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Maneuvering target tracking algorithm based on cubature Kalman filter with observation iterated update①

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Abstract

Reasonable selection and optimization of a filter used in model estimation for a multiple model structure is the key to improve tracking accuracy of maneuvering target. Combining with the cubature Kalman filter with iterated observation update and the interacting multiple model method, a novel interacting multiple model algorithm based on the cubature Kalman filter with observation iterated update is proposed. Firstly, aiming to the structural features of cubature Kalman filter, the cubature Kalman filter with observation iterated update is constructed by the mechanism of iterated observation update. Secondly, the improved cubature Kalman filter is used as the model filter of interacting multiple model, and the stability and reliability of model identification and state estimation are effectively promoted by the optimization of model filtering step. In the simulations, compared with classic improved interacting multiple model algorithms, the theoretical analysis and experimental results show the feasibility and validity of the proposed algorithm.

Key word: maneuvering target tracking, nonlinear filtering, cubature Kalman filter (CKF), interacting multiple model (IMM)

0 Introduction

Target tracking is used by subjects that realize a process of state modeling, estimation and tracking about the objects observed by means of various observation and calculation methods. As an emerging technique, target tracking is widely applied to the military, civilian and economic fields[1]. Target tracking is classified to maneuvering target tracking and non-maneuvering target tracking by the type and intensity of motions. In the maneuvering case, owing to the variety and complexity of target motion features, it is difficult to describe precisely the motion state of via the single and stationary models, therefore, the multiple model structure is commonly adopted[2]. Compared with the single-model approach, a set for describing the behavior pattern of the system is selected or designed through the multi-model approach where each model matches with a specific system pattern, and the estimation for system state is the reasonable synthesis of filtering results of parallel running filters[3]. Among various multiple model methods and their improved methods, the interacting multiple model (IMM) algorithm is recognized as an effective approach to handle the system model switch problem, which adopts the modeling soft switch mechanism and effectively keeps the balance between model identification and state estimation precision[4,5]. In the conventional IMM structure, the Kalman filter meeting the criterion of the linear minimum variance estimation is selected as the model filter which is able to obtain high precision for linear Gaussian system. However, the performance of the selected nonlinear filter will directly determine the estimation precision of the system state and computation complexity of the algorithm when the estimated system has strong nonlinear or non-Gaussian feature[6,7].

In recent years, the research in nonlinear filter catches much attention by domestic and international experts and scholars in related fields, and some phasic achievements are made. Considering the superiority of Kalman filter in realizing recursive Bayesian estima-

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tion, combining with local linearization techniques, the extended Kalman filter (EKF) is constructed under the framework of KF[8,9]. Its basic idea is to linearize the nonlinear systems by taking the advantage of the Taylor series, but this linearization error is large, and it is difficult to get the Jacobian matrix from nonlinear function in many practical problems. To solve such problems, some new nonlinear filtering methods have been proposed combining with the UT transform or numerical differencing technique, such as Unscented Kalman filter (UKF)[10,11], central difference filter (CDF)[12], Ensemble Kalman filter (EKF)[13,14], etc. However, these methods can hardly meet the engineering demands because they lead to sharp decline of estimation accuracy or even filter divergence when handling strong nonlinear and non-Gaussian problems. With the rapid improvement in computer performance, combining with sequential Monte-Carlo simulation method and recursive Bayesian thought, Gordon et al. proposed the particle filter (PF) which consists of two basic steps of prediction and update. Unlike Kalman filter, the prediction step combines a priori model information with sequential Monte-Carlo simulation technique (SMC), and the update step is completed through re-sampling technique. PF can achieve a better filtering accuracy than the EKF and UKF, also it is suitable for the nonlinear systems with arbitrary noise distribution[15]. However, the implementation mechanism sequential of importance sampling and re-sampling makes PF cannot effectively overcome the problems of particle degeneracy and re-sampling particle diversity impoverishment. Moreover, the filtering precision of PF is closely related with the number of system dimension and the amount of particles which limits the universality of its parameters for application objects[16]. In addition, based on the third cubature rule, Arasaratnam et al, proposed the cubature Kalman filter (CKF)[17]. CKF approximates the weighted Gaussian integration by numerical integration, which takes the advantage of high efficiency of calculating the multi-dimensional function integration by using cubature integration numerical value. With $2n$ equal weighted cubature points (n is the number of system state dimension), Cubature Kalman filtering is proved that its probability distribution precision is better than UKF's after approximating nonlinear transformation.

Based on the above analysis, in the framework of CKF, combining with the mechanism of observation iterated update, a novel improved cubature Kalman filter (ICKF) is constructed to improve the estimation precision of CKF. Then applying ICKF into the algorithm framework of IMM, that is, ICKF is used as the model filter to improve the performance of IMM. On the basis of that, this paper proposes a novel maneuvering target tracking algorithm based on cubature Kalman filter with observation iterated update (IMM-ICKF). The simulations have verified the superiority of the algorithm.

1 Cubature Kalman filter with observation iterated update

1.1 Cubature Kalman filter

The key idea of CKF is to calculate the normal weighted Gaussian integration of function $f(\boldsymbol{x})$ by the third cubature integration rule[18], that is

$$\int_{\mathbb{R}^n} f(\boldsymbol{x}) N(\boldsymbol{x};\boldsymbol{\mu},\boldsymbol{P})\,\mathrm{d}x \approx 1/L\sum_{i=1}^{L} f(\boldsymbol{\mu}+\sqrt{\boldsymbol{P}}\xi_i) \tag{1}$$

where $N(\boldsymbol{x};\boldsymbol{\mu},\boldsymbol{P})$ denotes that the random variable $\boldsymbol{x}$ is subject to the normal distribution with mean $\boldsymbol{\mu}$ and covariance matrix $\boldsymbol{P}$. $L=2n$ denotes the number of cubature points, and ξ_i represents the ith cubature point.

$$\xi_i = \sqrt{n}\left[\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix}\cdots\begin{pmatrix}0\\0\\\vdots\\1\end{pmatrix}\begin{pmatrix}-1\\0\\\vdots\\0\end{pmatrix}\cdots\begin{pmatrix}0\\0\\\vdots\\-1\end{pmatrix}\right] \tag{2}$$

To demonstrate the recursive realization of CKF, assuming that the posteriori probability density function $p(\boldsymbol{x}_{k-1}\mid \boldsymbol{Z}_{1:k-1})$ which is subject to $N(\boldsymbol{x}_{k-1};\hat{\boldsymbol{x}}_{k-1/k-1},\boldsymbol{P}_{k-1/k-1})$, has been given at time $k-1$. Both are based on the recursive Bayes framework, CKF is similar to KF and their realization consists of two steps, the state one-step prediction and observation update. Firstly, based on the filtering result of CKF at time $k-1$ and combining with priori modeling information of state evolution, the state one-step prediction is realized. The concrete realization is as follows: assuming that the prediction error covariance matrix $\boldsymbol{P}_{k-1/k-1}$ is positive definite, $\boldsymbol{S}_{k-1/k-1}$ is obtained by implementing the Cholesky decomposition onto $\boldsymbol{P}_{k-1/k-1}$, that is

$$\boldsymbol{P}_{k-1/k-1} = \boldsymbol{S}_{k-1/k-1}(\boldsymbol{S}_{k-1/k-1})^{\mathrm{T}} \tag{3}$$

Then the estimation for cubature points in the mechanism of state one-step prediction is achieved by $\boldsymbol{S}_{k-1/k-1}$.

$$\boldsymbol{x}^i_{k-1/k-1} = \boldsymbol{S}_{k-1/k-1}\xi_i + \hat{\boldsymbol{x}}_{k-1/k-1} \tag{4}$$

The diffusion of cubature points in the mechanism of state one-step prediction is realized by the state transform equation.

$$\boldsymbol{x}^i_{k/k-1} = f(\boldsymbol{x}^i_{k-1/k-1}) \tag{5}$$

And the state one-step prediction $\hat{\boldsymbol{x}}_{k/k-1}$ is solved.

$$\hat{\boldsymbol{x}}_{k/k-1} = \frac{1}{L}\sum_{i=1}^{L}\boldsymbol{x}^i_{k/k-1} \tag{6}$$

Then the prediction error covariance matrix $\boldsymbol{P}_{k/k-1}$ is calculated.

$$\boldsymbol{P}_{k/k-1} = \frac{1}{L}\sum_{i=1}^{L}\boldsymbol{x}_{k/k-1}^{i}(\boldsymbol{x}_{k/k-1}^{i})^{\mathrm{T}} - \hat{\boldsymbol{x}}_{k/k-1}(\hat{\boldsymbol{x}}_{k/k-1})^{\mathrm{T}} + \boldsymbol{\sigma}_{u,k-1}^{2} \tag{7}$$

where $\boldsymbol{\sigma}_{u,k-1}^{2}$ denotes the system noise covariance matrix. Secondly, the observation update step is realized by combining with sensor observation information at time k as follows, the Cholesky decomposition on $\boldsymbol{P}_{k/k-1}$ is implemented.

$$\boldsymbol{P}_{k/k-1} = \boldsymbol{S}_{k/k-1}(\boldsymbol{S}_{k/k-1})^{\mathrm{T}} \tag{8}$$

Next, according to $\boldsymbol{S}_{k/k-1}$, the estimation of cubature point in the mechanism of observation update is realized.

$$\bar{\boldsymbol{x}}_{k/k-1}^{i} = \boldsymbol{S}_{k/k-1}\xi_{i} + \hat{\boldsymbol{x}}_{k/k-1} \tag{9}$$

The diffusion of cubature points in the mechanism of observation update is achieved by observation equation.

$$z_{k/k-1}^{i} = h(\bar{\boldsymbol{x}}_{k/k-1}^{i}) \tag{10}$$

On the basis of that, the observation one-step prediction $\hat{z}_{k/k-1}$ is solved.

$$\hat{z}_{k/k-1} = \frac{1}{L}\sum_{i=1}^{L} z_{k/k-1}^{i} \tag{11}$$

To solve the filtering gain matrix $\boldsymbol{K}_k$, the observation partial error variance $\boldsymbol{P}_k^{zz}$ and the covariance matrix $\boldsymbol{P}_k^{xz}$ of state and observation prediction error need to be calculated.

$$\boldsymbol{P}_k^{zz} = \frac{1}{L}\sum_{i=1}^{L} z_{k/k-1}^{i}(z_{k/k-1}^{i})^{\mathrm{T}} - \hat{z}_{k/k-1}(\hat{z}_{k/k-1})^{\mathrm{T}} + \boldsymbol{\sigma}_{v,k}^{2} \tag{12}$$

$$\boldsymbol{P}_k^{xz} = \sum_{i=1}^{L}\bar{\boldsymbol{x}}_{k/k-1}^{i}(z_{k/k-1}^{i})^{\mathrm{T}} - \hat{\boldsymbol{x}}_{k/k-1}(\hat{z}_{k/k-1})^{\mathrm{T}} \tag{13}$$

where $\boldsymbol{\sigma}_{v,k}^{2}$ denotes the observation noise covariance matrix. $\boldsymbol{K}_k$ is calculated by

$$\boldsymbol{K}_k = \boldsymbol{P}_k^{xz}(\boldsymbol{P}_k^{zz})^{-1} \tag{14}$$

Finally, according to Eq. (15) and Eq. (16), the system state estimation $\hat{\boldsymbol{x}}_{k|k}$ and its estimation error covariance matrix $\boldsymbol{P}_{k/k}$ can be obtained.

$$\hat{\boldsymbol{x}}_{k/k} = \hat{\boldsymbol{x}}_{k/k-1} + \boldsymbol{K}_k(z_k - \hat{z}_{k/k-1}) \tag{15}$$

$$\boldsymbol{P}_{k/k} = \boldsymbol{P}_{k/k-1} - \boldsymbol{K}_k\boldsymbol{P}_k^{zz}(\boldsymbol{K}_k)^{\mathrm{T}} \tag{16}$$

1.2 The observation iterated update strategy

As we know from the implementing process of CKF, $\hat{\boldsymbol{x}}_{k/k}$ and $\boldsymbol{P}_{k/k}$ obtained in the observation update step are the reasonably correction results of $\hat{\boldsymbol{x}}_{k/k-1}$ and $\boldsymbol{P}_{k/k-1}$ by latest observation information z_k using $\boldsymbol{K}_k$, so the approximate degree of $\hat{\boldsymbol{x}}_{k/k}$ and $\boldsymbol{P}_{k/k}$ to the true state will be inevitably superior to $\hat{\boldsymbol{x}}_{k/k-1}$ and $\boldsymbol{P}_{k/k-1}$. The directly utilizing of $\hat{\boldsymbol{x}}_{k/k}$ and $\boldsymbol{P}_{k/k}$ in the observation update step can improve the filtering estimation precision necessarily. To repeat the observation update step with $\hat{\boldsymbol{x}}_{k/k}$ and $\boldsymbol{P}_{k/k}$ in filtering realization is the idea of observation iterated update. According to the above analysis, the flow of ICKF is as follows.

Firstly, combining with the construction principle of CKF, the state one-step prediction $\hat{\boldsymbol{x}}_{k/k-1,J}$ and state one-step prediction error covariance matrix $\boldsymbol{P}_{k/k-1,J}$ can be achieved.

$$\boldsymbol{P}_{k-1/k-1,J} = \boldsymbol{S}_{k-1/k-1,J}(S_{k-1/k-1,J})^{\mathrm{T}} \tag{17}$$

$$\boldsymbol{x}_{k-1/k-1,J}^{i} = \boldsymbol{S}_{k-1/k-1,J}\xi_i + \hat{\boldsymbol{x}}_{k-1/k-1,J} \tag{18}$$

$$\boldsymbol{x}_{k/k-1,J}^{i} = f(\boldsymbol{x}_{k-1/k-1,J}^{i}) \tag{19}$$

$$\hat{\boldsymbol{x}}_{k/k-1,J} = \frac{1}{L}\sum_{i=1}^{L}\boldsymbol{x}_{k/k-1,J}^{i} \tag{20}$$

$$\boldsymbol{P}_{k/k-1,J} = \frac{1}{L}\sum_{i=1}^{L}\boldsymbol{x}_{k/k-1,J}^{i}(\boldsymbol{x}_{k/k-1,J}^{i})^{\mathrm{T}} - \hat{\boldsymbol{x}}_{k/k-1,J}(\hat{\boldsymbol{x}}_{k/k-1,J})^{\mathrm{T}} + \boldsymbol{\sigma}_{u,k-1}^{2} \tag{21}$$

Let j denote the jth iterated implementation in observation update, $j = 1, 2, \cdots, J$, and J is the maximum number of iterations. From the above process, compared with conventional CKF, ICKF obtains state one-step prediction and state one-step prediction error covariance matrix through the Lth iteration (the last iteration) at time $k - 1$ in the observation update step. To realize the iterated update process, $\hat{\boldsymbol{x}}_{k/k}$ and $\boldsymbol{P}_{k/k}$ need to be solved as the initial value $\hat{\boldsymbol{x}}_{k/k,0}$ and $\boldsymbol{P}_{k/k,0}$ at time k for the iterated calculation by putting $\hat{\boldsymbol{x}}_{k/k-1,J}$ and $\boldsymbol{P}_{k/k-1,J}$ into observation update step. To calculate the estimation of cubature point in the observation iterated update, the estimation error covariance matrix needs to carry out the Cholesky decomposition.

$$\boldsymbol{P}_{k/k,j-1} = \boldsymbol{S}_{k/k,j-1}(\boldsymbol{S}_{k/k,j-1})^{\mathrm{T}} \tag{22}$$

$$\bar{\boldsymbol{x}}_{k/k,j}^{i} = \boldsymbol{S}_{k/k,j-1}\xi_i + \hat{\boldsymbol{x}}_{k/k,J} \tag{23}$$

$$z_{k/k,j}^{i} = h(\bar{\boldsymbol{x}}_{k/k,j}^{i}) \tag{24}$$

$$\hat{z}_{k/k,j} = \frac{1}{L}\sum_{i=1}^{L} z_{k/k,j}^{i} \tag{25}$$

$$\boldsymbol{P}_{k,j}^{zz} = \frac{1}{L}\sum_{i=1}^{L} z_{k/k,j}^{i}(z_{k/k,j}^{i})^{\mathrm{T}} - \hat{z}_{k/k,j}(\hat{z}_{k/k,j})^{\mathrm{T}} + \boldsymbol{\sigma}_{v,k}^{2} \tag{26}$$

$$\boldsymbol{P}_{k,j}^{xz} = \sum_{i=1}^{L}\bar{\boldsymbol{x}}_{k/k,j}^{i}(z_{k/k,j}^{i})^{\mathrm{T}} - \hat{\boldsymbol{x}}_{k/k,J}(\hat{z}_{k/k,j})^{\mathrm{T}} \tag{27}$$

$$\boldsymbol{K}_{k,j} = \boldsymbol{P}_{k,j}^{xz}(\boldsymbol{P}_{k,j}^{zz})^{-1} \tag{28}$$

$$\hat{\boldsymbol{x}}_{k/k,j} = \hat{\boldsymbol{x}}_{k/k,j-1} + \boldsymbol{K}_{k,j}(z_k - \hat{z}_{k/k,j}) \tag{29}$$

$$\boldsymbol{P}_{k/k,j} = \boldsymbol{P}_{k/k,j-1} - \boldsymbol{K}_{k,j}\boldsymbol{P}_{k,j}^{zz}(\boldsymbol{K}_{k,j})^{\mathrm{T}} \tag{30}$$

The repeating utilization of observation iterated update for improving the estimation performance is limited, and in the practical applications, in view of the balance between the filtering precision and the computation complexity, the number of iterations should not be too large, and J is usually 1 or 2.

2 Maneuvering target tracking algorithm based on iterated cubature Kalman filter with observation iterated update

2.1 Interacting multiple model

Consider the following multi-model nonlinear system with model switching.

$$\boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}, r_k, \boldsymbol{u}_{k-1}) \tag{31}$$

$$z_k = h(\boldsymbol{x}_k, r_k, \boldsymbol{v}_k) \tag{32}$$

$$r_k \sim p(r_k \mid r_{k-1}) \tag{33}$$

x_k and z_k denote the system state variable and observation, respectively. u_k and v_k denote the system process noise and the observation noise with the independent and identical distribution characteristic, respectively. r_k denotes the system model state, and $D \triangleq \{1, 2, \cdots, d\}$ is defined as the set of first order Markov chain model state satisfying the discrete time, homogeneous and limited state. $\mu_0^a = P_r\{r_0 = a\}$ denotes the initial probability of the model, and the priori transform probability of model state is $\boldsymbol{\pi}_{ab} = P_r\{r_{k+1} = b \mid r_k = a\}$. $\boldsymbol{\Pi} = [\boldsymbol{\pi}_1, \boldsymbol{\pi}_2, \cdots, \boldsymbol{\pi}_d]^{\mathrm{T}}$ denotes the model transform probability matrix, where $\boldsymbol{\pi}_a = [\pi_{a1}, \pi_{a2}, \cdots, \pi_{ad}]$, and $\sum_{b=1}^{d} \boldsymbol{\pi}_{ab} = 1$, and $a, b, d \in D$. The basic principle of IMM lies on keeping all the models in system parallel running and the estimation synthesis of each model filtering results through calculating their model probability weight. IMM consists of four parts which include input interaction, model filtering, model probability update and output interaction. The part of input interaction calculates the prediction probability of each model $\bar{\mu}_{k-1}^a$, the model mixture probability $\mu_{k-1}^{b/a}$, the model mixture state estimation of each model $\hat{\boldsymbol{X}}_{k-1/k-1}^a$ and the mixture state estimation error covariance $\hat{\boldsymbol{P}}_{k-1/k-1}^a$. The part of model filtering implements the filtering process on each model, the state estimation $\hat{\boldsymbol{x}}_{k/k,J}^a$ of each model, the state estimation error covariance matrix $\boldsymbol{P}_{k/k,J}^a$ and the partial error covariance matrix $\boldsymbol{P}_{k,J}^{zz,a}$ are obtained in this part. Using $\boldsymbol{P}_{k,J}^{zz,a}$, the model probability update part calculates each model likelihood l_k^a and model probability μ_k^a of each model from the model set at time k. According to μ_k^a, $\hat{\boldsymbol{x}}_{k/k,J}^a$ and $\boldsymbol{P}_{k/k,J}^a$ obtained through the above three parts, the output interaction part realizes the calculation of system state estimation $\hat{\boldsymbol{X}}_{k/k}$ and the state estimation error covariance matrix $\boldsymbol{P}_{k/k}$.

2.2 Interacting multiple model based on cubature Kalman filter with observation iterated update

In the practical application of IMM, the improvement of filtering precision lies on the reasonable selection of sub-filter according to the feature and performance requirements of estimated system. Considering that ICKF has high estimation precision and universality, ICKF is selected as sub-filter in the filtering part of the IMM framework and it promotes the overall performance of IMM by improving the state estimation result of each model. On the basis of that, this section proposes the interacted multi-model algorithm based on cubature Kalman filter with observation iterated update (IMM-ICKF). The recursive implementation process of IMM-ICKF is as follows.

While each model probability μ_{k-1}^a, $a \in D$ and the transform matrix $\boldsymbol{\Pi}$ of the model state are given, combining with the filtering result of ICKF for each model at time $k-1$, the prediction probability $\bar{\mu}_{k-1}^a$ of each model, the mixture probability $\mu_{k-1}^{b/a}$, the mixture state estimation $\hat{\boldsymbol{X}}_{k-1/k-1}^a$ and the mixture state estimation error covariance matrix $\hat{\boldsymbol{P}}_{k-1/k-1}^a$ are solved, and then the input interaction process of IMM is achieved.

$$\bar{\mu}_{k-1}^a = \sum_{b=1}^{d} \pi_{ab} \mu_{k-1}^b \tag{34}$$

$$\mu_{k-1}^{b/a} = \pi_{ab} \mu_{k-1}^b / \bar{\mu}_{k-1}^a \tag{35}$$

$$\hat{\boldsymbol{X}}_{k-1/k-1}^a = \sum_{b=1}^{d} \hat{\boldsymbol{x}}_{k-1/k-1,J}^b \mu_{k-1}^{b/a} \tag{36}$$

$$\hat{\boldsymbol{P}}_{k-1/k-1}^a = \sum_{b=1}^{d} \{\boldsymbol{P}_{k-1/k-1,J}^b + (\hat{\boldsymbol{X}}_{k-1/k-1}^a - \hat{\boldsymbol{x}}_{k-1/k-1,J}^b)(\hat{\boldsymbol{X}}_{k-1/k-1}^a - \hat{\boldsymbol{x}}_{k-1/k-1,J}^b)^{\mathrm{T}}\} \mu_{k-1}^{b/a} \tag{37}$$

where $\hat{\boldsymbol{x}}_{k-1/k-1,J}^b$ and $\boldsymbol{P}_{k-1/k-1,J}^b$ denote respectively the state estimation and the state estimation error covariance of model b obtained by ICKF, J denotes the above results obtained from the last iteration of ICKF (similarly hereinafter). Then, using $\hat{\boldsymbol{X}}_{k-1/k-1}^a$ and $\hat{\boldsymbol{P}}_{k-1/k-1}^a$ from the input interaction process as the initial value of ICKF filtering at time k, each model will be implemented with ICKF. And by using the Eq. (25), Eq. (26), Eq. (29) and Eq. (30), $\hat{z}_{k/k,j}$, $\boldsymbol{P}_{k,J}^{zz,a}$, $\hat{\boldsymbol{x}}_{k/k,J}^a$, and $\boldsymbol{P}_{k/k,J}^a$ of each model is calculated to achieve the IMM model filtering. Next, according to Eq. (38) and Eq. (39), the model likelihood l_k^a and model probability μ_k^a of each model is solved to realize the model probability update part of IMM.

$$l_k^a = (2\pi)^{-1/2} \mid \boldsymbol{P}_{k,J}^{zz,a} \mid^{-1/2} \exp\left(-\frac{1}{2}(z_k - \hat{z}_{k/k,J}^a)^{\mathrm{T}} (\boldsymbol{P}_{k,J}^{zz,a})^{-1} (z_k - \hat{z}_{k/k,J}^a) \right) \tag{38}$$

$$\mu_k^a = \bar{\mu}_{k-1}^a l_k^a / \sum_{b=1}^{d} \bar{\mu}_{k-1}^b l_k^b \tag{39}$$

Finally, combining with $\hat{\boldsymbol{x}}_{k/k,J}^a$ and $\boldsymbol{P}_{k/k,J}^a$ and μ_k^a, the system state estimation $\hat{\boldsymbol{X}}_{k/k}$ and the system state estimation error covariance matrix $\boldsymbol{P}_{k/k}$ are calculated.

$$\hat{\boldsymbol{X}}_{k/k} = \sum_{a=1}^{d} \hat{\boldsymbol{x}}_{k/k,J}^a \mu_a \tag{40}$$

$$\boldsymbol{P}_{k/k} = \sum_{a=1}^{d} \{ \boldsymbol{P}_{k/k,L}^{a} + (\hat{\boldsymbol{X}}_{k/k} - \hat{\boldsymbol{x}}_{k/k,L}^{a})(\hat{\boldsymbol{X}}_{k/k} - \hat{\boldsymbol{x}}_{k/k,L}^{a}) \} \mu_k^a \quad (41)$$

3 Simulation result and analysis

To verify the feasibility and availability of the proposed algorithm, the observations based on two-coordinate radar are adopted to realize the typical maneuvering target tracking setting in the X-Y plane. The motion of the observed target in Radar scanning area is as follows: uniform circular motion with the turning angular velocity $+0.4\text{rad/s}^2$ in the first 10 sampling periods; uniform circular motion with the turning angular velocity -0.2rad/s^2 in 11th to 25th sampling periods; uniform circular motion with the turning angular velocity -0.4rad/s^2 in the following 10 sampling periods, where plus sign and minus sign denote the different uniform turning directions and plus sign represent the clockwise direction and minus sign counterclockwise. Combining with dynamic characteristics of maneuvering target motion and physical properties of Radar sensors, maneuvering target tracking system state equation and the observation equation are as follows.

$$\boldsymbol{x}_k = \begin{cases} \boldsymbol{F}_1 \boldsymbol{x}_{k-1} + \boldsymbol{\Gamma} \boldsymbol{u}_{1,k-1} & 1 \leqslant k \leqslant 10 \\ \boldsymbol{F}_2 \boldsymbol{x}_{k-1} + \boldsymbol{\Gamma} \boldsymbol{u}_{2,k-1} & 11 \leqslant k \leqslant 25 \\ \boldsymbol{F}_1 \boldsymbol{x}_{k-1} + \boldsymbol{\Gamma} \boldsymbol{u}_{1,k-1} & 26 \leqslant k \leqslant 35 \end{cases}$$

$$\boldsymbol{z}_k = [\gamma_k \quad \theta_k]^{\mathrm{T}} + \boldsymbol{v}_k$$

$$\gamma_k = \mathrm{sqrt}(x_k^2 + y_k^2)$$

$$\theta_k = \tan^{-1}(y_k / x_k)$$

where $\boldsymbol{x}_k = [x_k \quad \dot{x}_k \quad y_k \quad \dot{y}_k]^{\mathrm{T}}$, x_k, $\dot{x}_k$, y_k and $\dot{y}_k$ denote the location components and velocity components of target state on the x-axis and y-axis, respectively.

$\boldsymbol{F}_1 =$

$$\begin{bmatrix} 1 & \sin(w_1\tau)/w_1 & 0 & -(1-\cos(w_1\tau))/w_1 \\ 0 & \cos(w_1\tau) & 0 & -\sin(w_1\tau) \\ 0 & (1-\cos(w_1\tau))/w_1 & 1 & \sin(w_1\tau)/w_1 \\ 0 & \sin(w_1\tau) & 0 & \cos(w_1\tau) \end{bmatrix}$$

and $\boldsymbol{F}_2 =$

$$\begin{bmatrix} 1 & \sin(w_2\tau)/w_2 & 0 & -(1-\cos(w_2\tau))/w_2 \\ 0 & \cos(w_2\tau) & 0 & -\sin(w_2\tau) \\ 0 & (1-\cos(w_2\tau))/w_2 & 1 & \sin(w_2\tau)/w_2 \\ 0 & \sin(w_2\tau) & 0 & \cos(w_2\tau) \end{bmatrix}$$

denote system state transform matrix in which $w_1 = 0.3\text{rad/s}^2$ and $w_2 = -0.2\text{rad/s}^2$ are the turning angular velocity of target motion. the sampling interval $\tau = 0.5$. The system noise $\boldsymbol{u}_{1,k}$ and $\boldsymbol{u}_{2,k}$ adopt the Gaussian white noise with means 0 and standard deviations $0.2\boldsymbol{I}$ and $0.4\boldsymbol{I}$ respectively, where $\boldsymbol{I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Observation noise $\boldsymbol{v}_k$ is Gaussian white noise with means 0 and standard deviation $\begin{bmatrix} R_\gamma & 0 \\ 0 & R_\theta \end{bmatrix}$, where the noise standard deviation of radial distance component R_γ is 0.2km and the noise variance of azimuth angle R_θ is $0.1°$. $\boldsymbol{\Gamma} = \begin{bmatrix} \tau\hat{}2/2 & \tau & 0 & 0 \\ 0 & 0 & \tau\hat{}2/2 & \tau \end{bmatrix}^{\mathrm{T}}$ denotes the process noise. the cycle index of Monte Carlo simulation is 50, and the number of simulation steps is 35. The number of particles in PF is 1000, and the maximum iteration time is 2, and the initial value of target state $\boldsymbol{x}_0 = [15 \quad 0.8 \quad 8 \quad 0.3]^{\mathrm{T}}$. The experimental platform adopts PC (Pentium4 (CPU) with main frequency 3.06GHZ, 2G memory, Windows XP), and the programming language is Matlab _ R2012a. Five algorithms, IMM-EKF, IMM-UKF, IMM-CKF, IMM-PF and IMM-ICKF are compared in the simulations, that is, EKF, UKF, CKF, PF and ICKF are used as sub-filter for the implementation of IMM.

Fig. 1 shows the real motion trajectory and the observation information of the target in the simulated experimental settings. With model probability as model identification reliability index, Fig. 2 to Fig. 6 give the model utilizations respectively of the filtering implementation of IMM-EKF, IMM-UKF, IMM-CKF, IMM-PF and IMM-ICKF. Fig. 7 and Fig. 8 show the comparison of root mean square error (RMSE) of state estimation of these five algorithms in 50 independent experiments. From the model identification effectiveness of these five algorithms given by Fig. 2 to Fig. 6, IMM-EKF is clearly shown to have the poorest stability of the accuracy, the essential reason of which is that IMM-EKF cannot provide state estimation result with high precision. Next, IMM-UKF is superior to IMM-EKF, while IMM-PF and IMM-CKF are superior to IMM-UKF to a certain degree, but the defect which these four algorithms mentioned above have in common is that there is large fluctuation of model identification in the filtering implementing process. Compared with the other four algorithms, IMM-ICKF improves the accuracy and stability of model identification obviously. As is known to all, in the IMM framework, the sub-filter with high precision will support IMM to achieve the effective identification of state evolution model at the current time, and the accurate model identification will support in turn sub-filter to obtain nice state estimation result in the next time filtering, and the feature is reflected in Fig. 7 and Fig. 8. Regarding to the filtering precision of algorithms, according to the state estimation precision, the ranking from the best to worst of all the five algorithms is as follows: IMM-ICKF, IMM-PF, IMM-CKF, IMM-UKF and IMM-EKF. It is worthy noting that the filtering precision of IMM-PF and IMM-CKF is similar, and IMM-ICKF is better than IMM-CKF, the fundamental reason of which is that ICKF realizes improvement of filtering estimation precision by introducing observation iterated update strategy. To quantitatively analyze the filtering precision and real-time performance of these five algorithms, their means of RMSE and average running time is compared in 50 independent simulations shown in Table1, and the data of means of RMSE describing algorithm filtering precision in the table verifies the results analyzed above. In addition, in the same simulation condition, regarding the time consumed of these algorithms, IMM-PF take the first place,

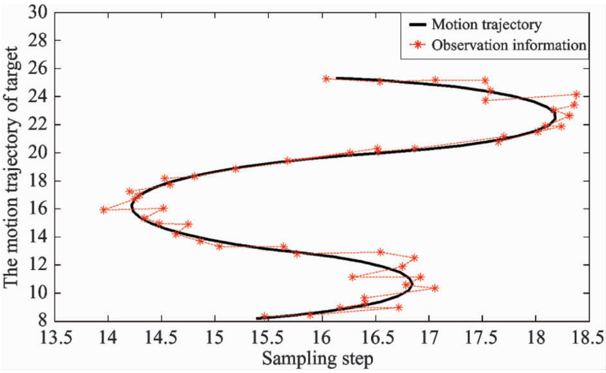

Fig. 1 The target trajectory and observation

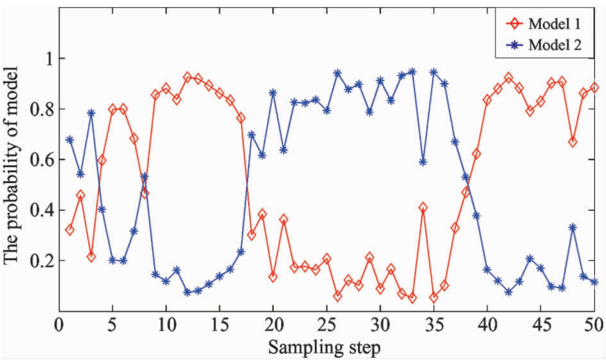

Fig. 2 Model probability in IMM-EKF

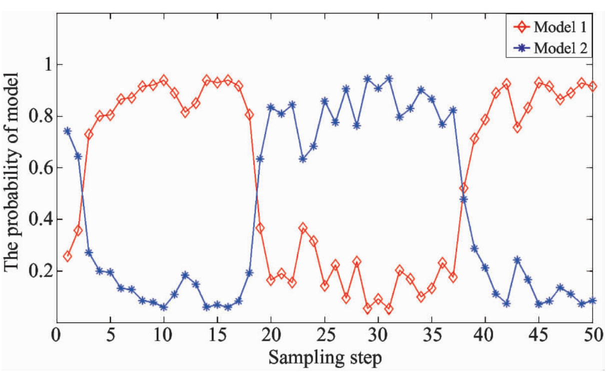

Fig. 3 Model probability in IMM-UKF

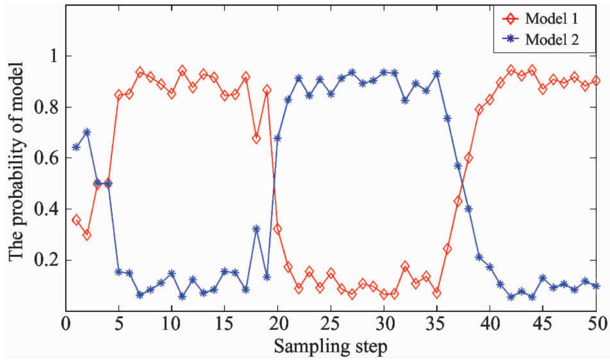

Fig. 4 Model probability in IMM-CKF

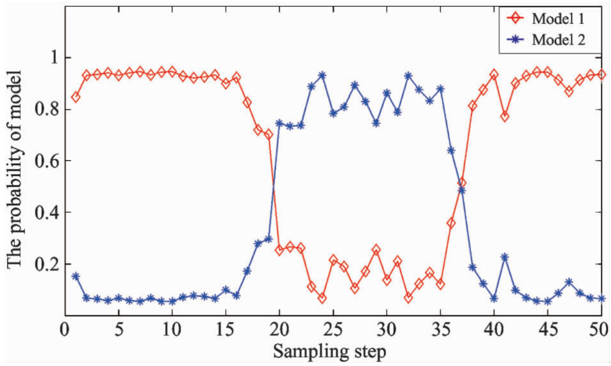

Fig. 5 Model probability in IMM-PF

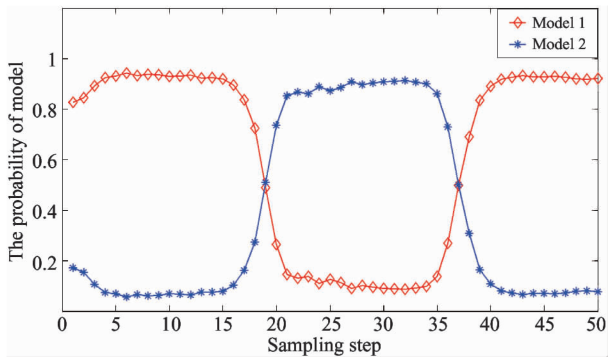

Fig. 6 Model probability in IMM-ICKF

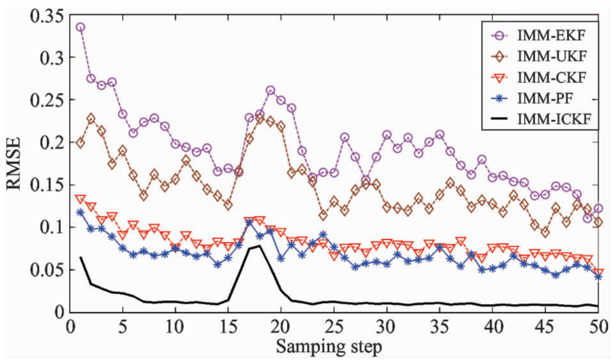

Fig. 7 Horizontal direction

and IMM-ICKF comes to the second but with the highest precision. The above results are conducive to reasonable selection of filters in practical engineering applications.

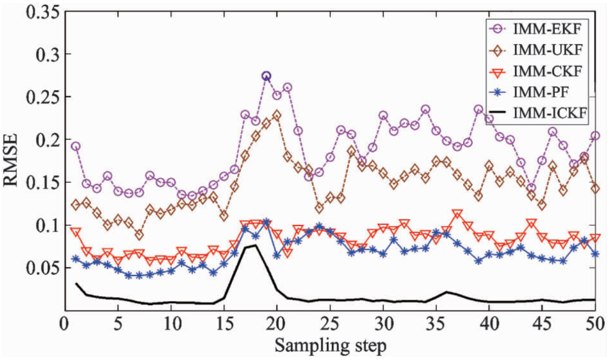

Fig. 8 Vertical direction

Table1 The comparison for the mean of RMSE and the average time over 50 independent runs

Algorithm	Horizontal direction	Vertical direction	Time-consuming
IMM-EKF	0.1925	0.1867	0.0054
IMM-UKF	0.1491	0.1478	0.0145
IMM-CKF	0.0830	0.0825	0.0182
IMM-PF	0.0685	0.0677	1.4480
IMM-ICKF	0.0174	0.0167	0.0276

4 Conclusions

Maneuvering target tracking is always the hot spot and difficulty of researches in target tracking field, this paper gives a maneuvering target tracking algorithm based on CKF with observation iterated update. CKF presented in recent years is an efficient handling method to solve the problem of nonlinear system estimation. In the framework of CKF, the CKF with observation iterated update is proposed by introducing the observation iterated update process. By synthesizing the results of multiple parallel running filters which match the system model, IMM can deal with the problems of uncertainty and variation of system structure and parameters. The novel algorithm realizes the effective identification and estimation of pattern and state by means of dynamically combining ICKF and IMM. Results from practical simulation examples have verified that the proposed algorithm with these effective measures is superior to the existing IMM and its improved algorithms.

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