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Two-stage prediction and update particle filtering algorithm based on particle weight optimization in multi-sensor observation①

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Abstract

The reasonable measuring of particle weight and effective sampling of particle state are considered as two important aspects to obtain better estimation precision in particle filter. Aiming at the comprehensive treatment of above problems, a novel two-stage prediction and update particle filtering algorithm based on particle weight optimization in multi-sensor observation is proposed. Firstly, combined with the construction of multi-senor observation likelihood function and the weight fusion principle, a new particle weight optimization strategy in multi-sensor observation is presented, and the reliability and stability of particle weight are improved by decreasing weight variance. In addition, according to the prediction and update mechanism of particle filter and unscented Kalman filter, a new realization of particle filter with two-stage prediction and update is given. The filter gain containing the latest observation information is used to directly optimize state estimation in the framework, which avoids a large calculation amount and the lack of universality in proposal distribution optimization way. The theoretical analysis and experimental results show the feasibility and efficiency of the proposed algorithm.

Key words: multi-sensor information fusion, particle filter, weight optimization, prediction and update

0 Introduction

In many scientific and practical problems, an estimation of the time-varying system state using a sequence of noisy measurements is required. The dynamic state space modeling approach is widely used in many applications, such as control, astronomy, economic data analysis, communication and radar surveillance. For the linear discrete system, several filtering methods have been reported, for example, Kalman filter(KF) and grid based filter[1]. In these filters, the posterior density probability was assumed to be Gaussian. However, in many real problems the posterior density is nonlinear and its performance is not as good as expected. To overcome this problem, many researches have been reported on the nonlinear filtering methods such as extended Kalman filter(EKF) and some sampling nonlinear filters[2,3]. One of the famous methods is the particle filter(PF)[4,5]. In the particle filter, any assumption on the functional form of the posterior is not made. Instead, the posterior probability density is approximated as a set of particles with associated weights. When these particles are properly placed, weighted and propagated, posteriors can be estimated sequentially over time. The density of particles represents the probability of posterior function. By using a finite number of particles, we can estimate almost any kind of system dynamics, even nonlinear system with non-Gaussian, or multimodal distributions.

As we all know, the effective sampling of particle state and reasonable measuring of particle weight are considered as two important aspects to obtain better estimation precision in realization of PF. The first is to optimize sampling particle by the introduction of current observation, and some existing solutions include the proposal distribution optimization, Markov Chain Monte Carlo[6], the construction of kernel function[7] and intelligent optimization methods[8], et al. The second is as far as possibly to reduce the adverse influence from random observation noise in the measuring process

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of particle weight, such as cost reference strategy[9]. Nevertheless, the above achievements mostly focus on the single sensor observation system, are mainly to resolve the own disadvantage in the particle filter theory. Based on the characteristics of multi-sensor fusion system, the studies of design and application for PF are relatively few. Xiong and others gave a new multi-sensor sequential particle filtering method by the dynamic combination of multi-sensor sequential fusion way and particle filter, and obtained better filtering precision[10]. But its calculation amount expands rapidly with the increase of sensor number in the estimation system. Under the wireless sensor network environment, Gu and others designed a kind of distributed improved particle filter for the target tracking problem[11], its main idea is to adaptively allot with the number of particles in the local node. But the cost of filtering precision improved is the deterioration of real-time. Achutegui and others proposed a distributed sampling strategy for independent node based on re-sampling technology with non-proportional allocation[12], and it utilized the existing observations to approximate the missing observation. This method has some reference values to improve the communication loads of node data and the filtering precision in wireless sensor networks. Nevertheless, the estimation accuracy of missing observations depends strongly on the sensor number of a measuring system, so more sensors are needed in the measured system. Armesto and others proposed the interpolation particle filter for the mobile robot localization and map-building system based on interpolation technique[13]. Because the observation relevant problems are effectively solved, its application areas are limited to some extent. Considering the comprehensive treatment of observation uncertainty and multiple sampling rates, Francois and others proposed the multi-sensor fusion particle filter based on observations Markov switching model[14]. However, the effectiveness depends on the accurate preset of observation model prior probability and state transition probability, meanwhile the state transition model is required to match sensors with different observation rate, these assumptions are difficult to achieve in the actual application.

According to the analysis above, a novel two-stage prediction and update particle filtering algorithm based on particle weight optimization in multi-sensor observation (TPF-PWO) is proposed in this paper. The remaining of the paper is organized as follows. The first section briefly introduces the basic features of particle filter. The second section provides the theoretical derivation on the particle weight optimization strategy and two stage-prediction and update method. In addition, concrete realization of TPF-PWO is given. The third section presents the experimental scene and simulations analysis. The final section lists the conclusions and recommendations.

1 Particle filter based on single sensor observation

Consider the following nonlinear state space model with the characteristic of multi-sensor observation.

$$\boldsymbol{x}_k = f(\boldsymbol{x}_{k-1}) + \boldsymbol{u}_{k-1} \tag{1}$$

$$z_{k,m} = h(\boldsymbol{x}_k) + \boldsymbol{v}_{k,m} \quad m = 1,2,\cdots,M \tag{2}$$

where $\boldsymbol{x}_k$ and $z_{k,m}$ denote the state variable and observation of sensor m at time k, respectively. f and h are known evolution function of state and observation function. $\boldsymbol{u}_k \sim N(0,\boldsymbol{Q}_k)$ and $\boldsymbol{v}_{k,m} \sim N(0,\sigma^2_{v,m})$ denote independently the system noise and the observation noise of sensor m, meanwhile, they meet identically distribution (i. i. d). Because the complete information of sequential estimation is in $p(\boldsymbol{x}_k \mid z_{1:k,m})$, and the problem of state estimation based on the observation sequences of sensor m, can be solved by calculating the posterior probability density function $p(\boldsymbol{x}_k \mid z_{1:k,m})$ of $\boldsymbol{x}_k$ based on all the available data of observation sequence. The main idea is to approximate $q(\boldsymbol{x}_k \mid z_{1:k,m})$ sampling particle $\boldsymbol{x}^i_k$ with associated importance weights $\omega^i_{k,m}$ from a known and easy-to-sample $q(\boldsymbol{x}_k \mid z_{1:k,m})$ in PF, where $q(\boldsymbol{x}_k \mid z_{1:k,m})$ is usually named as proposal distribution, which should be approximate $p(\boldsymbol{x}_k \mid z_{1:k,m})$ as much as possible, and the associated importance weight of particle is defined as

$$\omega^i_{k,m} \propto p(\boldsymbol{x}^i_k \mid z_{1:k,m})/q(\boldsymbol{x}^i_k \mid z_{1:k,m}) \tag{3}$$

In the practical application, the proposal distribution is commonly selected as prior distribution $p(\boldsymbol{x}^i_k \mid \boldsymbol{x}^i_{k-1})$. And particle weight $\omega^i_{k,m}$ can be obtained by solving the observation likelihood degree of every particle for the observation of sensor m.

$$\omega^i_{k,m} = \omega^i_{k-1,m} p(z_{k,m} \mid \boldsymbol{x}^i_k) \tag{4}$$

where $\omega^i_{k,m}$ is normalized and $\varpi^i_{k,m}$ denotes the normalized weights, and then the re-sampling step is introduced. In the re-sampling step, the particles with different weights are sampled again with replacement according to their weights, and the particles with larger weights are more likely to be selected than the particles with smaller weights.

2 Two stage-prediction and update particle filtering algorithm based on particle weight optimization in multi-sensor observation

In this section, firstly, we give the principle and process of the particle weight optimization strategy and the two stage-prediction and update method in detail. Next, the concrete realization of TPF-PWO is constructed in the framework of PF.

2.1 The particle weight optimization strategy in multi-sensor observation

In view of the characteristic of multi-sensor observation system, objectively the necessary condition is provided to improve the influence of random observations noise by the utilization of multi-sensor observations. According to the realization principle of particle filter and the characteristic of sensor accuracy, meanwhile, according to the construction of multi-senor likelihood function and the weight fusion ideology, the particle weight optimization strategy is given to improve the variance of particle weight. And then we give the principle and process of particle weight optimization strategy in detail. It is known that weight $\omega_{k,l}^{i}$ of particle i can be calculated by

$$\begin{aligned}\omega_{k,m}^{i} &= \omega_{k-1,m}^{i}\exp(-(z_{k,m}-h(\boldsymbol{x}_k^i))^2/2\boldsymbol{\sigma}_{v_{k,m}}^2)/\sqrt{2\pi}\boldsymbol{\sigma}_{v_{k,m}} \\ &= \omega_{k-1,m}^{i}\exp(-(\boldsymbol{v}_{k,m}-(h(\boldsymbol{x}_k^i)-h(\boldsymbol{x}_k)))^2/2\boldsymbol{\sigma}_{v_{k,m}}^2) \\ &\quad /\sqrt{2\pi}\boldsymbol{\sigma}_{v_{k,m}} \end{aligned} \tag{5}$$

According to Eq. (5), $\omega_{k,m}^{i}$ is subject to the Gaussian distribution with mean $h(\boldsymbol{x}_k^i) - h(\boldsymbol{x}_k)$ and variance $\boldsymbol{\sigma}_{v_{k,m}}^2$. Secondly, weight $\hat{\omega}_k^i$ of particle i after fusion is calculated at time k, and $\lambda_{k,m}$ is used as the weight coefficient.

$$\begin{aligned}\hat{\omega}_k^i &= \hat{\omega}_{k-1,m}^{i}\sum_{m=1}^{M}\lambda_{k,m}(\exp(-(\boldsymbol{v}_{k,m}-(h(\boldsymbol{x}_k^i) \\ &\quad -h(\boldsymbol{x}_k)))^2/2\boldsymbol{\sigma}_{v_{k,m}}^2)/\sqrt{2\pi}\boldsymbol{\sigma}_{v_{k,m}})\end{aligned} \tag{6}$$

In accordance with its characteristics of Gaussian distribution, we obtain

$$\hat{\omega}_k^i \sim N\left(\sum_{m=1}^{M}\lambda_{k,m}(h(\boldsymbol{x}_k^i)-h(\boldsymbol{x}_k)),\ \sum_{m=1}^{M}\lambda_{k,m}^2\boldsymbol{\sigma}_{v_{k,m}}^2\right) \tag{7}$$

Eq. (7) shows that the standard deviation of $\hat{\omega}_k^i$ can be written as

$$\boldsymbol{\sigma}_{\hat{\omega}_k^i} = \sqrt{\sum_{m=1}^{M}\lambda_{k,m}^2\boldsymbol{\sigma}_{v_{k,m}}^2} \tag{8}$$

where $\boldsymbol{\sigma}_{\hat{\omega}_k^i}$ is smaller, which indicates that the higher the accuracy of fusion output is. Obviously, when $\boldsymbol{\sigma}_{v_{k,m}}$ is set, $\boldsymbol{\sigma}_{\hat{\omega}_k^i}$ is closely related to the distribution of $\lambda_{k,m}$. In order to obtain the highest fusion accuracy, $\boldsymbol{\sigma}_{\hat{\omega}_k^i}$ should be minimized. Combined with the information conservation principle, the calculation of $\boldsymbol{\sigma}_{\hat{\omega}_k^i}$ can be further attributed to the solving problem of conditional extreme value. That is when $\boldsymbol{\sigma}_{k,j}$ and $\sum_{m=1}^{M}\lambda_{k,m} = 1\ (\lambda_{k,m} \geqslant 0)$ are known, and to find the conditions that the value of $\boldsymbol{A}(\lambda_{k,1},\lambda_{k,2},\cdots,\lambda_{k,M}) = \sum_{m=1}^{M}\lambda_{k,m}^2\boldsymbol{\sigma}_{v_{k,m}}^2$ is the minimum. Considering that the above is a constraint condition equation of multivariable conditions extremum problems, the solution can be calculated by the Lagrange multiplier method. After the modified functions $\vartheta(\sum_{m=1}^{M}\lambda_{k,m} - 1)$ is introduced, and the function expression of $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = \sum_{m=1}^{M}\lambda_{k,m}^2\boldsymbol{\sigma}_{v_{k,m}}^2 + \vartheta\left(\sum_{m=1}^{M}\lambda_{k,m} - 1\right) \tag{9}$$

The partial derivative of $\lambda_{k,m}$ is calculated on both sides of this function respectively. If and only if $\partial\boldsymbol{A}/\partial\lambda_{k,m}$ is equal to zero, and $\boldsymbol{A}$ can be taken as the minimum. The expression of $\lambda_{k,m}$ is written as

$$\lambda_{k,m} = -\vartheta/(2\boldsymbol{\sigma}_{v_{k,m}}^2) \tag{10}$$

In view of $\sum_{m=1}^{M}\lambda_{k,m} = 1$, and

$$\vartheta = -2/\left(\sum_{m=1}^{M}1/\boldsymbol{\sigma}_{v_{k,m}}^2\right) \tag{11}$$

Then Eq. (11) is substituted into Eq. (10), and

$$\lambda_{k,m} = 1/\left(\boldsymbol{\sigma}_{v_{k,m}}^2\sum_{m=1}^{M}1/\boldsymbol{\sigma}_{v_{k,m}}^2\right) \tag{12}$$

After $\lambda_{k,m}$ is solved, the fusion precision $\boldsymbol{\sigma}_{\hat{\omega}_k^i}$ can be calculated by Eq. (8).

$$\boldsymbol{\sigma}_{\hat{\omega}_k^i} = 1/\sqrt{\sum_{m=1}^{M}1/\boldsymbol{\sigma}_{v_{k,m}}^2} \tag{13}$$

According to Eq. (13), when the observation accuracies of sensors are the same and their values are all $\boldsymbol{\sigma}_{v_k}$, we obtain

$$\boldsymbol{\sigma}_{\hat{\omega}_k^i} = \boldsymbol{\sigma}_{v_k}^2/\sqrt{M} \tag{14}$$

The above equation also shows that the precision of particles weight can be improved $\sqrt{M}$ times than the single sensor, while M sensors with the same observation accuracy are used. When the observation accuracy of each sensor is different, and the highest accuracy and the worst accuracy is $\boldsymbol{\sigma}_{k,\max}^2$ and $\boldsymbol{\sigma}_{k,\min}^2$, respectively, and then

$$\boldsymbol{\sigma}_{\hat{\omega}_k^i} \leqslant 1/\left(\boldsymbol{\sigma}_{v_{k,\min}}^2 + \sqrt{\sum_{m=1}^{M-2}1/\boldsymbol{\sigma}_{v_{k,m}}^2}\right) \tag{15}$$

Based on Eq. (15), the observation accuracy of sensor will also be help to improve the variance of particle weight no matter how bad it is, when the observation likelihood degrees are weighted by means of the above way in the multi-sensor fusion structure. And so it provides the important theoretic basis that multi-sensor observations can be used to promote the precision of particles weight.

2.2 Two-stage prediction and update method

The optimization sampling particle is one of the important ways to improve filtering precision, and the proposal distribution optimization and MCMC are two classical methods. The objective of MCMC is to make particles tend to the stationary distribution so as to weaken the correlation among particles and expand particles diversity, and it is usually applied after re-sampling steps[6]. Nevertheless, it needs to accomplish the process of particle sampling and weight measuring three times, and inevitably lead to the dramatic increase of computational complexity. The objective of proposal distribution optimization is to improve sampling particles by the reasonable utilization of latest observation information. Some existing algorithms, such as EPF, IEPF, UPF and GHPF, CDPF, etc[4,15-17], are its concrete realization. The common features need run independently a suboptimal filter for every particle, especially when the dimension of estimated state is high, which aggravates computational burden undoubtedly. In addition, some suboptimal filters are often limited by system nonlinear intensity and Gaussian noise hypothesis, so their filtering precisions are closely related to application object, which undoubtedly results in the lack of universality.

UKF and PF adopt all one-step prediction and observation update mechanism, and their differences are only in the realization steps. Moreover, the filter gain has the property which can preferably describe the utilization degree of the latest observation in UKF. By the dynamic combination of UKF and PF, we put forward a kind of two-stage prediction framework of particle filter. The objective is to optimize directly state estimation, not to optimize single sampling particle. Compared with some algorithms adopted on the proposal distribution optimization technology, the differences between proposal distribution optimization and two-stage prediction and update in the structure are given in Fig. 1 and Fig. 2.

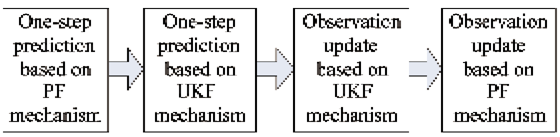

Fig. 1 The flow of proposal distribution optimization

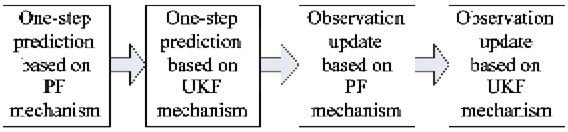

Fig. 2 The flow of two stages prediction and update

According to the above figures, it is known that two-stage prediction and update way directly updates the state estimation not to optimize the single particle by suboptimal filter, so the increase of calculated amount is less relative to general PF. In addition, the situation of update process designed is more back-end of filter process relative to the proposal distribution optimization, so that it can avoid the loss and pollution of the latest observation and original particle information.

2.3 The algorithm realization of TPF-PWO

In the two-stage prediction and update framework of particle filter, and combined with particle weight optimization strategy in multi-sensor observations, the concrete construction of TPF-PWO is as follows. Firstly, particles are sampled from proposal distribution by the prior modeling information, and then particle weight is calculated by Eq. (10). Hence, one-step prediction of state and observation can be solved by x_{k-1}^i and $\hat{\omega}_k^i$. The process can be considered as one-step prediction of system state estimation in PF framework.

$$\boldsymbol{x}_k^i = f(\boldsymbol{x}_{k-1}^i) + \boldsymbol{u}_{k-1}^i \tag{16}$$

$$\hat{\boldsymbol{x}}_{k/k-1} = \sum\nolimits_{i=1}^{N} \hat{\omega}_k^i \boldsymbol{x}_k^i \tag{17}$$

$$z_{k/k-1}^i = h(\boldsymbol{x}_k^i) \tag{18}$$

$$\hat{z}_{k/k-1} = \sum\nolimits_{i=1}^{N} \hat{\omega}_k^i z_{k/k-1}^i \tag{19}$$

where $z_{k/k-1}^i$ denotes one-step prediction of particle observation. $\hat{\boldsymbol{x}}_{k/k-1}$ and $\hat{z}_{k/k-1}$ denote the one-step prediction of system state and system observation, respectively. In order to construct the filter gain matrix $\boldsymbol{\Theta}_k$, it needs furthermore to calculate state and observation prediction covariance $\boldsymbol{P}_{xz}$ and observation prediction covariance $\boldsymbol{P}_{zz}$.

$$\boldsymbol{P}_{xz} = \sum\nolimits_{i=1}^{N} \hat{\omega}_k^i [\boldsymbol{x}_k^i - \hat{\boldsymbol{x}}_{k/k-1}][z_{k/k-1}^i - \hat{z}_{k/k-1}]^{\mathrm{T}} \tag{20}$$

$$\boldsymbol{P}_{zz} = \sum\nolimits_{i=1}^{N} \hat{\omega}_k^i [z_{k/k-1}^i - \hat{z}_{k/k-1}][z_{k/k-1}^i - \hat{z}_{k/k-1}]^{\mathrm{T}} + \boldsymbol{\sigma}_v^2 \tag{21}$$

$$\boldsymbol{\Theta}_k = \boldsymbol{P}_{xz}(\boldsymbol{P}_{zz})^{-1} \tag{22}$$

The construction process of variables including $\hat{\boldsymbol{x}}_{k/k-1}$, $\hat{z}_{k/k-1}$, $\boldsymbol{P}_{xz}$ and $\boldsymbol{P}_{zz}$, can be considered as the one-step prediction of system state estimation in UKF framework. $\boldsymbol{\Theta}_k$ can be calculated by Eq. (22). In addition, considering the improvement of particles degeneracy and the promotion of particles utilization efficiency, re-sampling step is used to realize the observation update of system state estimation in PF framework and particles set $\{\boldsymbol{x}_k^j\}_{j=1}^N$ after the re-sampling stage are sampled. According to the Monte Carlo simulation technology, the state estimation can be ultimately obtained by the calculation of arithmetic mean of $\{\boldsymbol{x}_k^j\}_{j=1}^N$.

$$\bar{\boldsymbol{x}}_{k/k} = \sum\nolimits_{j=1}^{N} \boldsymbol{x}_k^j / N \tag{23}$$

$$\hat{\boldsymbol{x}}_{k/k} = \bar{\boldsymbol{x}}_{k/k} + \boldsymbol{\Theta}_k(z_k - \hat{z}_{k/k-1}) \tag{24}$$

$$z_k = \sum\nolimits_{m=1}^{M} z_{k,m} / M \tag{25}$$

where $\bar{\boldsymbol{x}}_{k/k}$ denotes the state estimation in PF and the updated state estimation respectively. On the basis, $\boldsymbol{\Theta}_k$ is used to modify $\bar{\boldsymbol{x}}_{k/k}$ by means of the reasonable utilization of the latest observation, so the updated state estimation is obtained. The modified method based on Eq. (25) is considered as the observation update of system state estimation in UKF framework. The algorithm flow of TPF-PWO is expressed by the pseudo-code, and the concrete algorithm description is as follows.

- At time step $k-1$

Suppose we have $\{x_{k-1}^i, \hat{\omega}_{k-1}^i\}_{i=1}^N$ and $\hat{x}_{k-1/k-1}$

- At time step k

With new observations, $z_{k,m}$, $m = 1,2,\cdots,M$

Generate particles x_k^i from the proposal according to Eq. (16)

Calculate the weight $\hat{\omega}_k^i$ of particle i using Eq. (6)

Then the one-step prediction $\hat{x}_{k/k-1}$ of system state, the one-step prediction of system observation $\hat{z}_{k/k-1}$ and the filter gain matrix $\boldsymbol{\Theta}_k$ are solved by Eq. (17) to Eq. (22).

The re-sampling step is used to realize the observation update of system state estimation in PF framework and particles set $\{x_k^j\}_{j=1}^N$ after the re-sampling stage are sampled.

Finally, the state estimation $\hat{x}_{k/k}$ is calculated by Eq. (23) to Eq. (24).

3 Simulation results and analysis

To illustrate the performance of TPF-PWO, two examples are given in a simulation experiment. The first example includes two typical one-dimensional nonlinear models taken from Refs [15] and [4], respectively. The second is a practical moving target tracking by the observation of two-coordinate radar, which is a typical high-dimensional nonlinear estimation problem. Personal computer is used as the experiment platform. CPU (Pentium4) is with a fast dual-core processor, and the basic frequency and memory are 3.06GHZ and 2GB, respectively. Operating System is Windows XP and Matlab6.5 is used as programming software.

3.1 One-dimensional nonlinear examples

Model 1

$$x_{k+1} = 0.5x_k + \sin(0.04\pi k) + 1 + u_k$$

$$z_{k,m} = \begin{cases} x_k^2/5 + v_{k,m} & 1 \leq k \leq 15 \\ x_k/2 - 2 + v_{k,m} & 15 < k \leq 30 \end{cases}$$

Model 2

$$x_{k-1} = 0.5x_k + 25[x_k/(1+x_k^2)] + 8\cos(1.2k) + u_k$$

$$z_{k,m} = x_k^2/20 + v_{k,m} \quad m = 1,2,3$$

The evolution parameter of the system state and the distribution of noise are set in accordance with the references, and three sensors are used. The first model is of segmentation nonlinear characteristics. System noise u_k is drawn from Gamma distribution $\mathbf{Ga}(3,2)$. $v_{k,m}$ is subject to Gaussian distribution, and its statistical characteristics are with $N(0,0.005)$, $N(0,0.002)$ and $N(0,0.01)$, respectively. The second is also named as the single variable non-stationary growth model and is strongly nonlinear. System noise u_k is drawn from Gaussian distribution $N(0,10)$. $v_{k,m}$ is subject to Gaussian distribution, and its statistical characteristics are with $N(0,1)$, $N(0,2)$ and $N(0,1.5)$, respectively. The number of Monte Carlo simulation is 30 and the number of particles is 1000. And the total simulation steps are 25, respectively. The six algorithms are compared in simulation including EKF, PF, PFMC, EPF, PF-PWO and TPF-PWO.

Fig. 3 and Fig. 4 show the comparison of RMSE on the state estimation of six algorithms in Monte Carlo simulation for two models. The data from Table 1 quantitatively show the mean and the variance of RMSE and the average time. According to Fig. 3 and Table 1, it is shown that the filter precision of PF and its improved algorithms are superior to EKF in the weak nonlinear and non-Gaussian noise. Due to the reliability improvement of particle weight and the utilization of observation in particle sampling process, and TPF-PWO can obtain better filter precision relative to PF,

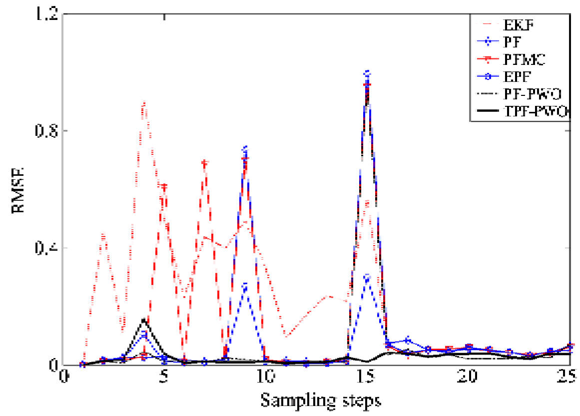

Fig. 3 The first model

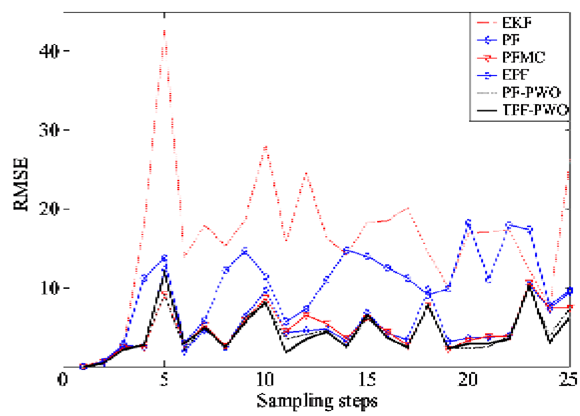

Fig. 4 The second model

PFMC, EPF and PF-PWO. According to Fig. 4 and Table 1, the performance of TPF-PWO is also superior to other five algorithms in strong nonlinearity and Gaussian noise. In addition, we find that the filtering precision and the real-time of EPF are worse than general PF. The main reason is that suboptimal filter is lack of the processing capability for strong nonlinearity. Namely, under the condition of strong nonlinearity, the adoption of EKF further aggravates the deviation between the modified proposal distribution and the true posterior distribution, and weakens the approximation efficiency of particles relative to true state. Moreover, compared with PFMC and EPF in real-time, the augment of computational complexity in TPF-PWO is less than general PF.

Table 1 The mean and variance of RMSE and the average time based on model 1 and model 2

Algorithm	RMSE(Mean)	RMSE(Var)	Time(s)	RMSE(Mean)	RMSE(Var)	Time(s)
EKF	0.29083	0.09543	----	16.343	78.396	----
PF	0.09976	0.05496	0.5205	5.0448	10.161	0.3292
PFMC	0.14386	0.07271	0.9736	4.8019	7.3386	0.7251
EPF	0.08102	0.04883	1.0404	10.146	25.631	1.1828
PF-PWO	0.05946	0.03410	0.5281	4.2363	7.1767	0.4343
TPF-PWO	0.02745	0.00091	0.6219	4.2214	6.3391	0.5072

3.2 High-dimensional nonlinear example

The simulation scenario is set to realize the target tracking in X-Y plane based on the three two-coordinate radars with different observation accuracy. The target motion equation and the observations equation are as follows.

$$\boldsymbol{x}_k = \boldsymbol{F}\boldsymbol{x}_{k-1} + \boldsymbol{\Gamma}\boldsymbol{u}_{k-1}$$

$$z_{k,m} = [\gamma_k \quad \theta_k]^{\mathrm{T}} + \boldsymbol{v}_{k,m} \qquad m = 1,2,3$$

where $\boldsymbol{x}_k = [x_k, \dot{x}_k, y_k, \dot{y}_k]^{\mathrm{T}}$ denotes system state vector. x_k, $\dot{x}_k$, y_k and $\dot{y}_k$ denote the position component and velocity component of target state based on the X-axis direction and the Y-axis direction. $\boldsymbol{F} = \begin{bmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{B} & \boldsymbol{A} \end{bmatrix}$ denotes the system state transition matrix, and $\boldsymbol{A} = \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix}$, $\boldsymbol{B} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$. The sampling period τ is 1. $\boldsymbol{\Gamma} = \begin{bmatrix} 0 & 0 & \tau/2 & \tau \\ \tau/2 & \tau & 0 & 0 \end{bmatrix}^{\mathrm{T}}$ denotes system noise matrix. $\boldsymbol{u}_k$ denotes the system noise vector which meets zero-mean Gaussian white noise with the standard deviation $0.15\mathbf{I}$, and $\mathbf{I}$ denotes a two dimensional unit matrix. $\mathbf{v}_{k,m}$ denotes the observations noise vector from three radars and is subject to zeros mean Gaussian white noise process with the standard deviation $\begin{bmatrix} \sigma_v^{\gamma} & 0 \\ 0 & \sigma_v^{\theta} \end{bmatrix}$. $\gamma_k = \sqrt{(x_k)^2 + (y_k)^2}$ and $\theta_k = \tan^{-1}(y_k/x_k)$ denote the radial and azimuth component of observation. Due to sensors adopted in observation system being with the same precision, σ_v^{γ} are 0.2km, 0.18km and 0.22km, and σ_v^{θ} are 0.15°, 0.12° and 0.18°, respectively. In this case, the number of Monte Carlo simulation is 50 and the number of particles is 2000, and the total simulation step is 25.

The comparisons of particle weight variance before the re-sampling and after the re-sampling are given in Fig. 5 and Fig. 6, respectively. The results clearly show that the stability and reliability of particle weight variance are improved by the utilization of the weight optimization strategy in PF-PWO and TPF-PWO. The comparison of RMSE from the X axis (horizontal direction) and the Y axis (vertical direction) are given in Fig. 7 and Fig. 8 under the same conditions, and the above results further verify the influence of particle weight improved for the filter precision. According to Fig. 5 and Fig. 6, we can know that the estimation precision of target state is also accordingly promoted along with the diminution of particle weight variance. The data in Table 2 quantitatively show the mean of RMSE and the average time for three algorithms.

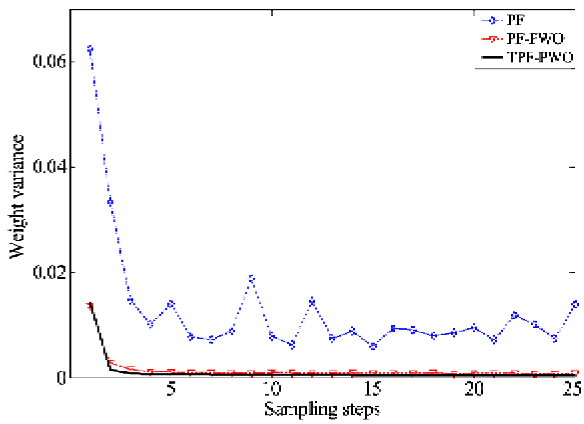

Fig. 5 Before the re-sampling

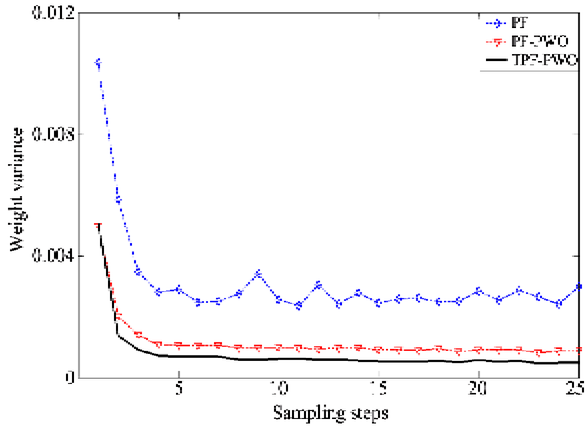

Fig. 6 After the re-sampling

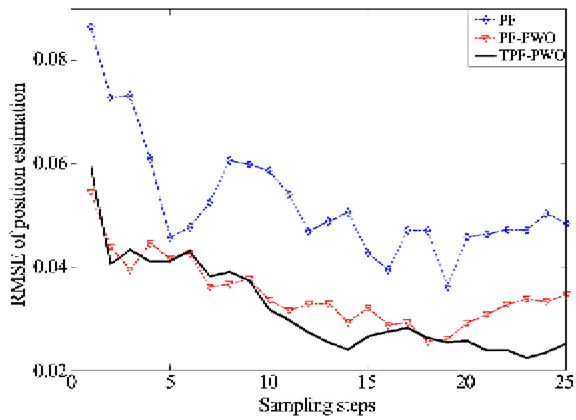

Fig. 7 Horizontal direction

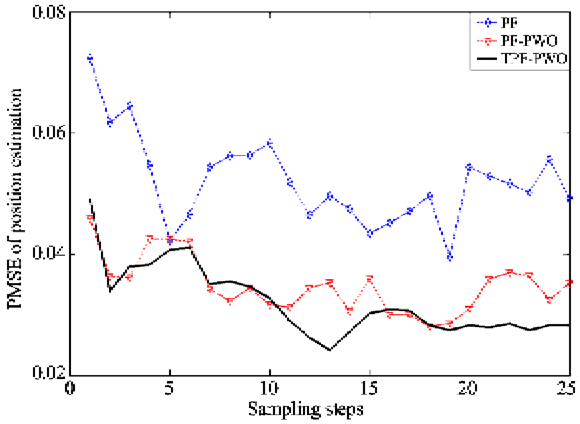

Fig. 8 Vertical direction

Table 2 The mean of RMSE for position component and the average time under 50 independent runs

Algorithm	PF	PF-PWO	TPF-PWO
Horizontal direction (km)	0.0527	0.0349	0.0290
Vertical direction (km)	0.0521	0.0348	0.0281
Time(s)	2.4169	3.1706	3.4425

4 Conclusions

The objective of this paper is to deal with reasonable measuring of particle weight and effective sampling of particle state in PF for multi-sensor fusion system. In the concrete construction process of algorithm, firstly, for the influence of random observations noise in the measuring process of particle weight, we construct the particle weight optimization strategy. Secondly, for the improvement of sampling particle reliability, we give a two stages prediction and update method. Moreover, they are dynamic adopted into the framework of PF. As expected, the proposed algorithm performs good compared with some other existing solutions. These have been confirmed by the simulations results. In addition, the algorithm retains the basic structure of PF, so it obtains a good scalability. For some specific nonlinear state estimation problem, some existing improved PF can be easily transplanted into the framework of algorithm proposed. In particular, the application object of algorithm proposed is multi-sensor information fusion system, and its application fields are widespread.

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