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Importance measure based optimal maintenance policy for deteriorating components①

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Abstract

An optimal maintenance policy for deteriorating components based on quasi renew-process model is presented. In this policy, the first $N-1$ failures of a component are maintained by repairs and the N^{th} failure is maintained by replacement. The policy takes replacement actions at component level by considering the fact that more and more components are designed to be field replaceable and maintenance activities are setting free from system halt. Concerning system structure impact, importance measure is employed in the optimization procedure which aims at maximizing the long-run profit per unit time. Two example series parallel systems are taken to illustrate the policy and it is proved to work well. According to importance analysis, components are classified into important ones and unimportant ones based on the system behavior under their failures. Simulation results show that the presented policy makes a clear distinction between them and takes effective maintenance actions to compensate the deteriorating of components.

Key words: deteriorating components, quasi-renew process, maintenance policy, series parallel systems, importance measure

0 Introduction

The steep raising demand of computing capability has fostered an unprecedented requirement for large scale computer systems with high reliability, particularly for the critical application such as air traffic control, telecommunication service and financial business, etc. Maintenance on these large scale computer systems plays a critical role in their efficient usage in terms of cost, reliability, and safety, and will be more important than redundancy, production, and construction in reliability theory[1].

In the past several decades, maintenance strategies have been extensively studied in the literature. Maintenance can be classified into two major categories: corrective maintenance (CM) and preventive maintenance (PM). CM is any maintenance that occurs when the system is failed, and some authors refer corrective maintenance as repair. PM is any maintenance that occurs when the system is operating[2].

Maintenance models of early years usually made an assumption that a system after repair can be restored to "as good as new". However, the perfect assumption does not always accord with field situation. In practice, most repairable systems are deteriorating because of the aging effect, degeneration of repair technology or the accumulative wear[2]. Barlow and Hunter[3] introduce a "minimal repair" model in which the repair activities do not change the failure rate of the system but just makes the system back to work. After minimal repair, the system is restored to the "as bad as old" state. Pham[2] classified maintenance activities into 5 categories according to the degree to which the operating condition of an item is restored by maintenance, and they are: perfect, minimal, imperfect, worse, and worst.

Deteriorating system model and imperfect maintenance are considered to be most practical in later research, and optimal maintenance policies are investigated on a variety of system types under different environments. Maintenance models make themselves different from each other primarily by the effect that maintenance activities are supposed to bring on the system. Nakagawa[4] presents the idea that the component is returned to the "as good as new" state with probability p and it is returned to the "as bad as old" state with probability $q = 1 - p$ after PM. This model is the basic

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(p, q) model. Block[5] extended the above model to $(p(t), q(t))$ model for the age-dependent imperfect repair: An item is repaired at failure. With probability $p(t)$ the repair is a perfect one and with $q(t) = 1 - p(t)$, the repair is a minimal one, where t is the age of the item in use at the failure time. Malik[6] introduces the improvement factor model in which maintenance changes the system time of the failure rate curve to some newer time but not all the way to zero. In addition, there are some other important models such as virtual age model[7], quasi-renew process[8], and some further research based on these models[9-11].

It is reasonable to take the assumption that for a deteriorating repairable system, the successive working time of the system after repair may become shorter and shorter, while the successive repair time of the system may become longer and longer. After Wang had introduced the quasi-renew process to describe such a system, much work was carried out to obtain optimal maintenance strategies[12-14].

Most previous maintenance policies take replacement actions at system level and solve the optimization problem without further investigation on system structure. While in critical applications, replacement of the whole system is not allowed. Many components are designed as field replaceable units (FRU) and are maintained and replaced separately. Thus, maintenance policy needs to be designed at component level concerning system structure impact.

This paper focuses on maintenance policy of deteriorating components with field replaceable design. We employ component importance measure to characterize impact of system structure and a failure number N based optimal maintenance policy is then presented aiming at maximizing the long-run average profit per unit time. The rest of this paper is organized as follows. Section 1 provides a description of the basic models and Section 2 analyzes the optimization problem. Numeral examples are given in Section 3 for illustration, and finally conclusions are drawn in Section 4.

1 General models

1.1 Importance measure

Importance measure (IM) is widely used to characterize the contribution of a component to the system performance such as reliability, availability, risk, etc., and proved to be an effective tool in identifying system weaknesses and prioritizing system improvement activities. The concepts of IM are firstly introduced by Birnhaum[15]. As the most widely used reliability importance indices, Birnbaum reliability importance is defined in Eq. (1):

$$I_i^B(t) = \frac{\partial R_S(t)}{\partial R_i(t)} = \frac{\partial F_S(t)}{\partial F_i(t)} \quad (1)$$

where $I_i^B(t)$ is the importance of component i at time t, $R_s(t)$ and $R_i(t)$ are respectively the reliability of system and reliability of component i at time t. $F_s(t)$ and $F_i(t)$ are the unreliability of the system and component i at time t. $I_i^B(t)$ can also be calculated with

$$I_i^B = R(S \mid r_i = 1) - R(S \mid r_i = 0) \quad (2)$$

For multi-state system with performance requirement d_k, the system reliability is defined as the probability of system performance being equal or greater than d_k. Let x_i be the state variable of component i. $x_i = 1$ means the component is working and $x_i = 0$ indicates its failure. The Birnhaum importance measure of multi-state system with binary components can be obtained with Eq. (3):

$$I_i^{BM} = R(\phi(x) \geqslant d_k \mid x_i = 1) - R(\phi(x) \geqslant d_k \mid x_i = 0) \quad (3)$$

1.2 Quasi-renew process

Let $\{N(t), t > 0\}$ be a counting process and let X_i be the time between the $(i-1)^{\text{th}}$ and the i^{th} event of the process for $i \geqslant 1$. Observe the sequence of non-negative random variables $\{X_1, X_2, X_3 \cdots\}$. The counting process $\{N(t), t > 0\}$ is said to be a quasi-renewal process with parameter a, if $X_1 = W_1$, $X_2 = aW_2$, $X_3 = a^2 W_3$, $\cdots$, where W_i are independently and identically distributed, and $a > 0$ is called the ratio of the process.

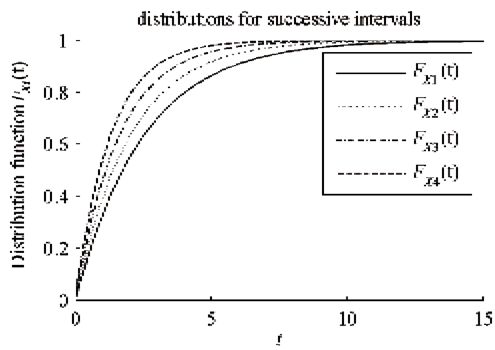

Fig. 1 Graph of $F_{Xi}(t)$ for a quasi renewal process

Quasi-renewal process describes such a scenario that successive intervals of events $\{X_1, X_2, X_3, \cdots\}$ are modeled stochastically as a fraction of the immediately preceding interval. The distribution function of the i^{th} interval is scaled by a factor a^{i-1}, while retaining the same shape. A graphical representation of this effect is shown in Fig. 1 with $a < 1$. $F_{Xi}(t)$ is the distribution of the i^{th} interval.

The cumulative distribution function (CDF) of subsequent intervals in a quasi-renewal process can be expressed as follows[16]:

$$F_{X_i}(t) = F_{X_1}(\frac{1}{a^{i-1}}t) \quad (4)$$

And, probability density function (PDF) can be obtained by changing variables:

$$f_{X_i}(t) = \frac{1}{a^{i-1}} f_{X_1}(\frac{1}{a^{i-1}}t) \quad (5)$$

2 System analyses

In this section, the system model with some assumptions is described, and an importance measure based maintenance policy is introduced for the deteriorating components. Profit evaluation is carried out for the optimization of the policy.

2.1 System assumptions and maintenance policy

Assumption 1: A new component i is installed and begins to operate at time 0, here time is measured by age of component i and the age of the whole system is ignored. When replacement takes place, the component will be replaced by a new and s-identical one.

Assumption 2: The system consists of an arbitrary number of deteriorating components in an arbitrary structure with a structure function Φ. Constant performance demand d_k is taken.

Assumption 3: When a component fails, repair commences as soon as possible. Successive working intervals of component i form a quasi-renew process with ratio $a < 1$. Repair time is also non-negligible and behaves according to another quasi-renew process with ratio $b > 1$. The two processes are independent.

Assumption 4: Components are designed to be field replaceable units which can be maintained and replaced separately. They are also independent.

Assumption 5: Both repairs and replacements take place at component level. Component failure results in a drop on the system performance and thus may lead to system unreliability.

Assumption 6: Policy N is employed. The first $N - 1$ failures of component are repaired, and replacement takes place at the N^{th} failure.

Assumption 7: Only one failure happens at the same time. The assumption means that during the repair or replacement interval of component i, other components don't fail.

Under the quasi-renew model and policy N, the alternative behavior of component working, failure and replacement is shown in Fig. 2. X_i represents the i^{th} working interval of the component, and Y_i is the i^{th} repair interval. The working interval becomes shorter and shorter while the repair interval becomes longer and longer. At the N^{th} failure, the component is replaced by a new one.

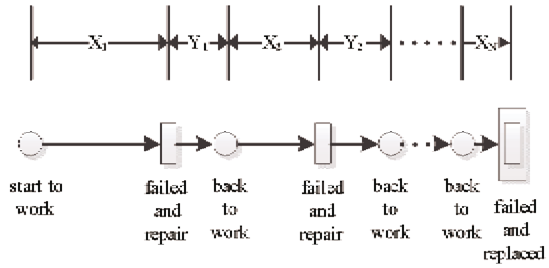

Fig. 2 Component behavior under policy N

2.2 Profit evaluation

$C(N)$ is used to denote the expected average long run profit per unit time of the component under policy N. Thus, according to the renewal reward theorem, the following is got:

$$C(N) = \frac{\text{The expected profit incurred in a renew cycle}}{\text{The expected length of a renew cycle}}$$

There are 4 portions needed to be taken into account in the expected profit.

1) Continuous reward of the whole system when it is functioning.

2) Cost for repair throughout the failure.

3) Possible cost of system unreliability as a result of component failure.

4) Cost of component replacement.

Let c_w, c_{ui}, c_{ri}, c_{ni} denote respectively the total system reward per unit of working time, cost of system unreliability per unit time as a result of component i failure, repair cost of the component per unit of failure time of component i, and cost of replacement of component i. Considering performance requirement d_k, when the system performance drops below d_k, it is taken as failed even it can still offer some services. Then c_{ui} can be evaluated according to c_w and I_i^{BM}. X_{ij} is the working time of component i after the $(j-1)^{th}$ repair, and Y_{ij} is the repair time of component i after the j^{th} failure, T_{ni} is the time of replacement for component i. $C(N)$ can be calculated with

$$C(N) = \frac{c_w E(\sum_{j=1}^{N} X_{ij} + \sum_{j=1}^{N-1} Y_{ij} + T_{ni}) - (c_{ri} + c_{ui}) E(\sum_{j=1}^{N-1} Y_{ij}) - c_{ui} T_{ni} - c_{ni}}{E(\sum_{j=1}^{N} X_{ij}) + E(\sum_{j=1}^{N-1} Y_{ij}) + T_{ni}} \quad (6)$$

With the definition of the quasi-renew process, we have:

$$E(\sum_{j=1}^{N} X_{ij}) = (\frac{1 - a_i^N}{1 - a_i}) E(X_{i1}) \quad (7)$$

$$E(\sum_{j=1}^{N-1} Y_{ij}) = (\frac{1-b_i^{N-1}}{1-b_i})E(Y_{i1}) \quad (8)$$

where a_i, b_i is respectively the ratio of X_{ij}, Y_{ij}. $E(X_{i1})$ is assessed by the *MTTF* (mean time to failure) of X_{i1}, and thus is calculated with

$$E(X_{i1}) = MTTF_{X_{i1}} = \int_0^\infty R_{X_{i1}}(t)\mathrm{d}t \quad (9)$$

$E(Y_{i1})$ is obtained in the same way:

$$E(Y_{i1}) = MTTF_{Y_{i1}} = \int_0^\infty R_{Y_{i1}}(t)\mathrm{d}t \quad (10)$$

The optimal N then can be determined by maximizing $C(N)$.

3 Numerical examples

3.1 Example system A

Consider a series parallel system described in Fig. 3. X_{i1} and Y_{i1} have exponential PDFs, $f(t) = \lambda e^{-\lambda t}$. Parameter λ, $E(X_{i1})$ and $E(Y_{i1})$ calculated with Eq. (9) and Eq. (10) are listed in Table 1. Fig. 4 shows the PDFs in graph.

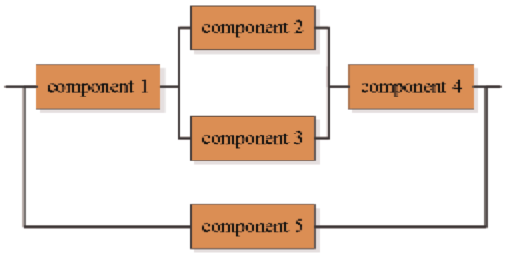

Fig. 3 Structure of system A

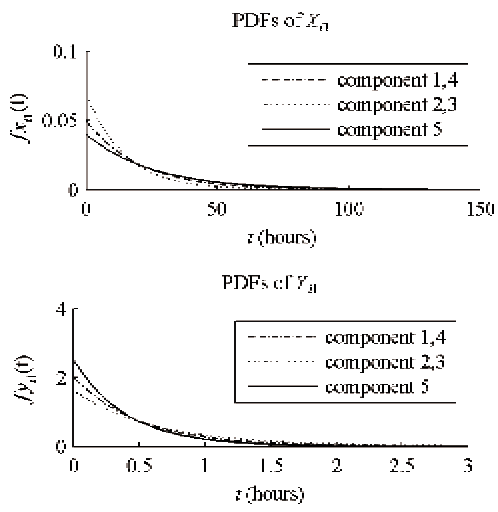

Fig. 4 Graph of PDFs of X_{i1}, Y_{i1}

Cost or profit per unit time can be measured in many ways such as the amount of money or number of transactions processed for OLTP systems. However, when it comes to the computation of $C(N)$, converting all parts into a same measure is necessary. Performances of components in transactions per hour, denoted by z_i, are also listed in Table 1.

Table 1 λ, $E(X_{i1})$, $E(Y_{i1})$ and z_i

Component	1	2	3	4	5
λ of X_{i1}	0.05	0.07	0.07	0.05	0.04
$E(X_{i1})$	20	14.3	14.3	20	25
λ of Y_{i1}	2.0	1.6	1.6	2.0	2.5
$E(Y_{i1})$	0.5	0.625	0.625	0.5	0.4
z_i	3000	1500	1500	3000	3000

From the structure of the system in Fig. 3, we can obtain the structure function of the system:

$$\phi(x) = \min(x_1 \cdot z_1, x_2 \cdot z_2 + x_3 \cdot z_3, x_4 \cdot z_4) + x_5 \cdot z_5 \quad (11)$$

According to Eq. (11), c_w can be obtained with all components working. It results $c_w = 6000$. The cost of system unreliability c_{ui} takes a simple form as Eq. (12). It implies that if the system doesn't meet demand d_k, then the rest performance is ignored. Otherwise, the performance loss is ignored.

$$c_{ui} = I_i^{BM} \cdot c_w \quad (12)$$

With **Assumption 7**, I_i^{BM} is assessed under the condition that no other components will fail. For $d_k = 4200$, which is set to 70% of the system full performance, the importance measures and other parameters are listed in Table 2. The cost and profit are measured by number of transactions, and time unit is hour.

Table 2 Maintenance parameters

Component	1	2	3	4	5
I_i^{BM}	1	0	0	1	1
a_i	0.98	0.97	0.97	0.98	0.96
b_i	1.02	1.01	1.01	1.02	1.03
c_{ri}	80	60	60	80	100
c_{ni}	2000	1800	1800	2000	2500
T_{ni}	0.5	0.3	0.3	0.5	0.6

Evaluation of expected average profit per unit time is shown graphically in Fig. 5. Component 1 and 4 have the same results and so do component 2 and 3.

Under policy N, a component is replaced at the failure marked with 'x', and failures before the replacement are maintained with repair.

The optimization results, maximum $C(N)$ denoted by $C(N)^*$, the corresponding N denoted by N^*, and the expected length of lifetime T^* in hours for each component are given in Table 3.

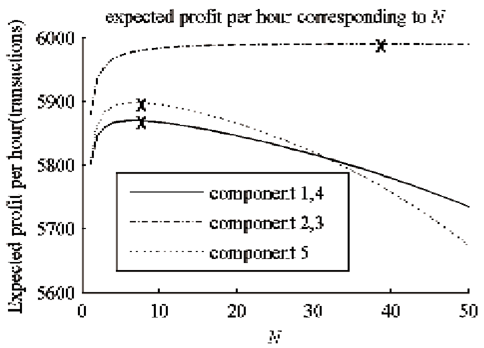

Fig. 5 Expected profit per hour of each component in system A according to N

Table 3 Optimization results of system A

Component	1	2	3	4	5
$C(N)^*$	5822.4	5990.2	5990.2	5822.4	5862.4
N^*	6	38	38	6	6
T^*	117.3	355	355	117.3	138.5

$C(N)$ under different N shows the competing process of decreasing average replacement cost and increasing repair cost. From the results we can see that the expected average long run profit differs greatly from important components to unimportant ones. For the component whose failure causes system performance to drop blow the required level, the increasing repair time as component deteriorating takes much more cost. The fact can be seen clearer when we execute the evaluation for a wide range of N. As shown in Fig. 6, the expected profit of component 1, 4 and 5 drops greatly with growing of N. Early replacement for such a component is necessary to compensate the deteriorating process.

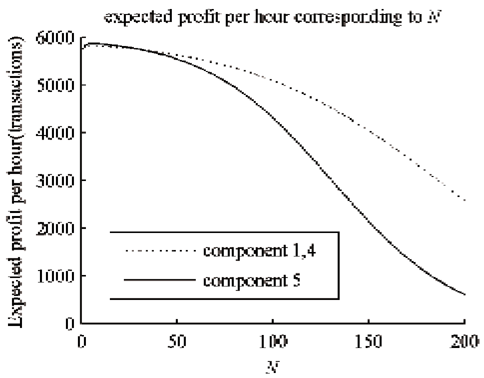

Fig. 6 Expected profit per hour of component 1, 4 and 5 in system A according to a wider range of N

Total lifetime T^* of unimportant component 2 or 3 is 355 hours, which is approaching 15 days with little average cost per hour, about 0.16% of system overall profit, for its failure. While lifetime of important component 1 is less than 6 days with about 2.96% average profit loss for its failure. The cost increases to 18.5 times of component 2 and lifetime reduces to 33.04%. Importance of a component shows great impact on the maintenance activity.

To evaluate the overall system profit under policy N with all components taken into account, the expected average cost per hour c^* for each component with $N = N^*$ can be calculated with $c^* = c_w - C(N)^*$. So the expected optimal overall profit $C(N)_S{}^*$ can be obtained with Eq. (13), where $C(N)_i{}^*$ is the $C(N)^*$ of component i, and c_i^* is the c^* of component i. Thus we have $C(N)_S^* = 5112.2$.

$$C(N)_s^* = c_w - \sum_{i=1}^{5} c_i^* = c_w - \sum_{i=1}^{5} (c_w - C(N)_i^*) \tag{13}$$

3.2 Example system B

Some modifications are made on series parallel system A, and another system is got described in Fig. 7. PDFs of X_{i1}, Y_{i1} follow the same exponential distributions in Table 1. And a_i, b_i, c_{ri}, c_{ni} and T_{ni} also are the same as system A as listed in Table 2. The structure function is obtained in Eq. (14). With adjusted z_i in Table 4, it yields s_w = 8000 transactions per hour. Under d_k = 5600 and **Assumption 7**, the corresponding I_i^{BM} is also listed in Table 4.

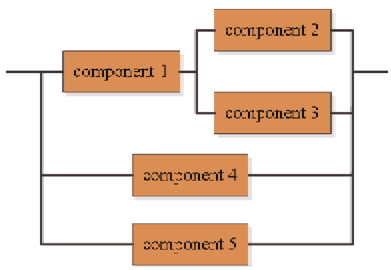

Fig. 7 Structure of system B

$$\phi(x) = \min(x_1 \cdot z_1, x_2 \cdot z_2 + x_3 \cdot z_3) + x_4 \cdot z_4 + x_5 \cdot z_5 \tag{14}$$

Table 4 I_i^{BM} and z_i of system B

Component	1	2	3	4	5
I_i^{BM}	0	0	0	1	1
z_i	3000	1000	1000	3000	3000

Evaluation of expected average profit per unit time is shown graphically in Fig. 8. Component 2 and 3 have the same results. The optimization results are listed in Table 5.

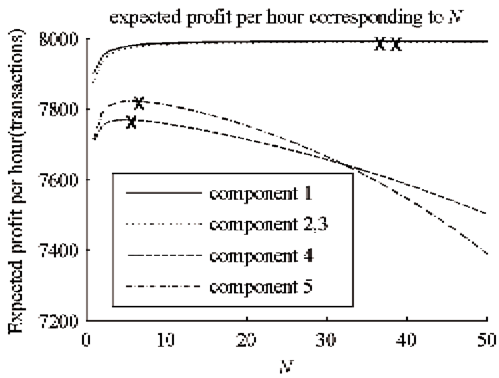

Fig. 8 Expected profit per hour of each component in system B according to N

Table 5 Optimization results of system B

component	1	2	3	4	5
$C(N)^*$	7992.6	7990.2	7990.2	7770.4	7823.1
N^*	36	38	38	5	6
T^*	542.3	354.8	354.8	98.6	138.5

Note that in system B, component 1 and 4 have all the same parameters except the positions in the system structure. For many policies such as periodic group replacement policy with minimal repair, they could be replaced at the same time. Considering the fact that failure of component 1 leads only to degradation on system performance while failure of component 4 leads to system unreliability, our importance measure based policy takes quite different maintenance activities on them. The optimal overall system profit is obtained according to Eq. (13), $C(N)_S^* = 7556.5$.

4 Conclusion

An optimal maintenance policy for deteriorating components modeled by quasi-renew process is presented in this paper. Maintenance activities take into account the system structure impact measured by component importance. Two numeral examples are illustrated to show the optimization procedure. From the results we can see that the expected average long run profit differs greatly from important components to unimportant ones. Maintenance activities performed on a component depend on how its failure will affect the system performance. Even for components with the same parameters, which may be replaced synchronously in another policy, our policy takes discriminative actions to achieve optimal profit considering the different system behaviors under their failure. The policy presented in this paper would be efficient in the case where components are designed to be field replaceable and system level replacement is not allowed to perform. The future work needs to take operation overhead into account and develop maintenance policy for systems with more field constrains.

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