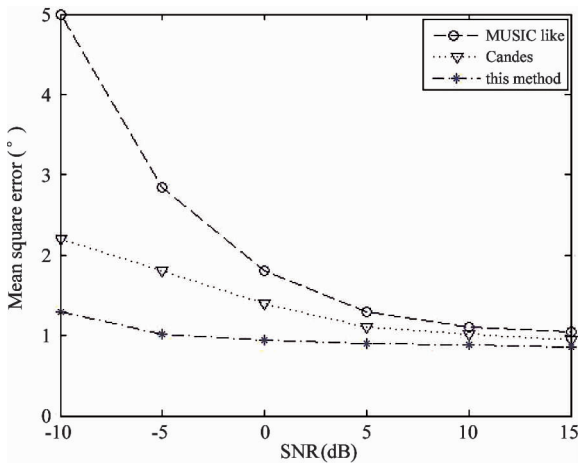


Fig. 10 Mean square error curve

Assuming that array antenna with M element receives L snapshots, the complexity is $O(LM^3)$ in ECA algorithm and $O(N^3 \log N)$ in sparse reconstruction. The total complexity is $O(LM^3 + N^3 \log N)$. MUSIC-like method needs a number of multiplications and addi-

tions, whose order of complexity is $O(M^{10} + LM^4 + NM^4)$. Therefore the complexity is lower based on sparse reconstruction.

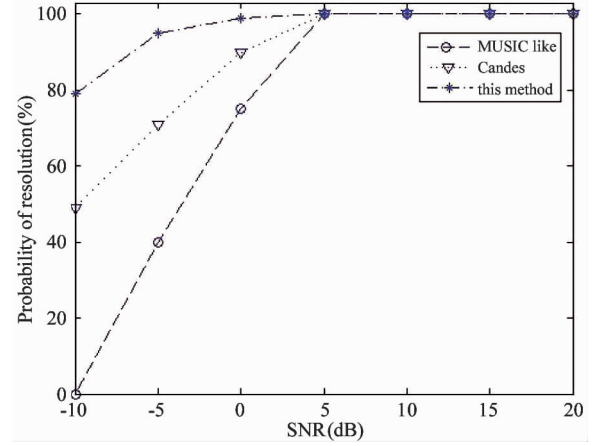


Fig. 11 Resolution probability curve

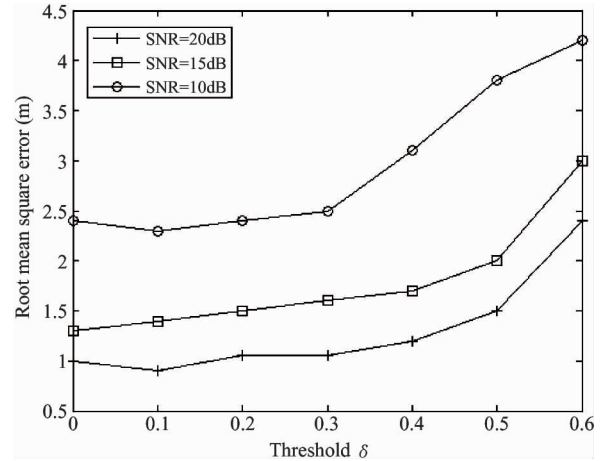


Fig. 12 Relationship between threshold and RMSE

5 Conclusion

By analyzing interference factors of GPS for passive radar system, a DOA estimation based on improved weight L1 norm constraints is proposed. In the first stage ECA method can eliminate the direct and multipath interference. In the second stage, by adjusting the weight, DOA estimate accuracy is high in low SNR with less computation complexity.

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Liu Nan, born in 1980. She received his Ph.D degrees in Information and Communications Engineering Department of Harbin Engineering University in 2007. She also received his B.S. and M.S. degrees from Harbin Institute of Technology University in 2002 and 2004 respectively. Her research interests include the radio navigation and radar system.