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A modified image matching algorithm based on robust Hausdorff distance①

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Abstract

Hausdorff distance measure is one of the widely adopted feature-based image matching algorithms due to its simplicity and accuracy. However, it is considered that its robustness still needs to be improved. In this paper, various forms of original and improved Hausdorff distance (HD) and their limitations are studied. Focusing on robust Hausdorff distance (RHD), an improved RHD with an adaptive outlier point threshold selection method is proposed. Furthermore, another new form of the Hausdorff distance which possesses the merits of RHD and M-HD is prsented. Finally, a recursive algorithm is introduced to accelerate the image matching speed of Hausdorff algorithms. Extensive simulation and experiment results are presented to validate the feasibility of the proposed Hausdorff distance algorithm.

Key words: Hausdorff distance (HD), robust Hausdorff distance (RHD), adaptive outlier point threshold, matching speed

0 Introduction

Image matching is a process to find the similar target image area within the reference image based on the known template image using a corresponding matching algorithm. Currently there are many image matching methods, such as gray correlation matching, feature-based matching, model-based matching, and matching based on transform domain. Hausdorff distance measure is a feature-based image matching algorithm, which does not need to establish the correspondence between points, just calculate the difference (Hausdorff distance) between two point sets. The position of the minimum Hausdorff distance is the best matching position. Meanwhile, the algorithm can handle images with multiple feature points, such as spurious feature points, missing feature points, etc. The advantages of using Hausdorff distance are small computational complexity and high real-time characteristic. Many modified Hausdorff distance measures have been proposed to improve the performances of efficiency or accuracy, and applied them to various applications[1-3]. The main problem is less robust when using the Hausdorff distance for image matching. This paper analyzes the limitations of various improved forms and then focuses on RHD which has better robust capabilities when compared with other forms on dealing with the outlier points, edge occlusions and spurious edge segments. However, the RHD method needs to determine an outlier point threshold. The paper presents a new selection method of outlier point threshold. When calculating with RHD, a recursive algorithm is given to accelerate the matching speed. Finally, the paper proposes a new form of the Hausdorff distance by combining RHD with M-HD, and validates the feasibility of the method through simulation experiments.

1 Hausdorff distance and its improved form

The HD measure computes the distance value between two sets of edge points extracted from template image A and reference image B. The HD measure between two point sets $A = (a_1, a_2, \cdots, a_n)$ and $B = (b_1, b_2, \cdots, b_m)$ is defined as

$$H(A,B) = \max[h(A,B), h(B,A)] \tag{1}$$

where $h(A,B)$ and $h(B,A)$ represent the directed distances between two sets A and B, of which the conventional definition is

$$h(A,B) = \max_{a \in A}\{\min_{b \in B} \| a - b \|\} \tag{2}$$

$$h(B,A) = \max_{b \in B}\{\min_{a \in A} \| b - a \|\} \tag{3}$$

The matching ability of the conventional HD is poor for the presence of noise, occlusion and outlier points. Huttenlocher[4] proposed the PHD (partial

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Hausdorff distance) measure in 1993, the directed distance of the partial HD using the kth ranked value instead of the maximum. Ref. [5] said that there were still some limitations of PHD, mainly in the following three aspects: When image B was matched with (1) spurious edge, (2) a strong presence of noise or outlier points, (3) phenomenon of occlusion, the results would not be the correct outputs.

Dubuisson and Jain proposed the MHD[6] measure (modified Hausdorff distance) based on the average distance value. But the robustness of this method is rather poor. Azencott proposed the CHD[7] measure (censored Hausdorff distance) on comparing binary images. However, the calculation of the method is much more complicated. Later Sim[8] proposed two different Hausdorff distance. One of which is called MHD (M-estimation Hausdorff distance), and the other is called LTS-HD (least trimmed squares Hausdorff distance). Both M-HD and LTS-HD can overcome the above limitations in (2) and (3), but still can not overcome (1). Ref. [2] proposed RHD (robust Hausdorff distance), considering factors such as the outlier points, edge occlusions and spurious edge segments. Assuming image A as the template image, which does not contain spurious edge, the directed distance of RHD is defined as

$$h_{RHD}(A,B) = \begin{cases} (\#(A)/\#(A'))^{p} \dfrac{1}{\#(A')} \sum\limits_{a \in A'} d_B(a) & \text{if } \#(A') \neq 0 \\ M & \text{else} \end{cases} \tag{4}$$

where M is a constant with big value. $A' = \{a \mid a \in A, d_B(a) \leqslant \beta\}$, and β is used to eliminate the outlier points threshold. $h_{RHD}(A,B)$ is set to M when A' is empty; Otherwise, $h_{RHD}(A,B)$ is determined by $(\#(A)/\#(A'))^{p}$ and $\frac{1}{\#(A')} \sum_{a \in A'} d_B(a)$. The first part considers the occlusions factor, and the second part is the average value of $d_B(a)$ in sets A'. The definition of $h_{RHD}(B,A)$ considers the problem of spurious edge, which is defined as

$$h_{RHD}(B,A) = \begin{cases} (\#(B)/\#(B'))^{p} \dfrac{1}{\#(B')} \sum\limits_{b \in B'} d_A(b) & \text{if } \#(B') \neq 0 \\ M & \text{else} \end{cases} \tag{5}$$

where $B' = \{b \mid b \in B, d_A(b) \leqslant \beta\}$, in sets B', the edge less than 3 (usually chosen as 3)[2] has been deleted by the edge tracking method.

The calculation of the directed distance is usually through the distance transform. Its purpose is to calculate the distance of each image pixel to its closest edge points. The steps of the algorithm:

(1) The distance value of edge point is zero, and the distance value of non-edge point is infinite.

(2) Forward recursion

Starting from the top left corner of the image, the recursive sequence is from left to right ,top to bottom

$$\begin{aligned} D(i,j) = \min(&D(i,j), D(i-1,j-1)+1, \\ &D(i-1,j)+1, D(i-1,j+1)+1, \\ &D(i,j-1)+1) \end{aligned} \tag{6}$$

(3) Backward recursion

Starting from the bottom right corner of the image, the recursive sequence is from right to left, bottom to top

$$\begin{aligned} D(i,j) = \min(&D(i,j), D(i+1,j-1)+1, \\ &D(i+1,j)+1, D(i+1,j+1)+1, \\ &D(i,j+1)+1) \end{aligned} \tag{7}$$

2 Improved RHD with adaptive outlier point threshold

In order to get the correct matching result, RHD needs to select the appropriate outlier point threshold. According to the calculation of distance transform, the distance transform image can be considered as a code table map, in which the code value of edge point is zero and the code value of non-edge point is the distance to its closest edge points. After the distance transforms on the template image and the reference image, two code table maps can be got and the directed distance $h_{RHD}(A,B)$ can be calculated. If the outlier points (multi-feature spots) exist in template map A, some of the larger distance in the reference image B will participate in the caculation of $h_{RHD}(A,B)$. Thus the value of $h_{RHD}(A,B)$ will be higher. If above area in reference image B is just the matching location, we will get higher $H(A,B)$ according to Eq. (1). But the actual matching location is the corresponding minimum $H(A,B)$. So if we do not overcome the impact from outlier points, we will get a mismatch result. The calculation of $h_{RHD}(B,A)$ is similar. This is the reason for reasonable setting the threshold. Meanwhile, the threshold should not be set too low. Much lower threshold will result in depressing the matching performance by ignoring too many edge points. So setting an appropriate threshold value plays a key role on the image with outlier points. The existence of the outlier points leads to smaller results of distance transform. We can get the mean of the overall distance transform image data. The selection of the outlier points can be based on the mean, as it is greater than or equal to the mean.

From the above analysis, the improved RHD with adaptive outlier point threshold proposed in this paper is defined as Eq. (4) and Eq. (5), while

$$A' = \{a \mid a \in A, d_B(a) \leqslant \beta 1\} \tag{8}$$

$$B' = \{b \mid b \in B, d_A(b) \leqslant \beta 2\} \tag{9}$$

$$\beta 1 = \frac{1}{M \times N}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} D_A(i, j) + \eta_1 \tag{10}$$

$$\beta 2 = \frac{1}{M \times N}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} D_B(i, j) + \eta_2 \tag{11}$$

where, the image is $M \times N$, $D_A(i, j)$, $D_B(i, j)$ is the distance transform data of image A and image B. η_1 and η_2 are thresholds and here they both take 2.

In order to verify the effectiveness of the method, five groups image data are taken for image matching, the results of RHD are shown in Table 1, and the results of the improved RHD are shown in Table 2.

Table 1 Matching position results by RHD

Image groups	Correct matching results	RHD	
		Parameters (ρ = 0.1 ~ 1)	Matching results
Image 1	(109,263)	β = 3	(110,271)
		β = 4	(109,264)
		β = 5	(115,264)
		β = 6 ~ 10	(109,263)
		β = 11 ~ 12	(109,264)
Image 2	(109,263)	β = 1 ~ 10	(109,263)
Image 3	(117,263)	β = 2	(174,272)
		β = 3	(120,271)
		β = 4	(122,263)
		β = 5 ~ 10	(117,263)
Image 4	(140,176)	β = 4	(150,174)
		β = 5	(141,174)
		β = 6 ~ 7	(140,176)
		β = 8	(94,86)
		β = 9	(94,85)
Image 5	(109,139)	β = 2	(172,162)
		β = 3	(174,167)
		β = 4	(171,182)
		β = 5 ~ 8	(109,139)
		β = 9 ~ 10	(108,139)

Table 2 Matching position results by improved RHD

Image groups	Correct matching results	Improved RHD	
		Parameters (ρ = 0.1 ~ 1)	Matching results
Image 1	(109,263)	$\beta 1$ = 6, $\beta 2$ = 6	(109,263)
Image 2	(109,263)	$\beta 1$ = 7, $\beta 2$ = 6	(109,263)
Image 3	(117,263)	$\beta 1$ = 6, $\beta 2$ = 6	(117,263)
Image 4	(140,176)	$\beta 1$ = 6, $\beta 2$ = 6	(140,176)
Image 5	(109,139)	$\beta 1$ = 6, $\beta 2$ = 5	(109,139)

Table 1 shows that the correct values of β is difficult to determine in RHD. Table 2 shows that the values of β_1 and β_2 calculated by Eqs(10) and (11) are determined in the improved RHD. Comparing Table 1 with Table 2, it is found that the matching position of Table 1 is correct when the values of β are around the values of Table 2. For group 2, the template is a part of the reference. So the values of β do not affect the matching position. For other groups, the template image and the reference image come from different sensors. When the values of β in Table 1 are larger or smaller than that in Table 2, we get the wrong matching results. This conclusion shows that the method of the outlier point selection is correct with more applicability.

3 An improved form of hausdorff distance—— RM-HD

In this paper, a new form of the Hausdorff distance RM-HD is proposed, which has the merits of RHD and M-HD. It is defined as

$$h_{RM\text{-}HD}(A,B) = \begin{cases} (\#(A)/\#(A'))^{\rho}\dfrac{1}{\#(A')}\sum_{a\in A'}\varphi_A(d_B(a)) & \text{if } \#(A') \neq 0 \\ M & \text{else} \end{cases} \tag{12}$$

$$\varphi_A(d_B(a)) = \begin{cases} |d_B(a)| & |d_A(a)| \leqslant \tau_A \\ \tau_A & |d_A(a)| > \tau_A \end{cases} \tag{13}$$

$$\tau_A = \frac{1}{M \times N}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} D_A(i, j) \tag{14}$$

$$h_{RM\text{-}HD}(B,A) = \begin{cases} (\#(B)/\#(B'))^{\rho}\dfrac{1}{\#(B')}\sum_{b\in B'}\varphi_B(d_A(b)) & \text{if } \#(B') \neq 0 \\ M & \text{else} \end{cases} \tag{15}$$

$$\varphi_B(d_A(b)) = \begin{cases} |d_A(b)| & |d_A(b)| \leqslant \tau_B \\ \tau_B & |d_A(b)| > \tau_B \end{cases} \tag{16}$$

$$\tau_B = \frac{1}{M \times N}\sum_{i=0}^{M-1}\sum_{j=0}^{N-1} D_B(i, j) \tag{17}$$

In the calculation of $h_{RM\text{-}HD}(A,B)$, τ_A is the mean of the distance transform data of image A and $D_A(i, j)$ is the distance transform data of image A. Similarly, in calculating $h_{RM\text{-}HD}(B,A)$, τ_B is the mean of the distance transform data of image B and $D_B(i, j)$ is the distance transform data of image B.

In order to verify the effectiveness of the method, five group image data are adopted to do image matching simulation experiments. The results shown in Table 3 are correct.

Table 3 Matching position results by RM-HD

Image groups	RM-HD Parameters (ρ = 0.1 ~ 1) τ_A, τ_B	RM-HD Matching results	Correct matching results
Image 1	4 , 4	(109,263)	(109,263)
Image 2	5 , 4	(109,263)	(109,263)
Image 3	4 , 4	(117,263)	(117,263)
Image 4	4 , 4	(140,176)	(140,176)
Image 5	4 , 3	(109,139)	(109,139)

4 Hausdorff recursive algorithm

The following work discusses the matching speed of HD. In the calculation process, double accounting is done on the number of edge points in the reference image under the coverage area of the template image. The recursive method could be used to reduce the amount of computation. The whole implementation process of the recursive method is shown in Fig. 1 which shows the entire region of the reference image.

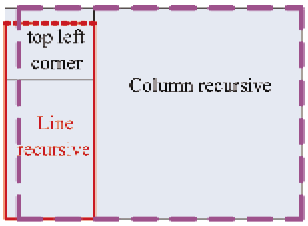

Fig. 1 Recursive process

If the size of the reference image under the coverage area of the template image is $m \times n$, $N_{x,y}$ is numbers of edge points of the reference image.

The Hausdorff recursive method is defined as:

(1) At first getting the numbers of edge points of the top left corner, and then calculating the numbers of edge points in other regions by the row recursive or the column recursive method.

(2) Fig. 2 shows the column recursive model, which gets the number of edge points in current region by calculating numbers of the column moved out and in.

$$\sum_{i=1}^{m}\sum_{j=1}^{n} N_{x,y+1}(i,j) = \sum_{i=1}^{m}\sum_{j=1}^{n} N_{x,y}(i,j) + \sum_{i=x}^{x+m-1}[N(i,y+n) - N(i,y)] \tag{18}$$

where, $N_{x,y+1}$ is the number of edge points of the new reference image in which the original reference image moves to the right column.

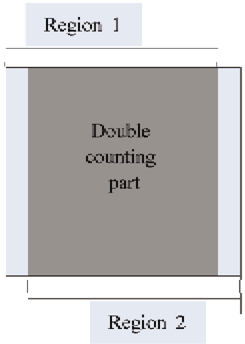

Fig. 2 Column recursive

(3) Fig. 3 shows the row recursive model, which gets the number of edge points in current region by calculating numbers of the rows moved out and in.

$$\sum_{i=1}^{m}\sum_{j=1}^{n} N_{x+1,y}(i,j) = \sum_{i=1}^{m}\sum_{j=1}^{n} N_{x,y}(i,j) + \sum_{j=y}^{y+n-1}[N(x+m,j) - N(x,j)] \tag{19}$$

where, $N_{x+1,y}$ is the number of edge points of the new reference image where the original reference image moves down one row.

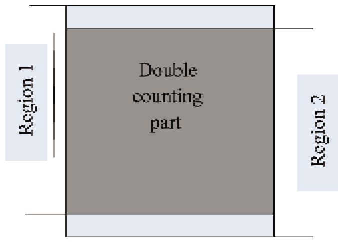

Fig. 3 Row recursive

The matching time of five groups image data is tested through simulation experiments using VC++ 6.0, in the CPU frequency of Pentium 3.0GHZ, 2GB of RAM on a computer. The size of the infrared reference image is 320 × 240, and the size of the infrared template image is 80 × 80. Table 4 gives the average matching time of the five groups image data.

Table 4 Time used of RHD & Recursive RHD

Algorithm	RHD	Recursive RHD	RM-HD	Recursive RM-HD
Average Time Used (second)	2.938 seconds	1.875 seconds	3.025 seconds	1.932 seconds

Table 3 shows that the matching speed has been improved by using the recursive method in Hausdorff.

5 Conclusions

The paper starts from the definition of Hausdorff distance, and focuses on the research of RHD. Based on RHD, the paper proposes an improved RHD with adaptive outlier point threshold selection method. Then, a new Hausdorff distance called RM-HD which possesses the merits of RHD and M-HD is presented. Finally, we give a recursive method of accelerating the image matching speed of Hausdorff distance calculation. Through the simulation experiment, the feasibility of the new algorithms is verified successfully.

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