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Application of interacting multi-model algorithm in gyro signal processing①

Wang Meng (王 萌)[* **], Wang Xiaofeng[*], Zhang He[*], Lu Jianshan②[* ***], Zhang Aijun[* ****]
([*] School of Mechanical Engineering, Nanjing University of Science & Technology, Nanjing 210094, P. R. China)
([**] Shanghai Research Institute of Aerospace Information, Shanghai 201109, P. R. China)
([***] Vehicle Engineering Research Institute, Zhejiang University of Technology, Hangzhou 310014, P. R. China)
([****] Key Laboratory of Intelligent Perception and Systems for High-Dimensional Information, Ministry of Education, Nanjing University of Science & Technology, Nanjing 210094, P. R. China)

Abstract

There is one problem existing in gyroscope signal processing, which is that single models can't adapt to change of carrier maneuvering process. Since it is difficult to identify the angular motion state of gyroscope carriers, interacting multiple model (IMM) is employed here to solve the problem. The Kalman filter-based IMM (IMMKF) algorithm is explained in detail and its application in gyro signal processing is introduced. And with the help of the Singer model, the system model set of gyro outputs is constructed. In order to demonstrate the effectiveness of the proposed approach, static experiment and dynamic experiment are carried out respectively. Simulation analysis results indicate that the IMMKF algorithm is excellent in eliminating gyro drift errors, which could adapt to the change of carrier maneuvering process well.

Key words: gyro, interacting multiple model (IMM), Kalman filter, singer model, signal processing

0 Introduction

The micro electro mechanical system (MEMS) gyroscope is a kind of inertial measurement devices widely used in stabilized platform control system, inertial navigation system, etc. But at present, the precision of MEMS gyroscope is not so good, therfore MEMS gyroscopes are always applied in the middle level systems. In order to extend the application range of MEMS gyroscopes, the performance should be improved with some methods of filtering or compensation, and now this has been the main research content[1].

All MEMS gyro output errors can be classified into two parts, the steady error and the stochastic noise. Steady error can be represented with algebraic equations, and is easy to be compensated because of its regularity[2]. Stochastic noise is the major influencing factor of gyroscope precision, and the main method of improving gyroscope performance is to carry out model identifying and filtering aiming at stochastic noise[1]. Efforts have been made to study eliminating methods for gyro random drift errors[1-7], and two main approaches are obtained. One is to find out the error mathematic characters, and compensate according to the error mathematic models. The other one is to filter for gyro output signal directly, and the elimination of gyro drift errors is achieved with the signal de-noising technology. When using the first method, the AR and ARMA models are always employed to create models for the zero-bias data of gyroscopes. But gyro random drift is weakly nonlinear, non-stationary and slowly time-varying. Hence in order to use AR model or ARMA model, the drift data pre-processing should be finished firstly. It is argued in Ref. [1] that when using Kalman filter with the AR(1) model, the mean value and standard deviation of errors after filtering are much smaller than before in the case of static state or constant angular rate, but when it is in an oscillating state, with the oscillating amplitude increasing, the mean value and standard deviation of errors would be increasing too. And it is pointed out in Ref. [7] that the signal de-noising method of filtering directly for gyro drift is feasible. The approaches of wavelet analy-

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② To whom correspondence should be addressed. E-mail: lujianshan@hotmail.com

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sis and neural network are always employed when filtering directly, but the model orders obtained with these approaches are usually high, hence they are not suitable for real-time estimation. Besides, it is difficult to identify the angular motion state of gyroscope carriers in the actual working progress, and using one single model with fixed parameters to model gyro outputs may cause a great error. Since single model can't adapt to the change of the carrier maneuvering process, the interacting multiple model (IMM) algorithm would be employed to deal with gyro signal processing. IMM algorithm is a software handover algorithm, and has been successfully applied in the fields of maneuvering target tracking and integrated navigation systems[8-11]. IMM algorithm uses two or more models to represent the possible states in the working progress, and the overall estimated state is obtained by weighted mixing all model estimates, hence the IMM algorithm can solve the inaccuracy problem of single model effectively. In the paper, after the gyro output model set of static state and dynamic state is constructed with Singer model, the Kalman Filter-based IMM (IMMKF) algorithm would be used to filter for gyro outputs, and at last some experiments would be carried out to test this newly proposed approach.

1 The Kalman filter-based IMM algorithm

IMM algorithm is a circular recursion algorithm, and in one cycle, different filters based on each single model run in parallel. The overall estimate is obtained by weighted mixing all model estimates. In each cycle, there are four major steps: initial states mixing, KF-based filtering, model probability update, and output estimates mixing[10]. The algorithm with two models is illustrated in Fig. 1.

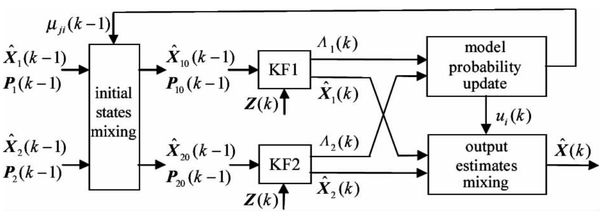

Fig. 1 Frame diagram of IMM algorithm (two models)

(1) Initial states mixing

When there are not restrained conditions, transformations between every two models are carried out based on the Markov chain theory. And the initial state $\hat{\boldsymbol{X}}_{i0}(k-1)$ and covariance matrix $\boldsymbol{P}_{i0}(k-1)$ for filter i at time k are calculated by

$$\hat{\boldsymbol{X}}_{i0}(k-1) = \sum_{j=1}^{N} \hat{\boldsymbol{X}}_j(k-1)\mu_{ji}(k-1) \tag{1}$$

$$\begin{aligned}\boldsymbol{P}_{i0}(k-1) = \sum_{j=1}^{N} \mu_{ji}(k-1)[\boldsymbol{P}_j(k-1) &+ (\hat{\boldsymbol{X}}_j(k-1) \\ &- \hat{\boldsymbol{X}}_{i0}(k-1))(\hat{\boldsymbol{X}}_j(k-1) \\ &- \hat{\boldsymbol{X}}_{i0}(k-1))^{\mathrm{T}}]\end{aligned} \tag{2}$$

where N is model number, $\hat{\boldsymbol{X}}_j$ is state estimate for filter j and $\boldsymbol{P}_j$ is the corresponding covariance matrix. μ_{ji} is model probability and can be represented as

$$\begin{cases}\mu_{ji}(k-1) = \dfrac{1}{\bar{C}_i}\pi_{ji}u_j(k-1) \\ \bar{C}_i = \sum\limits_{j=1}^{N}\pi_{ji}u_j(k-1)\end{cases} \text{ with } \begin{cases}0 < \pi_{ji} < 1 \\ \sum \pi_{ji} = 1\end{cases}.$$

(2) KF-based filtering

For model i, with the mixed initial state $\hat{\boldsymbol{X}}_{i0}(k-1)$ and the corresponding covariance matrix $\boldsymbol{P}_{i0}(k-1)$, Kalman filter is employed to give the state estimate and the updated covariance matrix at time k. Here we give the Kalman filtering equations.

①Initializing state vector and state covariance matrix

$$\hat{\boldsymbol{X}}_i(k \mid k-1) = \boldsymbol{\Phi}_i(k,k-1)\hat{\boldsymbol{X}}_{i0}(k-1) \tag{3}$$

$$\begin{aligned}\boldsymbol{P}_i(k \mid k-1) = \boldsymbol{\Phi}_i(k,k-1)&\boldsymbol{P}_{i0}(k-1) \\ &(\boldsymbol{\Phi}_i(k,k-1))^{\mathrm{T}} + \boldsymbol{\Gamma}_i(k-1) \\ &\boldsymbol{Q}_i(k-1)(\boldsymbol{\Gamma}_i(k-1))^{\mathrm{T}}\end{aligned} \tag{4}$$

② Computing Kalman gain matrix

$$\begin{aligned}\boldsymbol{K}_i(k) = \boldsymbol{P}_i(k \mid k-1)&(\boldsymbol{H}_i(k))^{\mathrm{T}}[\boldsymbol{H}_i(k) \cdot \\ &\boldsymbol{P}_i(k \mid k-1)(\boldsymbol{H}_i(k))^{\mathrm{T}} + \boldsymbol{R}_i(k)]^{-1}\end{aligned} \tag{5}$$

③ Multiplying prediction error vector by Kalman gain matrix to get state correction vector and update state vector

$$\begin{aligned}\hat{\boldsymbol{X}}_i(k) = \hat{\boldsymbol{X}}_i(k \mid k-1) &+ \boldsymbol{K}_i(k)[\boldsymbol{Z}_i(k) \\ &- \boldsymbol{H}_i(k)\hat{\boldsymbol{X}}_i(k \mid k-1)]\end{aligned} \tag{6}$$

④ Updating error covariance

$$\boldsymbol{P}_i(k) = [\boldsymbol{I} - \boldsymbol{K}_i(k)\boldsymbol{H}_i(k)]\boldsymbol{P}_i(k \mid k-1) \tag{7}$$

where $\boldsymbol{\Phi}_i$, $\boldsymbol{\Gamma}_i$ and $\boldsymbol{Q}_i$ are state transition matrix, noise drive matrix, system noise covariance respectively, and $\boldsymbol{H}_i$, $\boldsymbol{R}_i$ are measurement matrix and measurement noise covariance respectively.

(3) Model probability update

When the residual of model i at time k is zero-mean Gaussian white noise, likelihood function $\boldsymbol{\Lambda}_i(k)$ can be represented as

$$\boldsymbol{\Lambda}_i(k) = \sqrt{2\pi \mid \boldsymbol{S}_i(k) \mid}\exp[-\frac{1}{2}(\boldsymbol{\varepsilon}_i(k))^{\mathrm{T}}\boldsymbol{S}_i(k)\boldsymbol{\varepsilon}_i(k)] \tag{8}$$

where $\boldsymbol{\varepsilon}_i$ is the residual and $\boldsymbol{S}_i$ is its covariance matrix.

Then the model probability u_i is updated by

$$u_i(k) = \frac{\Lambda_i(k)\bar{C}_i}{\sum_{i=1}^{N}\Lambda_i(k)\bar{C}_i} \quad (9)$$

(4) Output estimates mixing

Finally the overall state $\hat{\boldsymbol{X}}(k)$ and covariance $\boldsymbol{P}(k)$ are obtained by weighted mixing the estimates of different filters.

$$\hat{\boldsymbol{X}}(k) = \sum_{i=1}^{N}\hat{\boldsymbol{X}}_i(k)u_i(k) \quad (10)$$

$$\boldsymbol{P}(k) = \sum_{i=1}^{N}u_i(k)[\boldsymbol{P}_i(k) + (\hat{\boldsymbol{X}}_i(k) - \hat{\boldsymbol{X}}(k)) \cdot (\hat{\boldsymbol{X}}_i(k) - \hat{\boldsymbol{X}}(k))^{\mathrm{T}}] \quad (11)$$

2 System model set of gyro outputs

The singer model is employed here to construct a system model set of gyro outputs. In the fields of maneuvering target tracking and integrated navigation systems, position, velocity and acceleration are always used as state variables to construct three-dimensional models while position is the observed value. When using the Singer model to construct system models for gyro outputs, the observed value is angular rate, and estimating for angle value obtained from integration of angular rate is meaningless[2]. Hence two-dimensional models which only use angular rate and angular acceleration as state variables will be presented in the paper, and dimension decrease will reduce calculation complexity and calculation time, too. Let T be the sampling period, the discrete Singer model can be represented as follows.

$$\boldsymbol{X}_i(k) = \boldsymbol{\Phi}_i(k,k-1)\boldsymbol{X}_i(k-1) + \boldsymbol{W}_i(k) \quad (12)$$

$$\boldsymbol{Z}_i(k) = \boldsymbol{H}_i(k)\boldsymbol{X}_i(k) + \boldsymbol{V}_i(k) \quad (13)$$

where $\boldsymbol{H}_i = [1 \quad 0]$, $\boldsymbol{\Phi}_i = \begin{bmatrix} 1 & \frac{1}{\alpha_i}(1 - e^{-\alpha_i T}) \\ 0 & e^{-\alpha_i T} \end{bmatrix}$, and α_i is the inverse of maneuvering time constant, namely maneuvering frequency. $\boldsymbol{W}_i$ is the process white noise vector and $\boldsymbol{V}_i$ is the measurement noise vector, $\boldsymbol{Q}_i = E[\boldsymbol{W}_i\boldsymbol{W}_i^{\mathrm{T}}] = 2\alpha_i\sigma_{ai}^2\begin{bmatrix} q_{11} & q_{12} \\ q_{12} & q_{22} \end{bmatrix}$, σ_{ai}^2 is the variance of maneuvering acceleration.

$$q_{11} = \frac{1}{2\alpha_i^3}(4e^{-\alpha_i T} - 3 - e^{-2\alpha_i T} + 2\alpha_i T),$$

$$q_{12} = \frac{1}{2\alpha_i^2}(e^{-2\alpha_i T} + 1 - 2e^{-\alpha_i T}),$$

$$q_{22} = \frac{1}{2\alpha_i}(1 - e^{-2\alpha_i T}).$$

Maneuvering frequency α_1 is a small positive number under the static condition (the first model), and the maximum of absolute value of angular acceleration would not be great, too. So the positive angular acceleration can be limited to $1.5\,\mathrm{deg/s^2}$[2]. When under the maneuvering condition (the second model), maneuvering frequency α_2 will be greater than α_1, and let α_2 be equal to $10\alpha_1$. Besides, thinking of the experimental situation, the maximum of positive angular acceleration would be taken as $300\,\mathrm{deg/s^2}$.

3 Experiment analysis

In order to test the effectiveness of IMMKF algorithm, experiments and analyzation are carried out. Fig. 2 is a gyroscope original signal curve under static condition and its power spectral density (PSD). From PSD curve, it can be seen that, besides the low – frequency signals of high energy density, there still exists high – frequency noises. After gyro outputs are collected by the serial interface of upper computer, MATLAB software is used to do filtering analysis. MATLAB software operates on a Windows system with Intel(R) Core (TM)2 Duo CPU and 2.0GB memory. When filtering with IMMKF algorithm, probabilities of the two models are set as $u_1(0) = u_2(0) = 0.5$ at the initial time. And Fig. 3 is part codes of the IMMKF algorithm in the main loop. In order to make a comparative analysis, Moving filter[12] and Kalman filter are applied to filter drift error, too. When filtering with Kalman filter, AR(2) model is used for random drift error modeling.

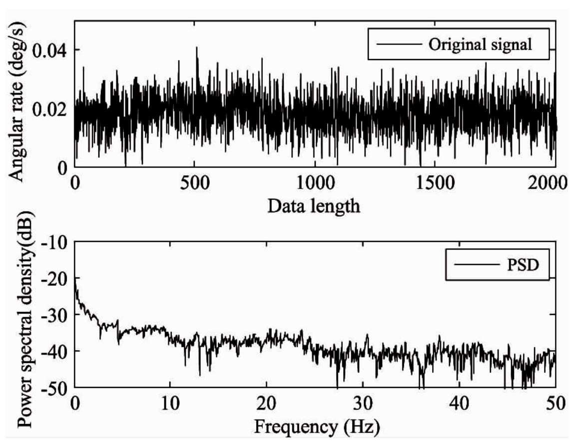

Fig. 2 Curves of gyro outputs and PSD

3.1 Static experiment

The static experiment is carried out firstly. An attitude and heading reference system (AHRS) is fixed on a stabilized platform, and kept motionless. The MEMS gyro outputs are sampled at a frequency of 10Hz after AHRS have been operating for one hour. Take 1000 data points of X-axis gyro to analyze. After finishing the constant drift compensation with mean estimation method, the algorithms mentioned above are employed to filter for gyro outputs. Error curves before and after filtering are shown in Fig. 4, and IMMKF probability changes in the static experiment are given in Fig. 5.

```
for k=1:m
    % Step1 Initial states mixing
    c_1=pai(1,1)*miu_CV+pai(2,1)*miu_CA;    c_2=pai(1,2)*miu_CV+pai(2,2)*miu_CA;
    miu11=pai(1,1)*miu_CV/c_1;    miu12=pai(1,2)*miu_CV/c_2;
    miu21=pai(2,1)*miu_CA/c_1;    miu22=pai(2,2)*miu_CA/c_2;
    X1=X11*miu11+X22*miu21;    P1=(P11+(X11-X1)*(X11-X1)')*miu11+(P22+(X22-X1)*(X22-X1)')*miu21;
    X2=X11*miu12+X22*miu22;    P2=(P11+(X11-X2)*(X11-X2)')*miu12+(P22+(X22-X2)*(X22-X2)')*miu22;
    %----------------------------------------------------------------------
    % Step2 KF-based filtering
    %%%%% Model 1 %%%%%
    ......
    Xe1(:,k)=X1; % Model 1 outputs
    %%%%% Model 2 %%%%%
    ......
    Xe2(:,k)=X2; % Model 2 outputs
    %----------------------------------------------------------------------
    % Step3 Model probability update
    like1=exp(-0.5*zk1'*inv(Pzz1)*zk1)/(sqrt(det(2*pi*Pzz1)));
    like2=exp(-0.5*zk2'*inv(Pzz2)*zk2)/(sqrt(det(2*pi*Pzz2)));

    c_1=pai(1,1)*miu_CV+pai(2,1)*miu_CA;    c_2=pai(1,2)*miu_CV+pai(2,2)*miu_CA;
    c=like1*c_1+like2*c_2;
    miu_CV=like1*c_1/c;    miu_CA=like2*c_2/c;
    %----------------------------------------------------------------------
    % Step4 Output estimates mixing
    X=X1*miu_CV+X2*miu_CA;
    Xe(:,k)=X; % Outputs
    miu1(k)=miu_CV;    miu2(k)=miu_CA;
    X11=X1;    X22=X2;
    P11=P1;    P22=P2;
end
```

Fig. 3 Part codes of IMMKF algorithm

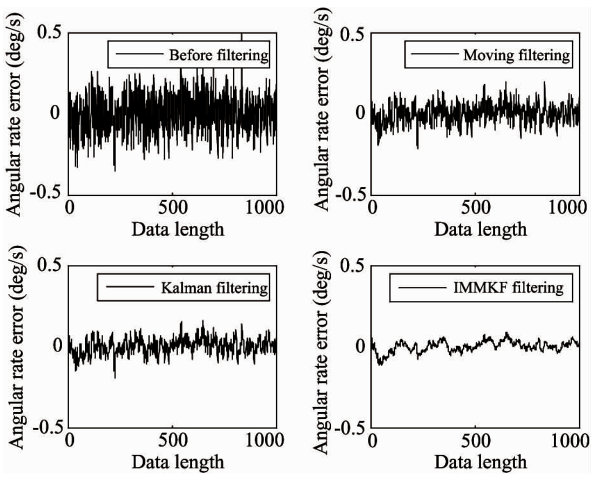

Fig. 4 Errors before and after filtering

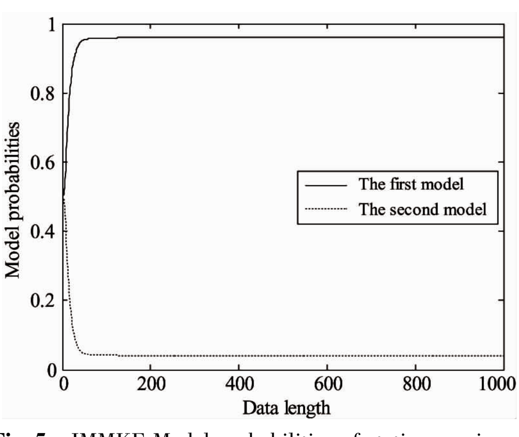

Fig. 5 IMMKF Model probabilities of static experiment

From Fig. 4 it can be found that errors after filtering are significantly decreased in the static experiment, and all the three filtering methods have a good eliminating performance for gyro static drift errors. The performance of Kalman filter is a little better than Moving filter, but IMMKF is the best obviously, and the error curve after IMMKF filtering is smoother than the ones after Moving filtering and Kalman filtering. From the results of error statistical analysis in Table 1 we can see that, maximum error and root mean square (RMS) error before filtering is 0.4860deg/s and 0.1136deg/s, but after IMMKF filtering they are down to 0.1142deg/s and 0.0326deg/s respectively, hence the precision is improved greatly. Fig. 5 illustrates that, when IMMKF filtering is started, probability of the first model (the static model) increases from initial probability of 0.5 to stable probability rapidly which is high to 0.959. And in the IMMKF filtering process, the first model is in the dominating position all the time, so it is concluded that the IMM algorithm can choose the right model and assign a higher model probability according to the system states.

Table 1 Results of error statistical analysis

Methods	Before Filtering	Moving Filter	KF	IMMKF
Maximum Error (deg/s)	0.4860	−0.2157	0.1961	0.1142
RMS Error (deg/s)	0.1136	0.0592	0.0484	0.0326

3.2 Dynamic experiment

In order to test the performance of IMM algorithm under the maneuvering condition, the dynamic experiment is carried out in an open water lane whose size is $110\text{m} \times 2.5\text{m} \times 2.5\text{m}$. The same gyroscope is used to collect information of the man-made wave at a frequency of 50Hz. Take 10000 data points of X-axis gyro to be analyzed. Original signal curve under the maneuvering condition and the signal curves after filtering are shown in Fig. 6. The PSD curves before and after filtering and the IMMKF probability changes in the filtering process are shown in Fig. 7 and Fig. 8 respectively.

By comparing curves in Fig. 6, and with Fig. 7, it can be seen that the performance of Kalman filter is still a little better than Moving filter, but both of their eliminating performance are not good enough. On the other hand, noises at the whole frequency range are all attenuated by IMMKF, and the noise PSD in high frequency band is decreased 10dB approximately after IMMKF filtering. Hence IMMKF still has a good eliminating performance under the maneuvering condition.

From Fig. 6 and the experiment process, one knows that at the beginning the wave is violent, and then turns to a relatively calm state at about the 5450th sampling point. Fig. 8 illustrates that, the probability of the second model is dominating under the seriously maneuvering condition, and its value changes with the wave amplitude oscillating. And then the probability of the first model becomes bigger under the relatively calm condition. Since the water surface is not absolutely quiet, the probability of the first model couldn't be as big as the probability shown in Fig. 5. So from model probabilities shown in Fig. 8, we can conclude that the IMM algorithm can adapt to the maneuvering condition well, and can overcome the shortcoming that single model can't adapt to the chang of carrier maneuvering process. The problem existing in Ref. [1] is solved that performance of Kalman filter with AR(1) model is becoming worse with the oscillating amplitude increasing.

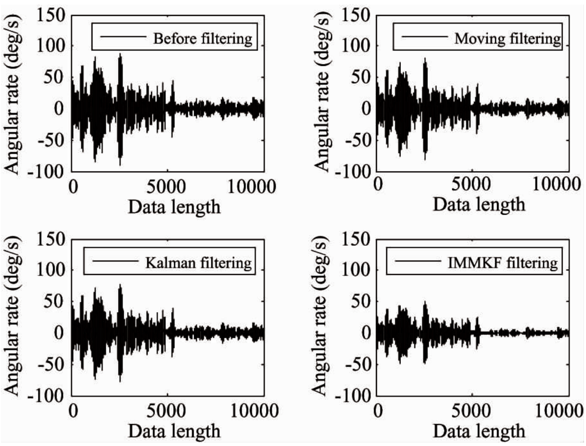

Fig. 6 Gyro outputs before and after filtering

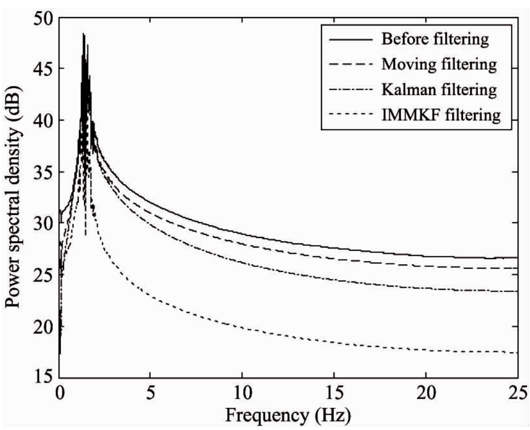

Fig. 7 Curves of power spectral density

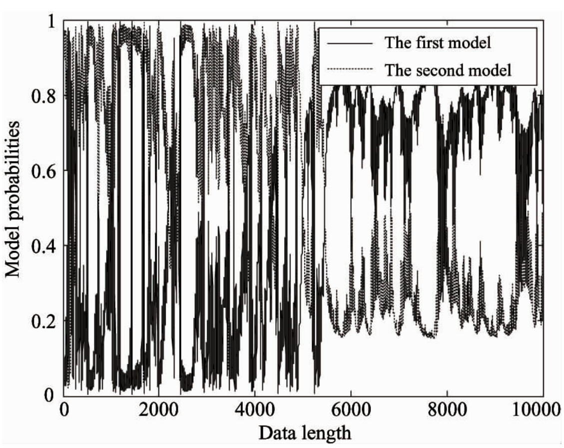

Fig. 8 IMMKF Model probabilities of dynamic experiment

4 Conclusion

The IMM algorithm proposed above widely used in the field of maneuvering target tracking system is introduced to deal with gyro signal processing, and the newly proposed filtering approach which combines IMM algorithm and Kalman filter is explained in detail. With the help of the Singer model, the gyro output models under the static and maneuvering conditions are constructed. In the static experiment, maximum error and RMS error are employed as the criteria, and the IMMKF algorithm is proved to be effective on improving the gyro precision. In the dynamic experiment, the power spectral density method is applied to demonstrate that the IMMKF algorithm has a good filtering performance at the whole frequency range, and the curves of model probabilities has shown that the IMMKF algorithm can adapt to the maneuvering condition well. All these analysis results indicate that the IMMKF filtering approach for gyro outputs is feasible.

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Wang Meng, born in 1981. He received his M. S degree from Nanjing University of Science and Technology (NUST) in 2008. He also received his B. S degree from Nanjing University of Science and Technology in 2003. He is currently working toward his Ph. D degree in NUST. His research interest is dexterity and intelligence of electromechanical system.